Deciding equivalence using type checking

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Methods for verifying finite and infinite state systems

- Notable success in Computer Science
- model checking + equivalence checking
- System = finite/infinite state transition graph

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- Model checking: does state s have property Φ ?
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- Equivalence checking: is state s equivalent to t ?
- mostly computing dyadic fixed points e.g. bisimulations to solve it. May need algebraic/combinatorial properties of reachability sets/generators of graph

Transfer these techniques to systems with binding

1. Model checking higher-order trees/schemes
   Using parity automata and geometry of interaction or game semantics or Krivine machines; alternative type checking
   [Knapik, Niwinski, Urzyczyn 2002; Caucal 2002; Ong 2006; Hague, Murawski, Ong, Serre 2008, Kobayashi 2009; Kobayshi, Ong 2009; Salvati, Walukiewicz 2011; Kobayashi 2011 . . .]

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2. Deciding observational equivalence for fragments of idealized Algol and ML (w.r.t. finite value sets)
   Reduce to equivalence of automata (such as deterministic pushdown automata)

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3. $\vdots \quad \vdots \quad \vdots$

Example scheme: second-order

$$
\begin{array}{lcl}
F x_1 x_2 x_3 & \stackrel{\mathrm{def}}{=} & f\,(F(G x_1)(H x_2) x_3)\, x_1 (x_2 x_3) \\
G y_1 y_2 & \stackrel{\mathrm{def}}{=} & g(y_1(y_2)) \\
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F g h a & & \text{Start}
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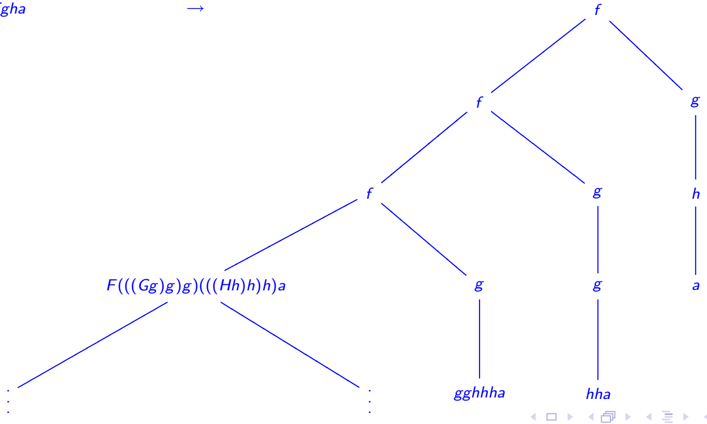
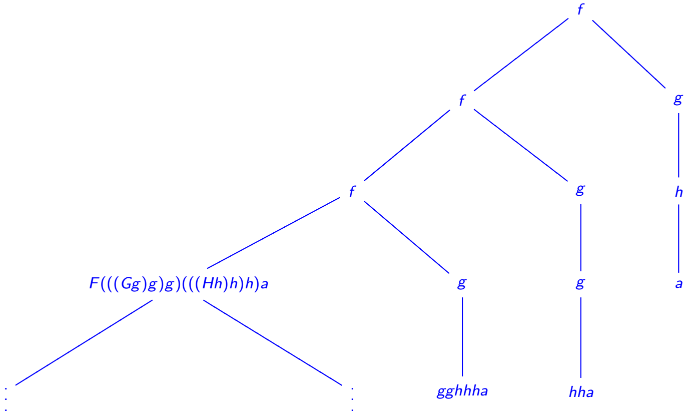
Fgha & \quad \textcolor{red}{\text{Start}}
\end{aligned}
$$

$Fgha \quad \longrightarrow$



Model checking a scheme



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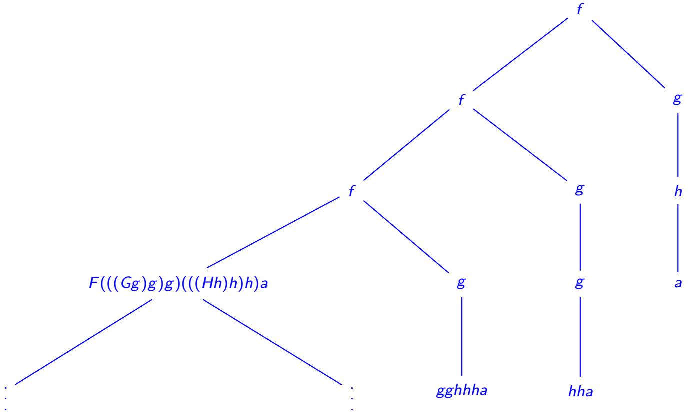

