Beyond Script Management

David Aspinall and Christoph Lüth

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Outline

State of the Union

The Proof Engineering Programme

Research

Implementation

Conclusions

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Proof General

```
Emacs@Bernays.local
State Context Goal Retract Undo Next Use Goto Q.E.D. Find Command Stop Restart Help

text {* Proper proof text -- \textit{advanced version}. *}

theorem "A & B --> B & A"
proof
  assume "A & B"
  then obtain B and A ..
  then show "B & A" ..
qed


text {* Unstructured proof script. *}

theorem "A & B --> B & A"
  apply (rule impI)
  apply (erule conjE)
  apply (rule conjI)
  apply assumption
  apply assumption
done
--:--  Example.thy     53% L38    CVS:8.3   (Isar script Scripting)-----------

proof (prove): step 3

goal (theorem, 2 subgoals):
 1. [| A; B |] ==> B
 2. [| A; B |] ==> A

-:--  *goals*          All L1      (proofstate)-----------
```

- *Generic* interface:
  - Isabelle,
  - Coq,
  - LEGO,
  - HOL, ACL2,
  - ...
- *Emacs*-based
- Est: 40k lines of code; 4 py effort
- Widely used since 1998
- **EOL** 2008?

Expert Users

- Large proofs developments certainly can be constructed:
  - the IP stack specification
  - type safety and programming logics for Java
  - Gonthier & Werner's proof of the Four Colour Theorem
  - Hales's ongoing project on formalising his proof of the Kepler Conjecture (20 person-years; 1/3 way through)
  - The Mizar Mathematical Library
  - ...

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  - ...
- But they are very difficult to construct and maintain
  — more than they ought to be, that is

New Users

[Interactive] proving is not just a slightly more fussy version of paper proving and neither (Curry-Howard notwithstanding) is it really like programming.
It's a strange new skill, much harder to learn than a new programming language or application.
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[Nick Benton, First Workshop on Mechanized Metatheory, 2006. Color emphasis mine.]

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Proof Engineering

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Proof Engineering

Support the *lifecycle* of large proof developments:

- **Construction**: importing, assisting, generating;
- **Maintenance**: changing, reusing, exploring choices;
- **Understanding**: navigating, documenting, exporting.

This is our characterisation of *Proof Engineering* (PE), by (loose) analogy with Software Engineering (SE).

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- Extraordinarily hard to learn and apply...

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- Currently there is a tragic fragmentation across systems:
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- $\Rightarrow$ a natural evolution of Proof General.

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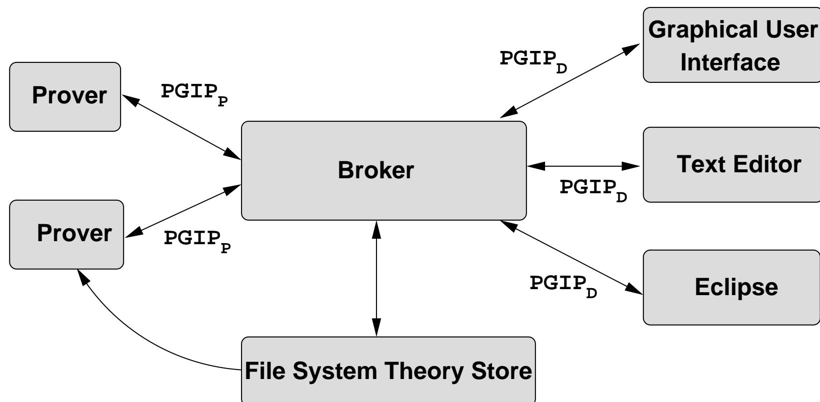
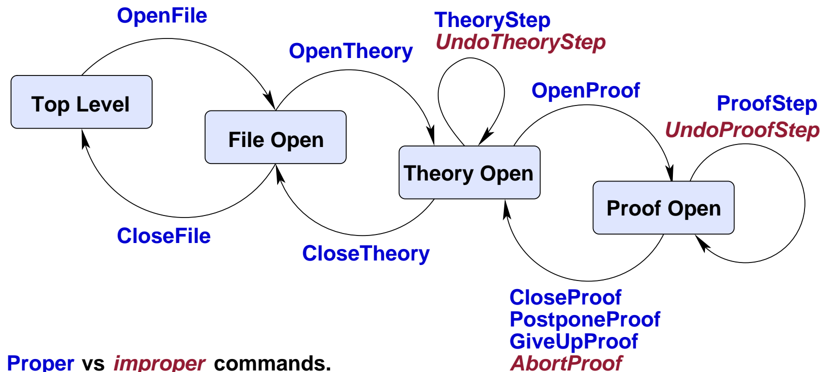
- **Development**: relationship between proofs, explaining development process in moving between versions. Possibility: use *development graphs*.
- **Dependency**: manifest or implied relationships between parts of a proof.
- **Interaction: how to construct proofs using interactive tools, connecting proof engines to interfaces.**

PG Kit [Bremen, Edinburgh, Munich 2003-]

