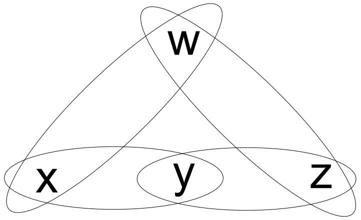
Semantic Optimization of Conjunctive Queries

Semantic Acyclicity

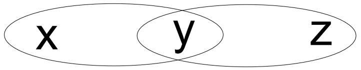
Definition: A CQ Q is semantically acyclic if there exists an acyclic CQ Q' such that $Q \equiv Q'$

Q(x,z) :- R(x,y), R(y,z), R(x,w), R(w,z)



$\{w \rightarrow y, z \rightarrow y\}$

Q(x,z) :- R(x,y), R(y,z)



Semantic Acyclicity

But, semantic acyclicity is rather *weak*:

- Not many CQs are semantically acyclic

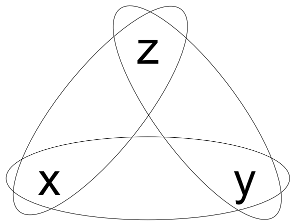
  $\Rightarrow$ consider acyclic approximations of CQs ✓

- Semantic acyclicity is not an improvement over usual optimization – both approaches are based on the core

  $\Rightarrow$ exploit semantic information in the form of constraints

Constraints Enrich Semantic Acyclicity

Q :- R(x,y), R(y,z), R(z,x)



- Assume that Q will be evaluated over databases that comply with the following set of inclusion dependencies

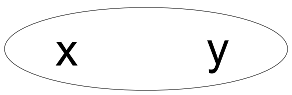
$$R[1,2] \subseteq P[1,2] \quad \equiv \quad \forall x \forall y\ (R(x,y) \rightarrow \exists z\ P(x,y,z))$$

$$P[2,3] \subseteq R[1,2] \quad \equiv \quad \forall x \forall y \forall z\ (P(x,y,z) \rightarrow R(y,z))$$

$$P[3,1] \subseteq R[1,2] \quad \equiv \quad \forall x \forall y \forall z\ (P(x,y,z) \rightarrow R(z,x))$$

- Then Q can be replaced by

Q' :- R(x,y)



Constraints Enrich Semantic Acyclicity

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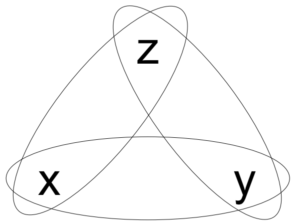

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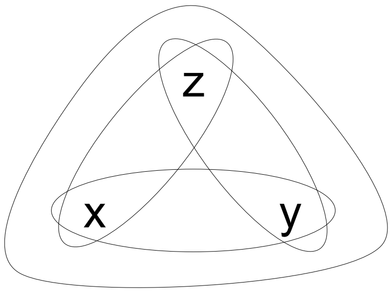
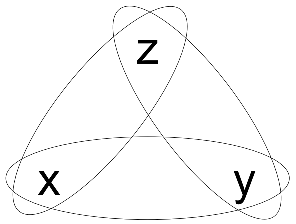
- Moreover, Q can be replaced by

Q' :- R(x,y), R(y,z), R(z,x), P(x,y,z)



Constraints Enrich Semantic Acyclicity

Q :- R(x,y), R(y,z), R(z,x), R(x,z)

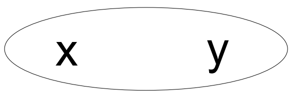


- Assume that Q will be evaluated over databases that comply with the following functional dependency

$$R : \{1\} \rightarrow \{2\} \quad \equiv \quad \forall x \forall y \forall z \, (R(x,y) \wedge R(x,z) \rightarrow y = z))$$

- Then Q can be replaced by

Q' :- R(x,y), R(y,y), R(y,x)



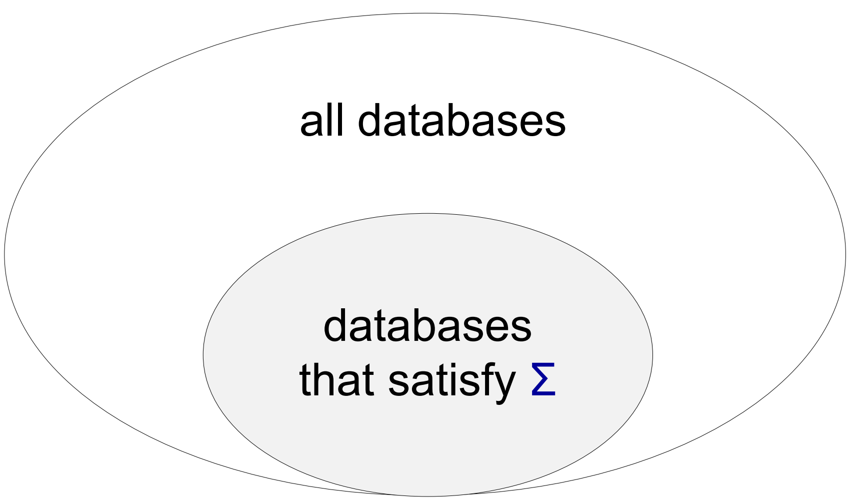
Semantic Acyclicity Under Constraints

(henceforth, by set of constraints we mean a set of inclusion or functional dependencies)