Want to run a finite state automaton on tree generated by scheme

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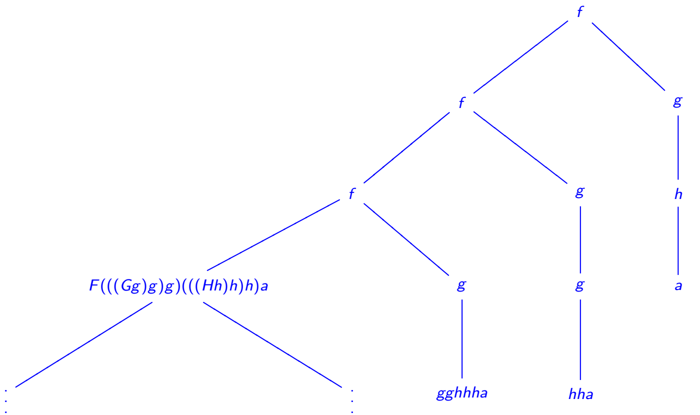

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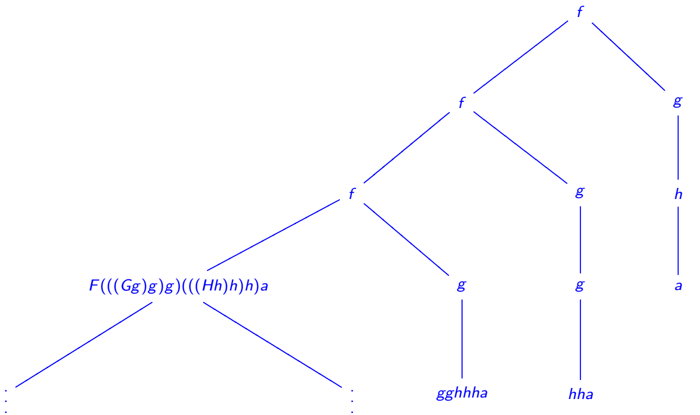

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Want to run a finite state automaton on tree generated by scheme

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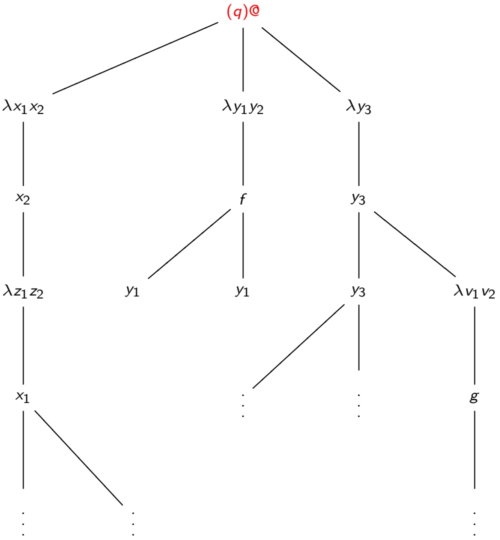
Leads to non-standard automata

Automaton running on lambda term applied to two terms



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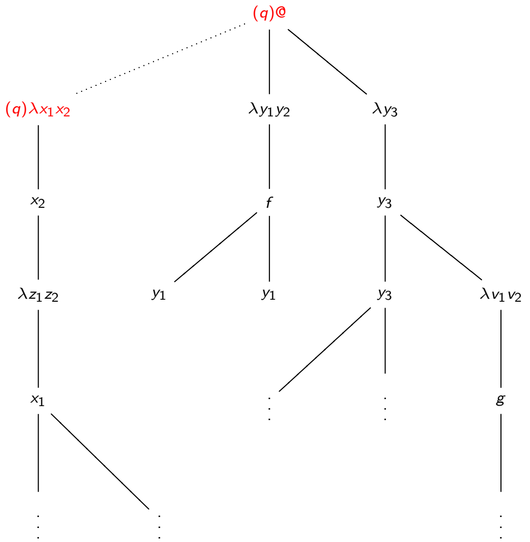

