Planned Comparisons and Post Hoc Tests

**Planned**: You define in advance a set of independent linear comparisons between the levels of a factor. This may reveal an internal difference even if there was no overall significance.

**Post-hoc**: After obtaining a significant effect for a factor, you carry out comparisons between specific levels to see which ones differ – you might consider all possible comparisons.

### Orthogonal contrasts

- Partition the sum of squares for a factor A with k levels into a set of k-1 orthogonal contrasts, each with two levels (df=1) formed by grouping the levels in A.
- The two groups contrasted are assigned, respectively, positive and negative coefficients, with any level not included in the contrast assigned 0.
- Coefficients of the contrast must add to 0, so use coefficients of 1/n where n is number in that group (or multiply through).
- Orthogonality: product of coefficients of any pair of contrasts sum to 0.

For example, for a one-way ANOVA with 4 groups (A B C D), you could first compare AB with CD, then compare AC with BD, then AD with BC. The contrast coefficients would be:

<table>
<thead>
<tr>
<th>Contrast</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB - CD</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>
A-B  1   -1   1   -1  
C-D  1   -1  -1   1  

But perhaps it is more relevant to compare A against BCD (perhaps A is the control) and then look for difference of B from CD and C from D

<table>
<thead>
<tr>
<th>Contrast</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-BCD</td>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>B-CD</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>C-D</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

There is a third possible set of orthogonal contrasts on the four levels, compare AB with CD, then A with B, and C with D. The contrast coefficients would be:

<table>
<thead>
<tr>
<th>Contrast</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB-CD</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>A-B</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C-D</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Which set of contrasts you choose would be determined by the logic of your experiment. Can then calculate for each contrast:

\[ \frac{\sum_{i=1}^{k} \text{coeff}_i \times \bar{x}_i}{\sqrt{M_{S_R} \sum_{i=1}^{k} \frac{\text{coeff}_i^2}{n_i}}} \]

Which is distributed as \( t[df_R] \).

**Post Hoc Tests**

- Used when you discover an unforeseen effect in ANOVA (or had no prior expectation about what differences might be seen) – comparisons not planned in advance.
- Multiple \( t \)-tests with Bonferroni correction – adjust for familywise error rate
  - Divide \( \alpha \) by number of comparisons made:
    - E.g. if making 5 comparisons, then \( p = \alpha/5 = .05/5 = .01 \)
      - Increase chance of Type II error – being too conservative / reducing \( \alpha \), and the power of the experiment to find where the actual differences lie.
- Modified Bonferonni, Dunn - Sidak's correction:
  - \( \alpha_{\text{comparison}} = 1 - (1 - \alpha_{\text{overall}})^{1/c} \)
    - Still too conservative if comparisons not independent (e.g. if doing all possible pairwise comparisons.
- All use some variation of \( t \)-test
  - The top of the formula uses the difference between cell means
  - The denominator uses \( M_{S_R} \) from the ANOVA table (i.e. estimates variance from all data, not just the means being compared).
- Other post hoc tests - some more conservative than others:
- Neuman-Keuls (S-N-K) – liberal on Type I and most likely to get a significant result
- Scheffé – strict on Type I – bad for Type II – less likely to show sig result
- Dunnett’s – for comparing all treatments to a single control
- Games-Howell – doesn’t assume equal variances
- Tukey HSD(Honestly Significant Difference) – preferred test; greatest power and readily available in many stats packages.
  - Power advantage of the Tukey test depends on the assumption that all possible pairwise comparisons are being made, which is usually the case for Post Hoc tests.