Definition: Given a CQ Q and a set of constraints Σ, we say that Q is semantically acyclic under Σ if there exists an acyclic CQ Q' such that $Q \equiv_{\Sigma} Q'$

for every database D that satisfies Σ, $Q(D) = Q'(D)$

(analogously, we define the notation $Q \subseteq_{\Sigma} Q'$)



in the above definition, we only care for the databases in the shaded part

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Two crucial questions: given a CQ Q and a set Σ of constraints

1. Can we decide whether Q is semantically acyclic under Σ, and what is the exact complexity?
2. Does this help query evaluation?

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Two crucial questions: given a CQ Q and a set Σ of constraints

1. Can we decide whether Q is semantically acyclic under Σ, and what is the exact complexity? *First, we need to understand CQ containment under constraints*
2. Does this help query evaluation?

CQ Containment Revisited

$Q \subseteq Q'$ $\Leftrightarrow$ there exists a query homomorphism from Q' to Q

$\Downarrow$ $\not\Uparrow$

$Q \subseteq_\Sigma Q'$

Q :- R(x,y), R(y,z), R(z,x)

Q' :- R(x,y), R(y,z), R(z,x), P(x,y,z)

$$\Sigma = \left\{ \begin{array}{l} R[1,2] \subseteq P[1,2] \\ P[2,3] \subseteq R[1,2] \\ P[3,1] \subseteq R[1,2] \end{array} \right\}$$

$Q \subseteq_\Sigma Q'$ but there is no query homomorphism from Q' to Q

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$Q \subseteq_{\Sigma} Q'$

Q :- R(x,y), R(y,z), R(z,x)

Q' :- R(x,y), R(y,y), R(y,x)

$\Sigma = \{ R : \{1\} \rightarrow \{2\} \}$

$Q \subseteq_{\Sigma} Q'$ but there is no query homomorphism from Q' to Q

CQ Containment Revisited

We need a result of the form:

Theorem: Let Q and Q' be conjunctive queries, and Σ a set of constraints. It holds that: $Q \subseteq_{\Sigma} Q' \Leftrightarrow$ there exists a query homomorphism from Q' to Q_{Σ}

a CQ that acts as a representative for all the specializations of Q that comply with Σ

Q_{Σ} can be constructed by applying a well-known algorithm – the chase

The Chase by Example

(inclusion dependencies)

$Q(x)$:- $R(x,y)$

$$\Sigma = \left\{ \begin{array}{c} R[2] \subseteq P[1] \\ P[1,2] \subseteq P[2,1] \end{array} \right\}$$

The Chase by Example

(inclusion dependencies)

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$Q(x)$:- $R(x,y)$

$Q(x)$:- $R(x,y), P(y,z)$

The Chase by Example

(inclusion dependencies)

$Q(x) \ :- \ R(x,y)$

$$\Sigma = \left\{ \begin{array}{c} R[2] \subseteq P[1] \\ P[1,2] \subseteq P[2,1] \end{array} \right\}$$

$Q(x) \ :- \ R(x,y)$

$Q(x) \ :- \ R(x,y), P(y,z)$

$Q(x) \ :- \ R(x,y), P(y,z), P(z,y)$ ✓

The Chase by Example

(inclusion dependencies)

$Q(x) \text{ :- } R(x,y)$

$$\Sigma = \left\{ R[2] \subseteq R[1] \right\}$$

The Chase by Example

(inclusion dependencies)

$Q(x)$:- $R(x,y)$

$\Sigma = \{ R[2] \subseteq R[1] \}$

$Q(x)$:- $R(x,y)$

$Q(x)$:- $R(x,y), R(y,z)$

$Q(x)$:- $R(x,y), R(y,z), R(z,w)$

⋮

we need to build an infinite CQ

The Chase by Example

(functional dependencies)

$Q(x,y)$:- $R(x,y), R(y,z), R(x,z)$

$$\Sigma = \left\{ R : \{1\} \rightarrow \{2\} \right\}$$

The Chase by Example

(functional dependencies)