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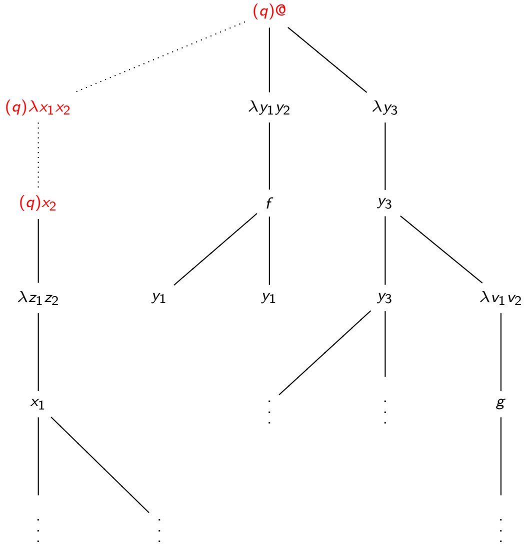

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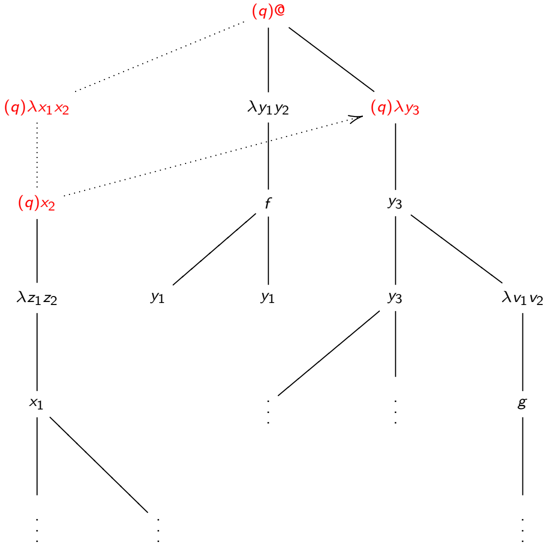

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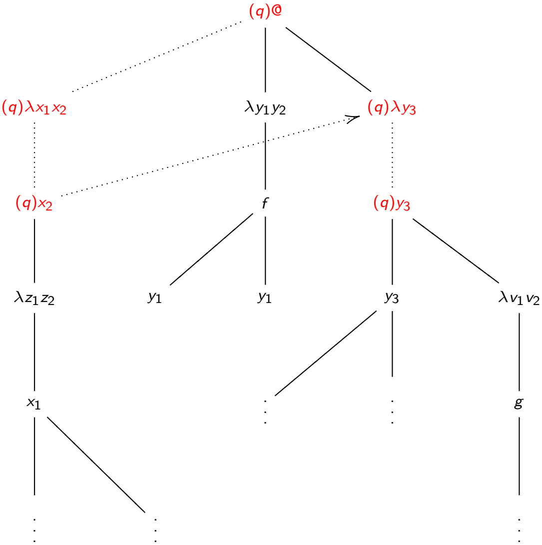

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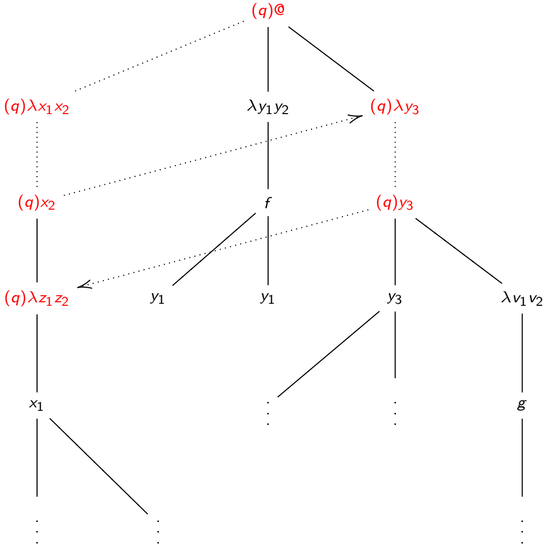

