Learning from Data: Decision Trees

Amos Storkey, School of Informatics
University of Edinburgh

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Decision Tree Learning - Overview

- Decision tree representation
- ID3 learning algorithm
- Entropy, Information gain
- Priors for Decision Tree Learning
- Overfitting and how to avoid it
- Reading: Mitchell, chapter 3

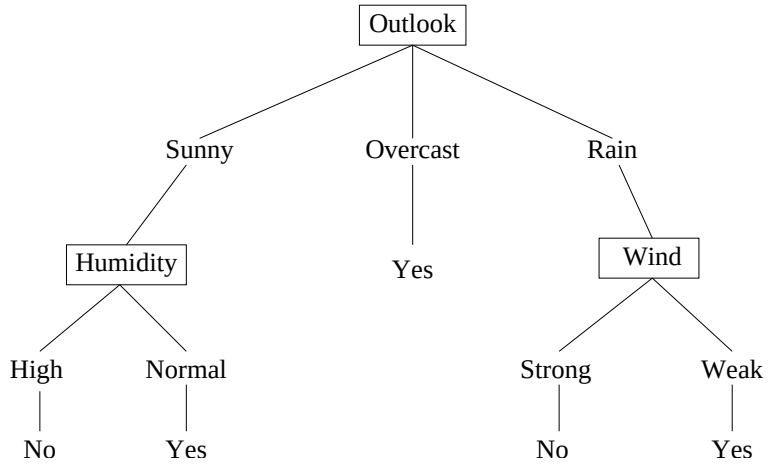
Acknowledgement: These slides are based on slides modified by Chris Williams and produced by Tom Mitchell, available from `http://www.cs.cmu.edu/~tom/`

Decision trees

- Decision tree learning is a method for approximating discrete-valued[1] target functions, in which the learned function is represented as a decision tree
- Decision tree representation:
  - Each internal node tests an attribute
  - Each branch corresponds to attribute value
  - Each leaf node assigns a classification
- Re-representation as if-then rules: disjunction of conjunctions of constraints on the attribute value instances

[1] The method can be extended to learning continuous-valued functions

Decision Tree for *Play Tennis*



Logical Formulation: $(Outlook = Sunny \wedge Humidity = Normal)$

$$\vee\ (Outlook = Overcast)$$

$$\vee\ (Outlook = Rain \wedge Wind = Weak)$$

When to Consider Decision Trees

- Instances describable by attribute–value pairs
- Target function is discrete valued
- Disjunctive hypothesis may be required
- Possibly noisy training data

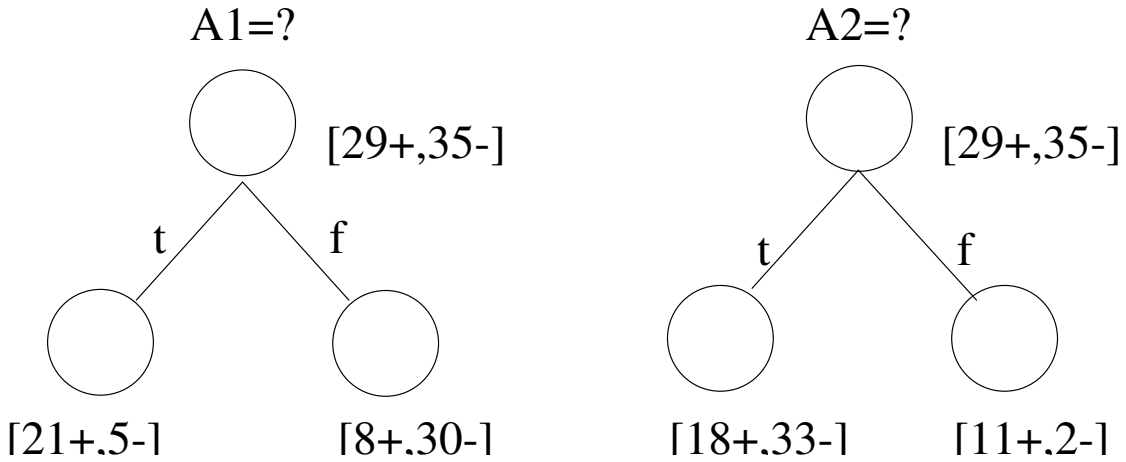
Examples:

- Equipment or medical diagnosis
- Credit risk analysis
- Modelling calendar scheduling preferences

Top-Down Induction of Decision Trees (ID3)

1. A is the “best” decision attribute for next $node$
2. Assign A as decision attribute for $node$
3. For each value of A, create new descendant of $node$
4. Sort training examples to leaf nodes
5. If training examples perfectly classified, Then STOP, Else iterate over new leaf nodes

Which attribute is best?