$Q(x,y)$:- $R(x,y), R(y,z), R(x,z)$ $\Sigma = \left\{ \mathbf{R : \{1\} \rightarrow \{2\}} \right\}$

$Q(x,y)$:- $R(x,y), R(y,z), R(x,z)$

The Chase by Example

(functional dependencies)

$Q(x,y)$:- $R(x,y), R(y,z), R(x,z)$

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$Q(x,y)$:- $R(x,y), R(y,z), R(x,z)$

$Q(x,y)$:- $R(x,y), R(y,y)$

✓

The Chase by Example

(functional dependencies)

$Q(x,y)$:- R(x,a), R(y,z), R(x,b)

(a,b are constants)

$$\Sigma = \left\{ R : \{1\} \rightarrow \{2\} \right\}$$

The Chase by Example

(functional dependencies)

$Q(x,y)$:- $R(x,a), R(y,z), R(x,b)$

(a,b are constants)

$\Sigma = \{ \mathbf{R : \{1\} \rightarrow \{2\}} \}$

$Q(x,y)$:- $R(x,a), R(y,z), R(x,b)$

$Q(x,y)$:-

the chase fails – constants cannot be unified

the empty query is returned

CQ Containment Under Functional Dependencies

Theorem: Let Q and Q' be conjunctive queries, and Σ a set of *functional dependencies*.

It holds that: $Q \subseteq_{\Sigma} Q' \Leftrightarrow$ there exists a query homomorphism from Q' to chase(Q,Σ)

the result of the chase algorithm starting from Q and applying the constraints of Σ

Proof hint: adapt the proof for the homomorphism theorem by exploiting the following:

- The canonical database of chase(Q,Σ) is a finite database that satisfies Σ
- Main property of the chase: there exists a homomorphism that maps the body of chase(Q,Σ) to every D that (i) can be mapped to the body of Q, and (ii) satisfies Σ

CQ Containment Under Inclusion Dependencies

- Things are much more difficult for inclusion dependencies. By following the same approach as for functional dependencies we only show the following:

Theorem: Let Q and Q' be conjunctive queries, and Σ a set of *inclusion dependencies*. It holds that: $Q \subseteq_{\Sigma,\infty} Q' \Leftrightarrow$ there exists a query homomorphism from Q' to chase(Q,Σ)

for every, possibly infinite, database D that satisfies Σ, $Q(D) \subseteq Q'(D)$

- Interestingly, the following highly non-trivial and deep theorem holds:

Theorem (Finite Controllability): $Q \subseteq_{\Sigma} Q' \Leftrightarrow Q \subseteq_{\Sigma,\infty} Q'$

CQ Containment Under Constraints

Theorem: Let Q and Q' be conjunctive queries, and Σ a set of constraints. The problem of deciding whether $Q \subseteq_{\Sigma} Q'$ is

- NP-complete, if Σ is a set of functional dependencies
- PSPACE-complete, if Σ is a set of inclusion dependencies

Proof Idea:

(NP-membership) (i) Construct chase(Q,Σ) in polynomial time, (ii) guess a substitution h, and (iii) verify that h is a query homomorphism from Q' to chase(Q,Σ)

(NP-hardness) Inherited form the constraint-free case

(PSPACE-membership) (i) Non-deterministically construct a subquery Q'' of chase(Q,Σ) with $|Q''| \leq |Q'|$, (ii) guess a substitution h, and (iii) verify that h is a query hom. from Q' to Q''

(PSPACE-hardness) Simulate a PSPACE Turing machine

Back to Semantic Acyclicity Under Constraints

Definition: Given a CQ Q and a set of constraints Σ, we say that Q is semantically acyclic under Σ if there exists an acyclic CQ Q' such that $Q \equiv_{\Sigma} Q'$

$Q \subseteq_{\Sigma} Q'$ and $Q' \subseteq_{\Sigma} Q$

Two crucial questions: given a CQ Q and a set Σ of constraints

1. Can we decide whether Q is semantically acyclic under Σ, and what is the exact complexity? *Now, we have the tools to study this problem*
2. Does this help query evaluation?

