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# Dynamic Conditional Random Fields for Jointly Labeling Multiple Sequences

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## Abstract

Conditional random fields (CRFs) for sequence modeling have several advantages over joint models such as HMMs, including the ability to relax strong independence assumptions made in those models, and the ability to incorporate arbitrary overlapping features. Previous work has focused on linear-chain CRFs, which correspond to finite-state machines, and have efficient exact inference algorithms. Often, however, we wish to label sequence data in multiple interacting ways—for example, performing part-of-speech tagging and noun phrase segmentation simultaneously, increasing joint accuracy by sharing information between them. We present *dynamic conditional random fields (DCRFs)*, which are CRFs in which each time slice has a set of state variables and edges—a distributed state representation as in dynamic Bayesian networks—and parameters are tied across slices. (They could also be called conditionally-trained *Dynamic Markov Networks*.) Since exact inference can be intractable in these models, we perform approximate inference using the tree-based reparameterization framework (TRP). We also present empirical results comparing DCRFs with linear-chain CRFs on natural-language data.

## 1 Introduction

The problem of labeling and segmenting sequences of observations arises in many different areas, including bioinformatics, music modeling, computational linguistics, speech recognition, and information extraction. Probabilistic finite state automata, such as hidden Markov models (HMMs), have been popular for such sequence labeling tasks. Finite-state Conditional Random Fields (CRFs) [4] are another sequence model that offers several advantages over HMMs, relaxing the strong dependence assumptions made in those models and allowing rich sets of overlapping features.

Many sequence-processing problems are traditionally solved by chaining errorful subtasks. The traditional language understanding task, for example, is often broken into parsing, semantic understanding, and contextual and discourse analysis. In information extraction, one often performs part-of-speech tagging and then shallow parsing as pre-processing steps before the main extraction task. In such an approach, however, errors early in processing

nearly always cascade through the chain, causing errors in the final output.

In this paper, we address this problem by representing the multiple label sequences in a single graphical model, explicitly modeling limited dependencies between them. We introduce *Dynamic CRFs*, which are CRFs that repeat structure and parameters over a sequence. For example, the factorial structure in Figure 1(b) models dependencies between cotemporal labels, allowing information to flow between the subtasks in both directions.

DCRFs are named after *Dynamic Bayesian Networks (DBNs)* [2], directed sequence models for which there is a large body of literature addressing representation, learning, and inference (see [7]). Particular classes of DBNs, such as factorial HMMs, have also been extensively studied [11, 8, 3]. Previous work with CRFs has used the linear-chain structure, depicted in Figure 1, in which a first-order Markov assumption is made among labels. DCRFs combine the modeling advantages of the distributed hidden state in DBNs with the rich feature sets allowed in conditional models.

First, we briefly describe the general framework of CRFs. Then, we describe DCRFs, including how to do approximate inference and parameter estimation. Finally, we compare DCRFs to combinations of linear-chain CRFs on a task that involves both part-of-speech tagging and noun-phrase segmentation.

## 2 CRFs

*Conditional Random Fields (CRFs)* [4] are undirected graphical models that encode a conditional probability distribution using a given set of features. CRFs are defined as follows. Let  $\mathcal{G}$  be an undirected model over sets of random variables  $\mathbf{y}$  and  $\mathbf{x}$ . As a typical special case,  $\mathbf{y} = \{y_t\}$  and  $\mathbf{x} = \{x_t\}$  for  $t = 1, \dots, T$ , so that  $\mathbf{y}$  is a labeling of an observed sequence  $\mathbf{x}$ .<sup>1</sup> If  $C = \{\{y_c, x_c\}\}$  is the set of cliques in  $\mathcal{G}$ , then CRFs define the conditional probability of a state sequence given the observed sequence as:

$$p_{\theta}(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \prod_{c \in C} \Phi(y_c, \mathbf{x}_c), \quad (1)$$

where  $\Phi$  is a potential function and  $Z(\mathbf{x}) = \sum_{\mathbf{y}} \prod_{c \in C} \Phi(y_c, \mathbf{x}_c)$  is normalization factor over all state sequences of length  $T$ . We assume the potentials factorize according to a set of features  $\{f_k\}$ , which are given and fixed, so that

$$\Phi(y_c, \mathbf{x}_c) = \exp \left( \sum_{t=1}^T \sum_k \lambda_k f_k(y_c, \mathbf{x}_c, t) \right) \quad (2)$$

The model parameters are a set of real weights  $\theta = \{\lambda_k\}$ , one weight for each feature.