Entropy

- S is a sample of training examples
- $p_\oplus$ is the proportion of positive examples in S
- $p_\ominus$ is the proportion of negative examples in S
- Entropy measures the impurity of S

$$Entropy(S) \equiv H(S) \equiv -p_\oplus \log_2 p_\oplus - p_\ominus \log_2 p_\ominus$$

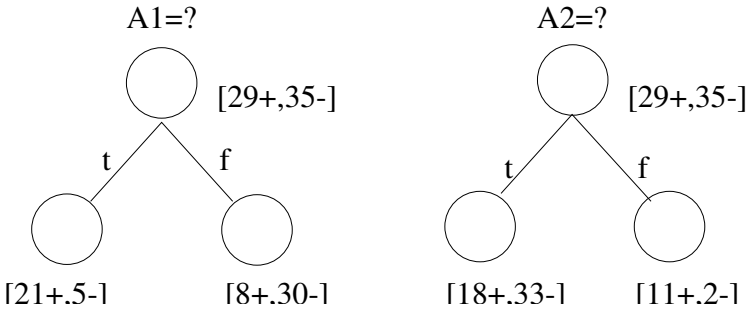
- $H(S) = 0$ if sample is pure (all + or all -), $H(S) = 1$ bit if $p_\oplus = p_\ominus = 0.5$

Information Gain

- $Gain(S, A)$ = expected reduction in entropy due to sorting on A

$$Gain(S, A) \equiv Entropy(S) \; - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

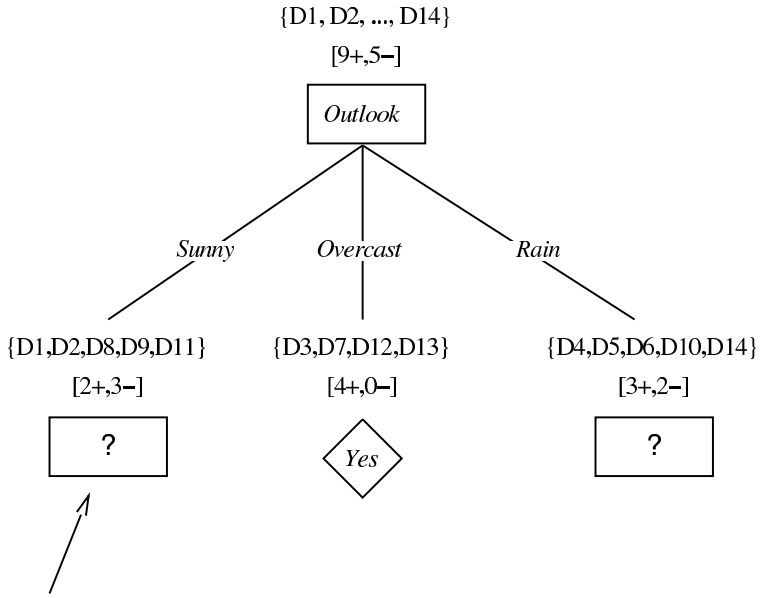
- Information gain is also called the *mutual information* between A and the labels of S



Training Examples

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Building the Decision Tree



Which attribute should be tested here?

S_{sunny} = {D1,D2,D8,D9,D11}

Gain (S_{sunny}, *Humidity*) = .970 − (3/5) 0.0 − (2/5) 0.0 = .970

Gain (S_{sunny}, *Temperature*) = .970 − (2/5) 0.0 − (2/5) 1.0 − (1/5) 0.0 = .570

Gain (S_{sunny}, *Wind*) = .970 − (2/5) 1.0 − (3/5) .918 = .019

Hypothesis Space Search by ID3

- Hypothesis space is complete!
  - Target function surely in there...
- Outputs a single hypothesis (which one?)
- No back tracking
  - Local minima...
- “Batch” rather than “on-line” learning
  - More robust to noisy data...
- Implicit prior: approx “prefer shortest tree”