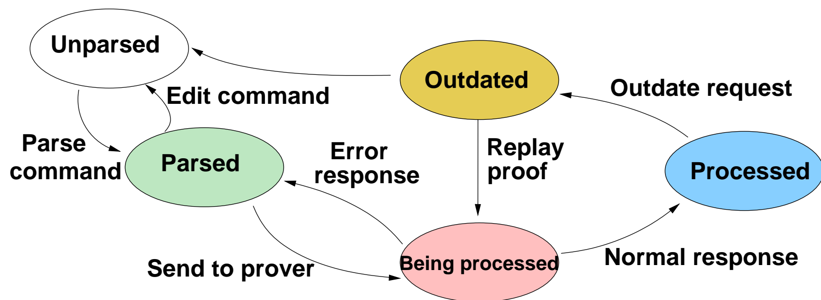


Interaction Model: Prover States in Development

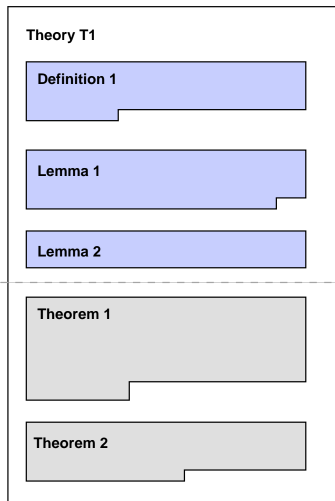


Proper vs ***improper*** **commands.**

Interaction Model: Document States in Development



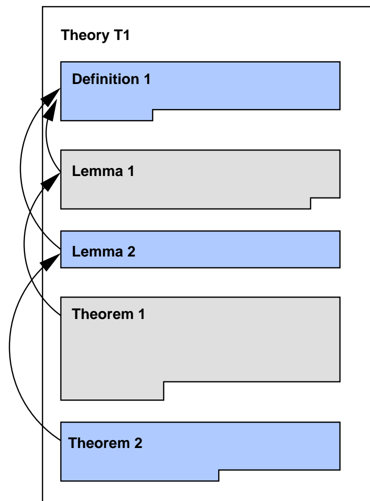
Interaction Model: Script Management



Linear dependency:

- Classical *script management*
- No *prover assistance*
- Potentially *expensive*

Dependencies for Script Management



Explicit dependencies:

- Requires *accurate prover assistance*
- Better *change management*

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- Do not give full logical meaning to language, just enough structure to do the job (c.f. *Isar* outer syntax without inner syntax).
- Semantics in terms of model notions.

Proof Language

Proof script: *native language*

```
lemma fn1: "(∃ x. P (f x)) ⟶ (∃ y. P y)"
proof
  assume "∃ x. P (f x)"
  thus "∃ y. P y"
  proof
    fix a
    assume "P (f a)"
    show ?thesis ..
  qed
qed
```

Proof Language Markup

Proof scripts: *native language* plus *PGIP markup*

```
<opengoal name="fn1">lemma fn1: &quot;(<sym name="exists">EX</sym
<openblock/><proofstep>proof</proofstep>
  <proofstep>assume &quot;<sym name="exists">EX</sym> x. P (f x)&qu
  <proofstep>thus &quot;<sym name="exists">EX</sym> y. P y&quot;</p
  <openblock/><proofstep>proof</proofstep>
    <proofstep>fix a</proofstep>
    <proofstep>assume &quot;P (f a)&quot;</proofstep>
    <proofstep>show ?thesis</proofstep><openblock/>
      <proofstep>..</proofstep><closeblock/>
  <proofstep>qed</proofstep><closeblock/>
<closegoal>qed</closegoal><closeblock/>
```

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- *Refactoring* proofs: simple but with complex pre-conditions (MoveTheorem) or complex operations (SplitProof, ClarifyProof).

PE Methods

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- *Refactoring* proofs: simple but with complex pre-conditions (MoveTheorem) or complex operations (SplitProof, ClarifyProof).
- *Proof Patterns*: recommended strategies for common problems (e.g. PartialFunction, Binder). Needs community effort.

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An IDE for Proof based on PGIP

Proof General - HOLex/PER.thy - Proof General Eclipse

```
   x ≅ y ==> f x ≅ g y"
unfolding eqv_fun_def by blast

text {*
  The class of partial equivalence relations is
  closed under function spaces (in \emph{both}
  argument positions).
*}

instance "fun" :: (partial_equiv,partial_equiv)
                 partial_equiv
proof
  fix f g h :: "'a::partial_equiv ⇒
                'b::partial_equiv"
  assume fg: "f ≅ g"
  show "g ≅ f"
  proof
  assume gh: "g ≅ h"
  show "f ≅ h"
  proof
    fix x y :: 'a
    assume x: "x ∈ domain" and y: "y ∈ domain"
       and "x ≅ y"
```

```
proof (state): step 21
goal (1 subgoal):
1. ⋀x y. ⟦x ∈ domain; y ∈ domain; x ≅ y⟧ ⟹ f x ≅ h y
```

- *Many features*
- 45k loc, 2-3py effort

Environments for Formal Proof

- An effective *environment* for theorem proving is much more than a proof engine.
- Rest of environment in total larger than core (UI, file system/database, document processor).
- As software environments gets more complex and yet still diverse, effective genericity like this is catching on: GCC, CIL, VS .NET, Eclipse (even UML).

Things We Need

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- rich output display (symbol and structure markup)
- exposing useful information (e.g. abstract syntax of terms and of proof scripts, dependencies in scripts).
- facilitating user interaction (e.g. classifying output, reporting status during long-running operations, querying user, informative error messages)
- providing useful operations (e.g. manipulation and examination of terms, templates for construction)
- facilitating behind-the-scenes programmatic interaction (e.g. parsing, querying, searching), ideally in an auxiliary thread
- ... provided in a stable protocol/API, e.g., PGIP

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- Proof Engineering: a research programme for bringing proof development into 21st century.
- It's needed to keep theorem proving viable: (Rushby: TP risks being replaced by "disruptive technology").
- It's needed now: growing number of people want accessible theorem proving technology.
- Proof General Kit with Isabelle: an ideal platform for our research.
- Also, we hope: a compelling Proof Development Environment for users.