Previous applications have used the *linear-chain CRF*, in which a first-order Markov assumption is made on the hidden variables. A graphical model for this is shown in Figure 1. In this case, the cliques of the conditional model are the nodes and edges, so that there are feature functions  $f_k(y_{t-1}, y_t, \mathbf{x}, t)$  for each label transition and  $g_k(y_t, \mathbf{x}, t)$  for each label. Feature functions can be arbitrary. For example, a feature function  $f_k(y_{t-1}, y_t, \mathbf{x}, t)$  on a pair of variables  $(y_{t-1}, y_t)$  could be a binary test that has value 1 if and only if  $y_{t-1}$  has the label “*adjective*”,  $y_t$  has the label “*proper noun*”, and  $x_t$  begins with a capital letter.

Linear-chain CRFs correspond to finite state machines, and can be roughly understood as conditionally-trained hidden Markov models (HMMs). This class of CRFs is also a

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<sup>1</sup>Note that in general, the set of labels may be different from the set of states of the FSM, in that multiple states can correspond to the same label. In practice, however, it is usually assumed that the set of states and labels are the same, or given the sequence of the labels, the set of states are unambiguously known.

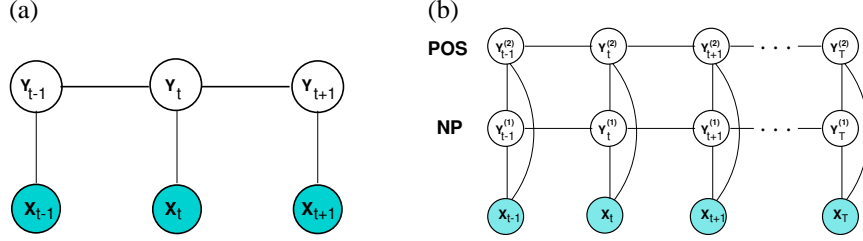


Figure 1: Graphical representation of (a) linear-chain CRF, and (b) factorial CRF for part-of-speech tagging and noun-phrase segmentation. Although the hidden nodes can depend on observations at any time step, for clarity we have shown links only to observations at the same time step.

globally-normalized extension to *Maximum Entropy Markov Models* [6] that avoids the label bias problem [4].

### 3 Dynamic CRFs

#### 3.1 Model Representation

A Dynamic CRF is a conditionally-trained undirected graphical model whose structure and parameters are repeated over a sequence. A DCRF is specified by a *2-CRF* that has a set of feature functions  $\{f_k\}$  and corresponding weights  $\{\lambda_k\}$  on each clique. A 2-CRF is a template of the graphical structure for two time steps, analogous to the 2-slice Temporal Bayes Network that specifies a DBN. Similarly to a DBN, a 2-CRF is unrolled into a full undirected model given an instance  $\mathbf{x}$ . A feature function  $f_k$  for a clique  $\mathbf{y}_c$  is an arbitrary function  $f(\mathbf{y}_c, \mathbf{x}, t)$ , that is,  $f_k$  can depend on any observations but only on labels from  $\mathbf{y}_c$ . The same set of features and weights is used at each time slice, so that the parameters are tied across the network. Then the conditional probability of a label sequence  $\mathbf{y}$  is given by:

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \exp \left( \sum_t \sum_k \lambda_k f_k(\mathbf{y}_{(k,t)}, \mathbf{x}, t) \right). \quad (3)$$

DCRFs generalize not only linear-chain CRFs, but more complicated structures as well. For example, in this paper, we use a *factorial DCRF*, which has linear chains of labels, with connections between cotemporal labels. Figure 1(b) is an example of an unrolled factorial DCRF. Of course more complicated structures, such as semi-Markov CRFs and hierarchical CRFs, are also possible.

#### 3.2 Inference in DCRFs

Inference in an unrolled DCRF can be done using any inference algorithm for undirected models. Because exact inference can be expensive in complex DCRFs, we use approximate methods. Here we describe approximate inference using tree-reparameterization (TRP) [10]. TRP is based on the fact that any exact algorithm for optimal inference on trees actually computes marginal distributions for pairs of neighboring nodes. For an undirected graphical model over variables  $\mathbf{x}$ , this results in an alternative parameterization of the distribution (Figure 2(a)) as:

$$p(\mathbf{x}) = \frac{1}{Z} \prod_{s \in V} \psi_s(x_s) \prod_{(s,t) \in E} \psi_{st}(x_s, x_t) \Rightarrow p(\mathbf{x}) = \prod_{s \in V} P_s(x_s) \prod_{(s,t) \in E} \frac{P_{st}(x_s, x_t)}{P_s(x_s)P_t(x_t)} \quad (4)$$

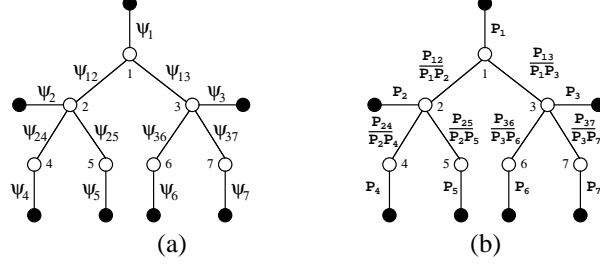


Figure 2: (a) A simple tree-structured graphical model and its original parameterization; (b) Alternative parameterization in terms of marginal distributions.