Implicit priors in ID3

- Searching space from simple to complex, starting with empty tree, guided by information gain heuristic
- Preference for short trees, and for those with high information gain attributes near the root
- Bias is a *preference* for some hypotheses, rather than a *restriction* of hypothesis space
- Occam's razor: prefer the shortest hypothesis that fits the data

Occam's Razor

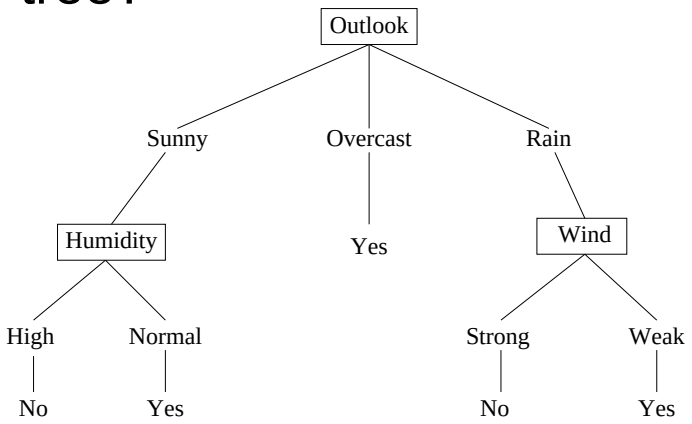
- Why prefer short hypotheses?
- Argument in favour:
  - Fewer short hypotheses than long hypotheses
  - a short hypothesis that fits data unlikely to be coincidence
  - a long hypothesis that fits data might be coincidence
- Argument opposed:
  - There are many ways to define small sets of hypotheses (notion of coding $length(X) = -\log_2 P(X)$, Minimum Description Length ...)
  - e.g., all trees with a prime number of nodes that use attributes beginning with "Z"
  - What's so special about small sets based on *size* of hypothesis??

Overfitting in Decision Trees

- Consider adding noisy training example #15:

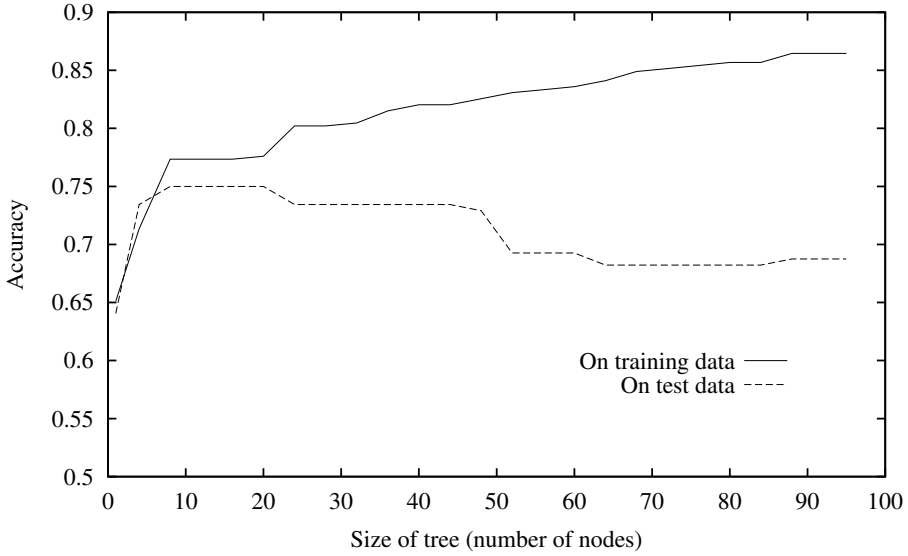
$$Sunny,\ Hot,\ Normal,\ Strong,\ PlayTennis = No$$

- What effect on earlier tree?