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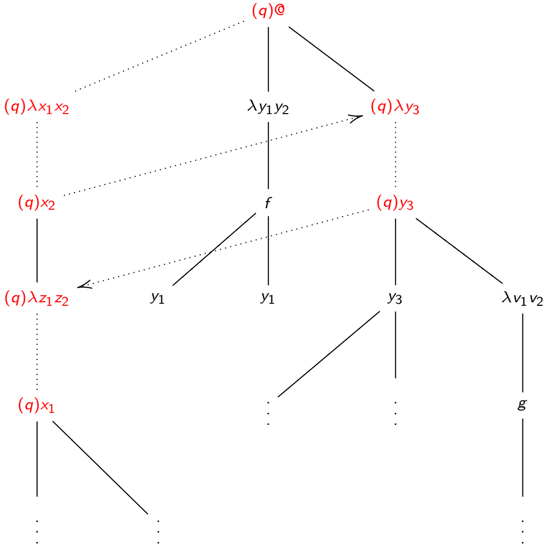

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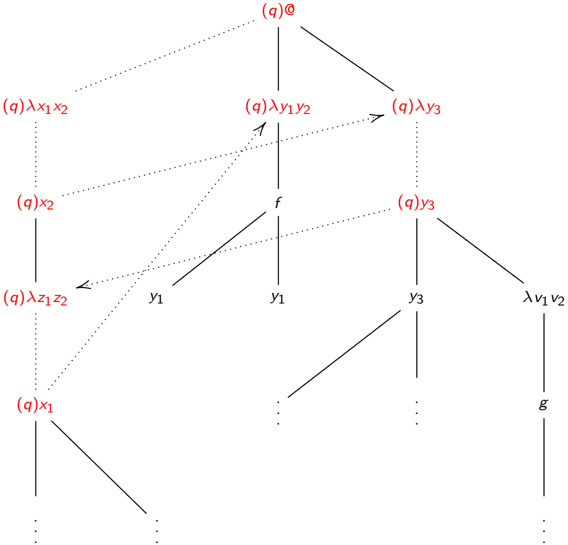

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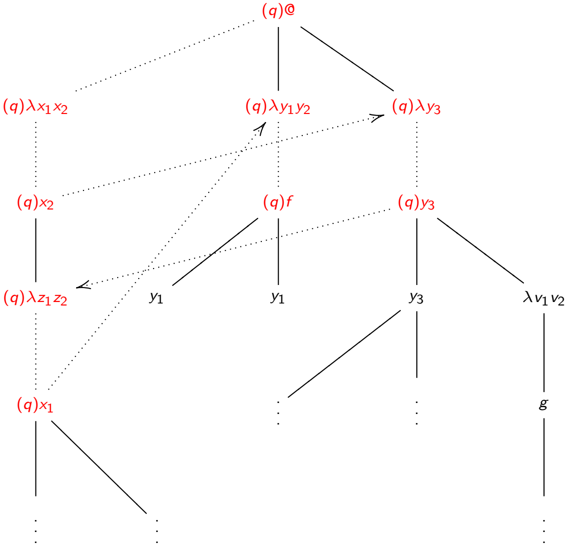

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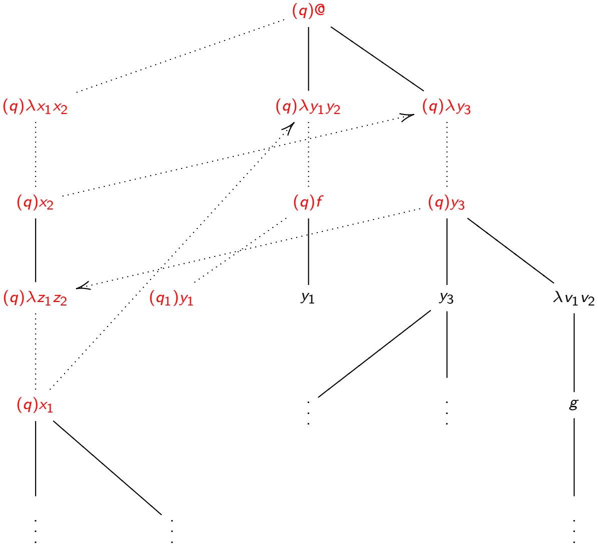