Figure 2(b) shows the reparameterized tree<sup>2</sup>.

Here we summarize the TRP algorithm as a sequence of updates  $\mathbf{T}^n \rightarrow \mathbf{T}^{n+1}$  on the graph  $\mathbf{U}$  with the edge set  $\mathcal{E}$ , where  $\mathbf{T}$  represents the set of marginal probabilities maintained by TRP consisting of single-node marginals  $\mathbf{T}_u^{n+1}(x_u)$  and pairwise joint distribution  $\mathbf{T}_{uv}^{n+1}(x_u, x_v)$ ; and  $n$  denotes the iteration number:

1. **Initialization:** for every node  $u$  and every pair of nodes  $(u, v)$ , initialize  $\mathbf{T}^0$  by  $T_u^0 = \kappa\psi_u$  and  $T_{uv}^0 = \kappa\psi_{uv}$ , with  $\kappa$  being a normalization factor. (Other initializations are also possible.)
2. **TRP updates:** for  $i = 1, 2, \dots$ , do:
  - Select some spanning tree  $\mathcal{T}^i \in \Upsilon$  with edge set  $\mathcal{E}^i$ , where  $\Upsilon = \{\mathcal{T}\}$  is a set of spanning trees.
  - Use any exact algorithm, such as belief propagation, to compute exact marginals  $p^i(x)$  on  $\mathcal{T}^i$ . For all  $(u, v) \in \mathcal{E}^i$ , set

$$\mathbf{T}_u^{i+1}(x_u) = p^i(x_u).$$

$$\mathbf{T}_{uv}^{i+1}(x_u, x_v) = \frac{p^i(x_u, x_v)}{p^i(x_u)p^i(x_v)}.$$

- Set  $\mathbf{T}_{uv}^{i+1} = \mathbf{T}_{uv}^i$  for all  $(u, v) \in \mathcal{E}/\mathcal{E}^i$  (i.e., all the edges not included in the spanning tree  $\mathcal{T}^i$ ).

When selecting spanning trees  $\Upsilon = \{\mathcal{T}\}$ , the only constraint is that the trees in  $\Upsilon$  cover the edge set of the original graph  $\mathbf{U}$ .

### 3.3 Parameter Estimation in DCRFs

The parameter estimation problem is to find a set of parameters  $\theta = \{\lambda_k\}$  given training data  $\mathcal{D} = \{x^{(i)}, y^{(i)}\}_{i=1}^N$ . More specifically, we optimize the conditional log-likelihood

$$\mathcal{O}(\theta) = \sum_i \log p_\theta(\mathbf{y}^{(i)} | \mathbf{x}^{(i)}). \quad (5)$$

The derivative of this is

$$\frac{\partial \mathcal{O}}{\partial \lambda_k} = \sum_i \sum_t \sum_{c \in C} f_k(\mathbf{y}_{t,c}^{(i)}, \mathbf{x}^{(i)}, t) - \sum_i \sum_t \sum_{\mathbf{y}} p_\theta(\mathbf{y} | \mathbf{x}_t^{(i)}) f_k(\mathbf{y}_{t,c}, \mathbf{x}_t^{(i)}, t), \quad (6)$$

<sup>2</sup>This figure is adopted from [10].

<b>Words</b>	Confidence	in	the	pound	is	widely	expected	...
<b>POS</b>	NN	IN	DT	NN	VBZ	RB	VRB	
collapsed	NOUN	OTHER	OTHER	NOUN	VERB	RBP	VERB	
<b>Phrases</b>	B-NP	B-PP	B-NP	I-NP	B-VP	I-VP	I-VP	
collapsed	B	O	B	I	O	O	O	

Table 1: Example document with POS and NP labels, before and after collapsing the labels.

where  $C$  is the set of cliques of the 2-CRF, and  $\mathbf{y}_{t,c}$  denotes the variables of  $\mathbf{y}$  at time step  $t$  in clique  $c$  of the 2-CRF. Although equation (6) seems to require summing over all possible label sequences, if we observe that each feature function depends only on a single clique, we obtain

$$\frac{\partial \mathcal{O}}{\partial \lambda_k} = \sum_i \sum_t f_k(\mathbf{y}_{t,c}^{(i)}, \mathbf{x}^{(i)}, t) - \sum_i \sum_t \sum_{c \in C} \sum_{\mathbf{y}_c} p_\theta(\mathbf{y}_c | \mathbf{x}_t^{(i)}) f_k(\mathbf{y}_{t,c}, \mathbf{x}^{(i)}, t), \quad (7)$$

where  $\mathbf{y}_c$  ranges over assignments to the clique  $c$ .