Overfitting in Decision Tree Learning

- Overfitting can occur with noisy training examples, and also when small numbers of examples are associated with leaf nodes ($\rightarrow$ coincidental or accidental regularities)



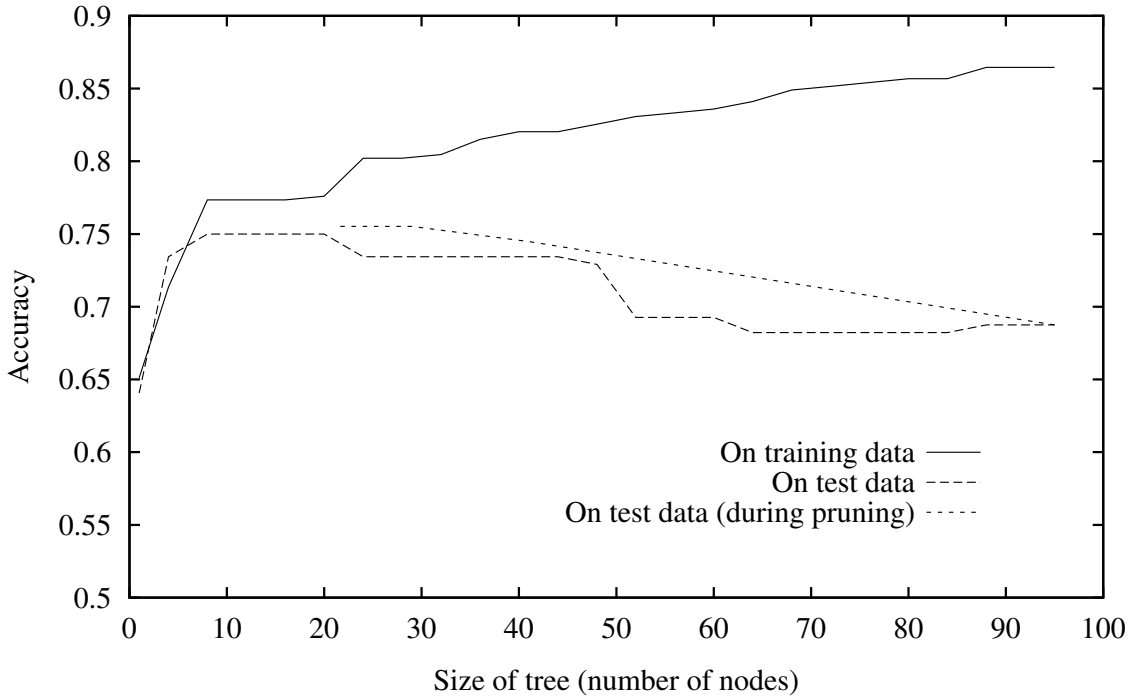
Avoiding Overfitting

- How can we avoid overfitting?
  - stop growing when data split not statistically significant
  - grow full tree, then post-prune
- How to select "best" tree:
  - Measure performance over training data
  - Measure performance over separate validation data set
  - MDL: minimize $size(tree) + size(misclassifications(tree))$

Reduced-Error Pruning

- Split data into $training$ and $validation$ set
- Do until further pruning is harmful:
  1. Evaluate impact on $validation$ set of pruning each possible node (plus those below it)
  2. Greedily remove the one that most improves $validation$ set accuracy
- produces smallest version of most accurate subtree
- What if data is limited?

Effect of Reduced-Error Pruning



Rule Post-Pruning

1. Convert tree to equivalent set of rules
2. Prune each rule independently of others, by removing any preconditions that result in improving its estimated accuracy
3. Sort final rules into desired sequence for use

- Perhaps most frequently used method (e.g., C4.5)

Alternative Measures for Selecting Attributes

- Problem: if an attribute has many values, $Gain$ will select it
- Example: use of dates in database entries
- One approach: use $GainRatio$ instead

$$GainRatio(S, A) \equiv \frac{Gain(S, A)}{SplitInformation(S, A)}$$

$$SplitInformation(S, A) \equiv -\sum_{i=1}^{c} \frac{|S_i|}{|S|} \log_2 \frac{|S_i|}{|S|}$$

where S_i is subset of S for which A has value v_i

Further points

- Dealing with continuous-valued attributes—create a split, e.g. $(Temperature > 72.3) = t, f$. Split point can be optimized (Mitchell, §3.7.2)
- Handling training examples with missing data (Mitchell, §3.7.4)
- Handling attributes with different costs (Mitchell, §3.7.5)

Summary

- Decision tree learning provides a practical method for concept learning/learning discrete-valued functions
- ID3 algorithm grows decision trees from the root downwards, greedily selecting the next best attribute for each new decision branch
- ID3 searches a complete hypothesis space, using a preference bias for smaller trees with higher information gain close to the root
- The overfitting problem can be tackled using post-pruning