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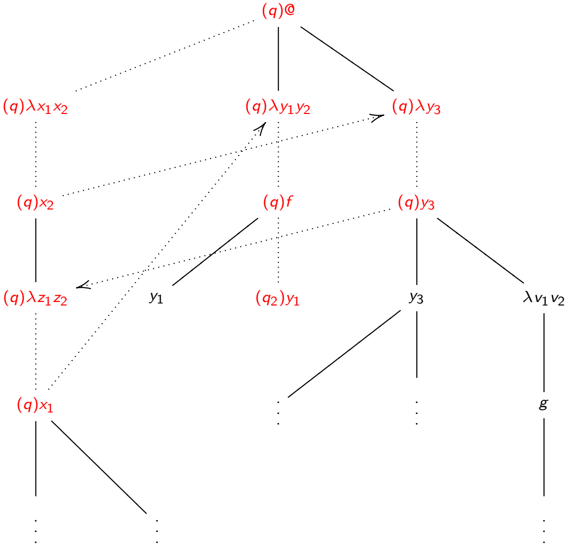

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- Encode running of automaton using intersection types

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\theta & := & q \mid \tau \to \theta \\
\tau & := & \bigwedge_{i_1 \in I_1} \theta_{i_1} \wedge \ldots \wedge \bigwedge_{i_m \in I_m} \theta_{i_m}
\end{array}
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q state of automaton

I_j finite

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- Decision procedure: via the (finite) typing rules
- Can this technique work for equivalence checking ?
- Use type checking to solve equivalence problem (for real-time strict deterministic pushdown automata)

Inspired by [Tsukada, Kobayshi 2012] which looks at special language inclusion problems

Real-time strict deterministic pushdown automata

Finite sets: Q states, Γ stack symbols, A alphabet and T basic transitions

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$pX \xrightarrow{a} q\alpha$ where $p, q \in Q$, $a \in A$, $X \in \Gamma$ and $\alpha \in \Gamma^*$

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Transitions of configuration: if $pX \xrightarrow{a} q\alpha \in T$ then $pX\beta \xrightarrow{a} q\alpha\beta$

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Language accepted $L(p\alpha) = \{w \mid p\alpha \xrightarrow{w} q\varepsilon \text{ for some } q\}$

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Equivalence: given $p\alpha$ and $q\beta$ is $L(p\alpha) = L(q\beta)$?

Example

$Q = \{p, p_1, p_2, p_3\}$, $\Gamma = \{X, Y\}$ and $A = \{a, b, c\}$. T is

$$
\begin{array}{lll}
pX \xrightarrow{a} p_1X & pX \xrightarrow{b} p_2\epsilon & p_2X \xrightarrow{c} p_3X \\
p_1X \xrightarrow{a} pXX & p_1X \xrightarrow{b} p_3X & p_3X \xrightarrow{c} p_2\varepsilon \\
pY \xrightarrow{a} pYY & pY \xrightarrow{b} p_1\epsilon & p_1Y \xrightarrow{c} p_1\varepsilon
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\end{array}
$$

$p_1YYY \xrightarrow{c} p_1YY$ because $p_1Y \xrightarrow{c} p_1\varepsilon \in T$

Example continued

$$
\begin{array}{ccccccccc}
pX & \xrightarrow{a} & p_1X & \xrightarrow{a} & pXX & \xrightarrow{a} & p_1XX & \xrightarrow{a} & \dots \\
\downarrow b & & \downarrow b & & \downarrow b & & \downarrow b & & \dots \\
p_2\varepsilon & \xleftarrow{c} & p_3X & \xleftarrow{c} & p_2X & \xleftarrow{c} & p_3XX & \xleftarrow{c} & \dots
\end{array}
$$

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pY & \xrightarrow{a} & pYY & \xrightarrow{a} & pYYY & \xrightarrow{a} & pYYYY & \xrightarrow{a} & \dots \\
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p_1\varepsilon & \xleftarrow{c} & p_1Y & \xleftarrow{c} & p_1YY & \xleftarrow{c} & p_1YYY & \xleftarrow{c} & \dots
\end{array}
$$

$$\text{For } n > 0, \quad L(pX^n) = L(pY^{2n-1})$$

Where is application?