This loss function is convex, and can be optimized by any number of techniques, as in other maximum-entropy models [4, 1]. In the results below, we use L-BFGS, which has previously outperformed other optimization algorithms for linear-chain CRFs [9, 5].

Note that this optimization requires computing marginal probabilities for every training instance at every iteration of the optimizer. In the experiments reported here, it was typical to need to compute marginals in 32 000 different graphical models. This intensifies the need for efficient inference.

## 4 Experiments

We used factorial DCRFs to perform simultaneous part-of-speech tagging and noun-phrase segmentation on data from the CoNLL 2002 shared task data set<sup>3</sup>. Table 1 shows example data. We considered each sentence to be a training instance, with single words as tokens. In these preliminary experiments, we used a subset of data  $\mathcal{D}_1$  containing 209 sentences, and we collapsed the original POS labels from 45 to 5. Table 2 shows some of the features we used.

The three NP labels—begin-phrase, inside-phrase, and other—were left unchanged. The original data contained 45 different POS labels, which we collapsed to 5 labels as follows:

- Collapse all different types of nouns into one label NOUN.
- Collapse all different types of verbs into one label VERB.
- Collapse all different types of adjectives into one label JADJ.
- Collapse all different types of adverbs into one label RBP.
- Collapse the remaining POS labels into one label OTHER.

We present two experiments: one comparing factorial DCRFs with linear-chain models, and one comparing different inference algorithms in DCRFs.

### 4.1 Comparison to linear-chain CRFs

We compared three approaches: a factorial DCRF; *cascaded* CRFs; and a *best-case* CRF. The cascaded CRFs used one linear-chain CRF to predict POS labels, and another linear-

<sup>3</sup>See <http://lcg-www.uia.ac.be/~erikt/research/np-chunking.html>.

word (collapsed: years, year-spans, fractions, numbers, ...)
contains-dash “-”
contains-dash-based “-based”
capitalized
all-caps
single-capital-letter
mixed-capitalization
contains-digits (and other symbols)

Table 2: Some of the features used in these experiments.

chain CRF to predict NP labels, using as a feature the Viterbi POS labeling from the first CRF. The best-case CRF predicted NP labels using the true POS labels.

The factorial DCRF used the graph structure in Figure 1(b), with one chain modeling the part-of-speech (POS) process and the other modeling the noun-phrase (NP) process. The vertical edges capture the dependencies between POS and NP labels.

We used L-BFGS to learn the parameters  $\theta$  of the DCRF. Computing the gradient requires computing the marginals over vertices and edges of the unrolled DCRF at different portions in time. We used the TRP approximation to compute these marginals.

Each TRP iteration selects a random spanning tree from the graphical model unrolled over the current training instance. To ensure that all the edges of the graph were covered by the TRP updates, we included eight hand-designed trees among the random spanning trees.

Next, we trained two cascaded linear-chain CRFs, where one CRF predicted the POS labels, and then the other CRF predicted the NP labels, using the POS predictions as input features. More specifically, we trained a POS-tagger (which we call  $CRF_{pos}$ ) using a training set  $\mathcal{D}_2$  that had 86 instances labeled by their POS tags. Then we substituted the POS labels of the original training set  $\mathcal{D}_1$  by the labels predicted by the learned model (i.e.,  $CRF_{pos}$ ) over the data in  $\mathcal{D}_1$ , resulting in the new training set  $\mathcal{D}_3$ . Note that  $\mathcal{D}_3$  has exactly the same observations as  $\mathcal{D}_1$ , and the same NP labels, but possibly different POS labels. Using  $\mathcal{D}_3$ , we trained a new CRF model (which we call  $CRF_{np}^+$ ) for predicting the NP labels, using as a feature the Viterbi POS labeling from  $CRF_{pos}$ . Finally, we trained a best-case linear-chain CRF (which we call  $CRF_{np}^*$ ) for predicting NP labels using the true POS labels along with the base features from Table 2. Of course, it is unrealistic to assume that the true POS labels are provided, however, this model gives an upper bound on how much POS knowledge can help noun-phrase segmentation.