Semantic Acyclicity Under Inclusion Dependencies

Proposition (Small Query Property): Consider a CQ Q and a set Σ of inclusion dependencies. If Q is semantically acyclic under Σ, then there exists an acyclic CQ Q' such that $|Q'| \leq 2 \cdot |Q|$ and $Q \equiv_{\Sigma} Q'$

Guess-and-check algorithm:

1. Guess an acyclic CQ Q' of size at most $2 \cdot |Q|$
2. Verify that $Q \subseteq_{\Sigma} Q'$ and $Q' \subseteq_{\Sigma} Q$

Theorem: Deciding semantic acyclicity under inclusion dependencies is:

- PSPACE-complete in general
- NP-complete for fixed arity (because containment is NP-complete)

Semantic Acyclicity Under Functional Dependencies

Proposition (Small Query Property): Consider a CQ Q and a set Σ of functional dependencies over unary and binary relations. If Q is semantically acyclic under Σ, then there exists an acyclic CQ Q' such that $|Q'| \leq 2 \cdot |Q|$ and $Q \equiv_{\Sigma} Q'$

Guess-and-check algorithm:

1. Guess an acyclic CQ Q' of size at most $2 \cdot |Q|$
2. Verify that $Q \subseteq_{\Sigma} Q'$ and $Q' \subseteq_{\Sigma} Q$

Theorem: Deciding semantic acyclicity under inclusion dependencies is NP-complete

Semantic Acyclicity Under Functional Dependencies

$R : \{1\} \rightarrow \{3\} \quad \equiv \quad R(x,y,z,w), R(x,y',z',w') \rightarrow z = z'$

only one attribute

Theorem: Semantic acyclicity under unary functional dependencies (over unconstrained signatures) is NP-complete

Open Problem: Deciding semantic acyclicity under arbitrary (or even binary) functional dependencies is a non-trivial open problem

Evaluating Semantically Acyclic CQs

- Recall that evaluating Q over D takes time $|D|^{O(|Q|)}$

- Evaluating a CQ Q that is semantically acyclic under Σ over D takes time

$$2^{O(|Q| + |\Sigma|)} + O(|D| \cdot |Q|)$$

time for computing an acyclic CQ Q' such that $|Q'| \leq 2 \cdot |Q|$ and $Q \equiv_{\Sigma} Q'$

time for evaluating Q'

- $|Q'| \leq 2 \cdot |Q|$
- Evaluation of an acyclic CQ Q_A is feasible in time $O(|D| \cdot |Q_A|)$

- Observe that $2^{O(|Q| + |\Sigma|)} + O(|D| \cdot |Q|)$ is dominated by $O(|D| \cdot 2^{O(|Q| + |\Sigma|)})$

  $\Rightarrow$ fixed-parameter tractable

Acyclic Approximations Under Constraints

- There are CQs that are not semantically acyclic even in the presence of constraints
- The small query properties lead to acyclic approximations

Theorem: Consider a CQ Q and a set Σ of constraints. There exists an acyclic CQ Q' of size at most $2 \cdot |Q|$ that is maximally contained in Q under Σ

$Q' \subseteq_{\Sigma} Q$ and there is no acyclic CQ Q'' such that $Q'' \subseteq_{\Sigma} Q$ and $Q' \subset_{\Sigma} Q''$

- We know that acyclic approximations of polynomial size always exist
- However, by exploiting the constraints we obtain more informative approximations

Semantic Optimization: Recap

- Constraints enrich semantic acyclicity
- We can decide semantic acyclicity in the presence of inclusion dependencies and functional dependencies over unary and binary relations
  - The underlying tool is CQ containment under constraints
- Semantic acyclicity under functional dependencies is an important open problem
- Semantically acyclic CQs can be evaluated “efficiently” (fixed-parameter tractability)
- For CQs that are not semantically acyclic, even in the presence of constraints, we can always compute (more informative) acyclic approximations

Semantic Acyclicity: Wrap-Up

- Semantic acyclicity is an interesting notion that allows us to replace a CQ with an acyclic one – this significantly improves query evaluation

- But, semantic acyclicity is rather *weak*:

  – Not many CQs are semantically acyclic
  $\Rightarrow$ consider acyclic approximations of CQs

  – Semantic acyclicity is not an improvement over usual optimization – both approaches are based on the core
  $\Rightarrow$ exploit semantic information in the form of constraints

Associated Papers

- Pablo Barceló, Georg Gottlob, Andreas Pieris: Semantic Acyclicity Under Constraints. PODS 2016: 343-354

  Sementic acyclcitiy under several classes of constraints

- Diego Figueira: Semantically Acyclic Conjunctive Queries under Functional Dependencies. LICS 2016: 847-856

  Semantic acyclicity under unary functional dependencies

- David S. Johnson, Anthony C. Klug: Testing Containment of Conjunctive Queries under Functional and Inclusion Dependencies. J. Comput. Syst. Sci. 28(1): 167-189 (1984)

  Containment of CQ under inclusion dependencies via the chase

- David Maier, Alberto O. Mendelzon, Yehoshua Sagiv: Testing Implications of Data Dependencies. ACM Trans. Database Syst. 4(4): 455-469 (1979)

  The paper that introduced the chase algorithm