- Assume states $\{p_1, \ldots, p_k\}$

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- Assume states $\{p_1, \ldots, p_k\}$
- Configuration $p\gamma\alpha$ is $p\gamma$ applied to $p_1\alpha, \ldots, p_k\alpha$
- Types $\tau ::= (\theta_1, \ldots, \theta_k) \rightarrow q\beta$
- $q\beta$ is a configuration and θ_i finite set of configurations

Meaning of a type

$p\alpha : (\theta_1, \ldots, \theta_k) \to q\beta$ iff

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$p\alpha : (\theta_1, \ldots, \theta_k) \to q\beta$ iff

1. $p\alpha \trianglelefteq q\beta$ (consonance)
2. if $p\alpha \xrightarrow{w} p_i\varepsilon$ and $q\beta \xrightarrow{w} q'\gamma$ then $q'\gamma \in \theta_i$

where $p\alpha \trianglelefteq q\beta$ iff

- if $w \in L(p\alpha)$, then there is a v, $wv \in L(q\beta)$
- if $w \in L(q\beta)$, then there is a prefix of w, $w' \in L(p\alpha)$

Meaning of a type

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Key property: there is a m, if $p\alpha \trianglelefteq q\beta$ then there is a prefix β' of β, $p\alpha \trianglelefteq q\beta'$ and $|\beta'| \leq m|\alpha|$

Proof system

Type assumptions $qX : \tau \in \Delta$

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Axiom

$\Delta \vdash pX : (\theta_1, \ldots, \theta_k) \to q\beta\delta$ if $pX : (\theta'_1, \ldots, \theta'_k) \to q\beta \in \Delta$ and $\theta_i = \{r\lambda\delta \mid r\lambda \in \theta'_i\}$

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Application rule

$$\frac{\Delta \vdash pX : (\theta'_1, \ldots, \theta'_k) \rightarrow q\beta \ \ldots, \Delta \vdash p_j Y\alpha : (\theta^{ji}_1, \ldots, \theta^{ji}_k) \rightarrow r_{ji}\lambda_{ji}}{\Delta \vdash pXY\alpha : (\theta_1, \ldots, \theta_k) \rightarrow q\beta}$$

for all j and $r_{ji}\lambda_{ji} \in \theta'_j$ and $\theta_m = \bigcup_j \bigcup_i \theta^{ji}_m$

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Δ needs to be closed under transitions

Δ closed under transitions

If $pX : (\theta_1, \ldots, \theta_k) \rightarrow q\beta \in \Delta$ then

Δ closed under transitions

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1. if $pX \xrightarrow{a} p'\alpha$ then $q\beta \xrightarrow{a} q'\beta'$ and vice versa

Δ closed under transitions

If $pX : (\theta_1, \ldots, \theta_k) \to q\beta \in \Delta$ then

1. if $pX \xrightarrow{a} p'\alpha$ then $q\beta \xrightarrow{a} q'\beta'$ and vice versa
2. if $pX \xrightarrow{a} p_i\varepsilon$ and $q\beta \xrightarrow{a} q'\beta'$ then $q'\beta' \in \theta_i$

Δ closed under transitions

If $pX : (\theta_1, \ldots, \theta_k) \rightarrow q\beta \in \Delta$ then

1. if $pX \xrightarrow{a} p'\alpha$ then $q\beta \xrightarrow{a} q'\beta'$ and vice versa
2. if $pX \xrightarrow{a} p_i\varepsilon$ and $q\beta \xrightarrow{a} q'\beta'$ then $q'\beta' \in \theta_i$
3. if $pX \xrightarrow{a} rZ\alpha$ and $q\beta \xrightarrow{a} q'\beta'$ then
   $\Delta \vdash rZ\alpha : (\theta_1', \ldots, \theta_k) \rightarrow q'\beta'$ for $\theta_i' \subseteq \theta_i$

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   $\Delta \vdash rZ\alpha : (\theta'_1, \ldots, \theta_k) \rightarrow q'\beta'$ for $\theta'_i \subseteq \theta_i$
4. θ_i is union of cases 2 and 3

Equivalence checking as type checking

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- $\Delta \vdash p\alpha : (\theta^0_1, \ldots, \theta^0_k) \rightarrow q\beta$? where each $\theta^0_i \subseteq \{p_1\varepsilon, \ldots, p_k\varepsilon\}$

Example

$$
\begin{array}{ccccccccc}
pX & \xrightarrow{a} & p_1X & \xrightarrow{a} & pXX & \xrightarrow{a} & p_1XX & \xrightarrow{a} & \ldots \\
\downarrow b & & \downarrow b & & \downarrow b & & \downarrow b & & \ldots \\
p_2\varepsilon & \xleftarrow{c} & p_3X & \xleftarrow{c} & p_2X & \xleftarrow{c} & p_3XX & \xleftarrow{c} & \ldots
\end{array}
$$

$$
\begin{array}{ccccccccc}
pY & \xrightarrow{a} & pYY & \xrightarrow{a} & pYYY & \xrightarrow{a} & pYYYY & \xrightarrow{a} & \ldots \\
\downarrow b & & \downarrow b & & \downarrow b & & \downarrow b & & \ldots \\
p_1\varepsilon & \xleftarrow{c} & p_1Y & \xleftarrow{c} & p_1YY & \xleftarrow{c} & p_1YYY & \xleftarrow{c} & \ldots
\end{array}
$$

