### Entailment for Structured Specifications (1988)

| \( SP \vdash \varphi_1 \) | \( \ldots \) | \( SP \vdash \varphi_n \) | \( \{ \varphi_1, \ldots, \varphi_n \} \vdash_{\text{Sig}(SP)} \varphi \) |
|-----------------|-----------------|-----------------|
| \( SP \vdash \varphi \) | \( \{ \varphi \} \vdash \varphi \) | \( \varphi \in \Phi \) |
| \( SP_1 \vdash \varphi \) | \( SP_2 \vdash \varphi \) | \( SP_1 \cup SP_2 \vdash \varphi \) |
| \( SP \vdash \varphi \) | \( SP \vdash \sigma(\varphi) \) | \( SP \text{ hide via } \sigma \vdash \varphi \) |

**Clarifications:** INS = \( \langle \text{Sign}, \text{Sen} : \text{Sign} \rightarrow \text{Set}, \text{Mod} : \text{Sign}^{op} \rightarrow \text{Cat}, \langle \|=_{\Sigma} \subseteq [\text{Mod}(\Sigma) \times \text{Sen}(\Sigma)]_{\Sigma \in \text{Sign}} \rangle \) is an institution that defines the logical system used for specifications. \( SP, SP_1 \) and \( SP_2 \) are structured \( \Sigma \)-specifications over INS, where \( \Sigma \) is a signature in the category \( \text{Sign} \), \( \varphi, \varphi_1, \ldots, \varphi_n \) are \( \Sigma \)-sentences, i.e. elements in \( \text{Sen}(\Sigma) \), \( \Phi \) is a set of \( \Sigma \)-sentences, and \( \sigma(\varphi) \) denotes \( \text{Sen}(\sigma(\varphi)) \), the translation of the sentence \( \varphi \) along \( \sigma : \Sigma \rightarrow \Sigma' \). Structured specifications in INS are built from basic specifications (\( \Sigma, \Phi \)), the union of \( \Sigma \)-specifications \( SP_1 \cup SP_2 \), the translation “\( SP \text{ with } \sigma' \)” of \( SP \) along a signature morphism \( \sigma : \Sigma \rightarrow \Sigma' \), and hiding “\( SP \text{ hide via } \sigma \)” for hiding the symbols in \( SP \) not occurring in the image of \( \sigma : \Sigma' \rightarrow \Sigma \). \( \text{Sig}(SP) \) is the signature of \( SP \). Translations of \( \Sigma \)-sentences and \( \Sigma' \)-models along \( \sigma : \Sigma \rightarrow \Sigma' \) are required to preserve satisfaction: for any \( \varphi \in \text{Sen}(\Sigma) \) and \( M' \in [\text{Mod}(\Sigma')] \), \( M' \models_{\Sigma'} \text{Sen}(\sigma)(\varphi) \iff \text{Mod}(\sigma(\varphi))(M') \models_{\Sigma} \varphi \). Finally, \( \langle \|=_{\Sigma} \subseteq [\text{Pow}(\text{Sen}(\Sigma)) \times \text{Sen}(\Sigma)]_{\Sigma \in \text{Sign}} \rangle \) is a sound entailment relation for the satisfaction relation \( \langle \|=_{\Sigma} \rangle_{\Sigma \in \text{Sign}} \).

The judgement \( SP \vdash \varphi \) is meant to capture the property that \( \varphi \) is satisfied in all models of \( SP \).

**History:** The first systems for proving entailment in structured specifications were given by Sannella and Burstall [1], Sannella and Tarlecki [2], and Wirsing [3]. The above presentation can be found in [6], Sect. 9.2.

**Remarks:** The system is sound; completeness is shown in [3] for the first-order logic instance and in [5][6] for an institution INS which is finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation \( \langle \|=_{\Sigma} \rangle_{\Sigma \in \text{Sign}} \). [7] shows that this is the most powerful sound proof system that is compositional in the structure of specifications. [6] provides additional rules for observability operators.

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