In the cascaded model  $CRF_{np}^+$  and the best case model  $CRF_{np}^*$ , we used POS labels as features, however  $CRF_{np}^+$  uses the POS labels predicted by  $CRF_{pos}$  whereas  $CRF_{np}^*$  uses the correct POS labels as originally provided with the training set  $\mathcal{D}_1$ .

Table 3 compares the performance of these models. We measured accuracy on POS labels, on NP labels, and also joint accuracy on (POS, NP) pairs. To compute the joint accuracy for  $CRF_{np}^+$  on the test set, we used the predicted POS tags from  $CRF_{pos}$  and the predicted NP tags using  $CRF_{np}^+$ . To compute the joint accuracy for  $CRF_{np}^*$  we used the true POS tags of the test set together with the predicted NP tags on the test set using  $CRF_{np}^*$ .

The factorial DCRF outperformed the cascaded  $CRF_{pos}^+$  in joint accuracy and POS accuracy, but had lower NP accuracy. We conjecture that because there are more POS labels than NP labels, L-BFGS is forced to minimize the error across POS with more weight. The best-case model  $CRF_{np}^*$  outperforms the other models in every category. The performance of 100% for POS labels is because this model was provided with true POS labels.

	$CRF_{np}^+$	<b>DCRF</b>	$CRF_{np}^*$
NP Accuracy	0.9084	0.8611	0.9249
POS Accuracy	0.7722	0.8203	1.0
Joint Accuracy	0.7197	0.7728	0.9249

Table 3: Comparison of performance of CRFs and DCRFs. Note that the feature set is restricted in these experiments.

Algorithm	Overall F1	Training time (hr)	LBFGS iterations
TRP	0.6740	5.342	87
Loopy	0.6756	14.728	81
Junction Tree	0.6675	8.614	83

Table 4: Comparison of inference algorithms for 2-chain factorial CRF on CoNLL 2002 data set. Overall F1 is the average of the F1 measure over all types of NP and POS labels. LBFGS iterations gives the number of iterations of the LBFGS gradient descent. Because these experiments are preliminary, we ran with a restricted feature set.

These results are low compared to the state of the art. To get initial evidence comparing DCRFs to cascaded approaches, we used a small training set and a restrictive set of features. However, our new implementation does scale to the large feature sets needed for best performance—in fact, because these models are conditional, the running time of inference scales not with the number of features but with the number of labels. We leave experiments with larger training and feature sets to future work.

## 4.2 Comparison of Inference Algorithms

Because DCRFs can have rich graphical structure, and require many marginal computations during training, inference is critical to efficient training with many labels and large data sets. We compared the performance and running time of three different propagation algorithms: TRP, loopy belief propagation, and junction tree.

We ran TRP with random spanning trees, stopping after 25 iterations whether the algorithm had converged or not. Loopy belief propagation was run until all marginal probabilities had converged to within  $10^{-4}$ , which usually took between 10 and 15 iterations of synchronous updates. Exact inference using junction tree was feasible because we used collapsed tags and only two chains. In this experiment, we used 410 training instances, a superset of the training set of the previous section. POS tags were collapsed as before. All experiments were run on an Intel Xeon 2.8 GHz machine with 3 GB RAM. We measured performance on a test set and total training time. The training times include a few non-inference tasks such as computing the gradient; however, the running time is dominated by the time used by inference.

The results are shown in Table 4. In overall F1 on a test set, the inference algorithms perform very similarly. For an unknown reason, junction tree has a slightly lower F-measure on this test set; in other experiments, exact inference has had slightly higher F-measure. However, TRP trains much faster, using only 62% of the time needed by junction tree. Synchronous loopy belief propagation performed very slowly on this data set. Using a less strict stopping criterion might allow it to run faster without sacrificing performance on the tagging task.

Although these results need to be replicated in other data sets, they suggest that TRP is a good choice for training this kind of model.

## 5 Conclusions

Dynamic CRFs are conditionally-trained undirected sequence models with repetitive graphical structure and tied parameters. Inference in DCRFs can be done efficiently using approximate methods, and training can be done within the maximum-entropy framework. Because of their factorized state, we can use DCRFs to do several labeling tasks at once, sharing information between them. On a joint noun-phrase segmentation / part-of-speech tagging task, factorial DCRFs have higher joint accuracy and POS accuracy than linear-chain CRFs, but lower NP accuracy.

More work is needed for training DCRFs where accuracy on certain labels is more important than others. We are currently running experiments that train on the marginal probability of one chain, e.g.,  $p(\text{NP}|\mathbf{x})$ .

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