Assume states are ordered p, p_1, p_2, p_3 and $\pi = (\emptyset, \emptyset, \{p_1\varepsilon\}, \emptyset)$

For any $n > 0$, $\quad \Delta \vdash pX^n : \pi \rightarrow pY^{2n-1}$

where $\Delta = \{pX : \pi \rightarrow pY, p_1X : \pi \rightarrow pYY, p_2X : \pi \rightarrow p_1YY, p_3X : \pi \rightarrow p_1Y\}$

Proof tree upside down

$$\cfrac{\Delta \vdash pX^4 : \pi \longrightarrow pY^7}{\Delta \vdash pX : (\emptyset, \emptyset, \{p_1 Y^6\}, \emptyset) \longrightarrow pY^7 \qquad \cfrac{\Delta \vdash p_2 X^3 : \pi \longrightarrow p_1 Y^6}{\Delta \vdash p_2 X : (\emptyset, \emptyset, \{p_1 Y^4\}, \emptyset) \longrightarrow p_1 Y^6 \qquad \ldots}}$$

where $\ldots$ is the subtree

$$\cfrac{\Delta \vdash p_2 X^2 : \pi \longrightarrow p_1 Y^4}{\Delta \vdash p_2 X : (\emptyset, \{p_1 Y^2\}, \emptyset, \emptyset) \longrightarrow p_1 Y^4 \qquad \Delta \vdash p_2 X : \pi \longrightarrow p_1 Y^2}$$

where $\Delta = \{pX : \pi \longrightarrow pY, p_1 X : \pi \longrightarrow pYY, p_2 X : \pi \longrightarrow p_1 YY, p_3 X : \pi \longrightarrow p_1 Y\}$ and $\pi = (\emptyset, \emptyset, \{p_1 \varepsilon\}, \emptyset)$

Δ closed under transitions

$$
\begin{array}{lcr}
pX & : \pi \longrightarrow & pY \\
\downarrow a & & \downarrow a \\
p_1X & : \pi \longrightarrow & pY^2
\end{array}
\qquad
\begin{array}{lcr}
pX & : \pi \longrightarrow & pY \\
\downarrow b & & \downarrow b \\
p_2\varepsilon & & p_1\varepsilon
\end{array}
\qquad
\begin{array}{lcr}
p_1X & : \pi \longrightarrow & pY^2 \\
\downarrow a & & \downarrow a \\
pX^2 & : \pi \longrightarrow & pY^3
\end{array}
$$

$$
\begin{array}{lcr}
p_1X & : \pi \longrightarrow & pY^2 \\
\downarrow b & & \downarrow b \\
p_3X & : \pi \longrightarrow & p_1Y
\end{array}
\qquad
\begin{array}{lcr}
p_2X & : \pi \longrightarrow & p_1Y^2 \\
\downarrow c & & \downarrow c \\
p_3X & : \pi \longrightarrow & p_1Y
\end{array}
\qquad
\begin{array}{lcr}
p_3X & : \pi \longrightarrow & p_1Y \\
\downarrow c & & \downarrow c \\
p_2\varepsilon & & p_1\varepsilon
\end{array}
$$

$$
\frac{\Delta \vdash pX : (\emptyset, \emptyset, \{p_1Y^2\}, \emptyset) \longrightarrow pY^3 \qquad \Delta \vdash p_2X : \pi \longrightarrow p_1Y^2}{\Delta \vdash pX^2 : \pi \longrightarrow pY^3}
$$

where $\Delta = \{pX : \pi \longrightarrow pY, p_1X : \pi \longrightarrow pYY, p_2X : \pi \longrightarrow p_1YY, p_3X : \pi \longrightarrow p_1Y\}$ and $\pi = (\emptyset, \emptyset, \{p_1\varepsilon\}, \emptyset)$

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- Does the technique naturally extend to schema?