1. INTRODUCTION

The title of this paper represents a somewhat arbitrary choice. Certainly a central portion of the subsequent text is concerned with an analysis of verb phrase ellipsis within Kamp's (1981) Discourse Representation (DR) Theory. Nevertheless, the starting point for the whole enterprise was an attempt to answer the following question: Can the rules for translating expressions of English into discourse representations be stated in parallel with the syntactic rules which assign a structural description to those expressions? As many readers will recognize, such a translation regime is embodied in Montague's (1974) fragments for English, and has been widely adopted in subsequent linguistic work. Assuming a context free phrase structure syntax, we get the following schematic pairing of syntactic with semantic rules:

\[ \alpha_0 \rightarrow \alpha_1, \ldots, \alpha_n \]
\[ \alpha_0' = f(\alpha_1', \ldots, \alpha_n') \]

where \( \alpha_i' \) is the meaning of the constituent \( \alpha_i \), and where \( \alpha_0' \) is defined as some function \( f \) of the meanings \( \alpha_1', \ldots, \alpha_n' \). Semantic rules stated in this format are compositional: all the information relevant to constructing a meaning for a complex expression \( \alpha_0 \) is locally present in the subconstituents of \( \alpha_0 \). The reason why this enterprise seems challenging is that DR construction rules have typically been formulated as requiring reference to non-local information; for example, the rule which assigns a representation to an NP-VP sentence varies according to the identity of the determiner of the NP.

What is the justification for trying a different formulation? At least two
reasons might be given. The first involves compositionality. As indicated above, DR theory appears to be less strictly compositional than, say, Montague grammar.\(^1\) It is not obvious that strict compositionality is an indispensable feature of any adequate theory of natural language semantics; it might be the case that the proper treatment of anaphora is one in which compositionality is relaxed. On the other hand, to the extent that compositionality is a constraint on possible grammars, it would be interesting to determine exactly where it should be relaxed. Kamp himself claims:

\[\ldots\text{the conception of a perfect rule-by-rule parallelism between syntax and semantics is one that must be proved rather than taken for granted. [footnote omitted] In fact the data here presented point towards the conclusion that this conception is ultimately untenable. (1981, p. 298)}\]

From a methodological point of view, it might be countered that it is the lack of parallelism which must be proved. One way of doing this is to cast DR construction rules in a compositional mould, and then to see at what point the mould becomes a Procrustean bed.

A second argument involves processing considerations. One of the appeals of DR theory is that it appears to provide an account of semantic representation in which the claims of model-theoretic semantics can be reconciled with those of automatic natural language parsing systems. It is far from clear, given the present state of our knowledge, what the optimal relation is between syntax and semantics in such systems. Nevertheless, the more flexibility there is between syntax and semantics, the better the chances of successfully incorporating DRSs in natural language parsers. The requirement that DR construction proceed top-down on syntactic parse trees would severely limit the design possibilities of any parser that incorporated DRSs as a semantic representation, and this again suggests that it is worthwhile exploring a rule-by-rule approach.

Despite these intentions, I shall not attempt to give a detailed reformulation of the required kind here (for some proposals in this direction, see Johnson and Klein (1985) and Reyle (1985)). Instead, I shall present the results of asking a subsidiary question: Assuming that each constituent in a phrase structure analysis of a sentence is to be assigned a DR translation, what is an appropriate representation and interpretation for a VP? It turns out that the evidence of VP ellipsis provides some useful clues about the answer to this. These issues form the subject matter of sections 3 and 4. They in turn raise further questions about the treatment of quantification and bound anaphora in DR theory, and this topic is briefly discussed in section 5. However, I start off in section 2 by giving a brief overview of some salient aspects of DR theory; the reader who is already familiar with Kamp (1981) may safely omit this section.
Discourse representation theory\(^2\) is a relatively new approach to the semantics of natural language. As the name suggests, a central tenet of the theory is that the basic unit of semantic analysis should be a discourse, rather than a sentence. Although this idea is not in itself very novel, DR theory is unusual in the way that it attempts to integrate techniques from model-theoretic semantics in a framework that also takes serious account of pragmatic aspects of language.

A key component of DR theory is the set of construction rules that convert natural language discourse into a formal representation of content, namely a Discourse Representation Structure. Existing proposals employ an algorithm that works top-down on syntactic parse trees. Starting off with a complex input, the procedure decomposes the parse tree into smaller and smaller units, while also indexing coreferential argument positions by means of reference markers. The procedure terminates when no further decompositions are possible.

In order to illustrate this, and other points, let us study some examples. We start by considering a two sentence discourse.

(1) Lee owns a cat. It loves him.

We assume these sentences are assigned syntactic phrase markers of the usual kind. Application of the construction algorithm to the first sentence of (1) leads successively to the structures \(K_0\) and \(K_1\).
In the first step, \( K_0 \), the NP *Lee* licenses the introduction of a discourse marker \( x_0 \) into the universe of the DRS. We also add two conditions: \( \text{Lee}(x_0) \) — representing the information that \( x_0 \) stands for the bearer of the name *Lee* — and \( x_0 \) *owns a cat*, which is derived by replacing the subject of the S by the marker \( x_0 \). In the next step, the VP in this second condition is reduced further. The NP *a cat* licenses the introduction of discourse marker \( x_1 \) into the universe, and we add the conditions *cat*(\( x_1 \)), and *owns* (\( x_0, x_1 \)).

Processing of the second sentence now has to proceed relative to \( K_1 \). The natural interpretation of the pronouns *it* and *him* of this sentence is that they are anaphorically connected with the "antecedents" *a cat* and *Lee*. In terms of the construction algorithm, this means that the discourse markers which are introduced when the anaphoric pronouns are processed must be linked with the discourse markers \( x_1 \) and \( x_0 \) of \( K_1 \) that were earlier introduced for *a cat* and *Lee*. Thus the next two steps yield the following structure.

\[
\begin{array}{c|cccc}
 & x_0 & x_1 & x_2 & x_3 \\
\hline
\text{Lee}(x_0) & & & & \\
\text{cat}(x_1) & & & & \\
\text{own}(x_0, x_1) & & & & \\
x_2 = x_1 & & & & \\
x_3 = x_0 & & & & \\
\text{love}(x_2, x_3) & & & & \\
\end{array}
\]

This illustrates how a given DRS can serve as context for the processing of the next sentence, and how its discourse markers are essential to this function. Note that, according to this approach, the indefinite *a cat* is not treated as a quantifier, but simply as an expression that introduces a discourse marker together with some conditions.

The semantic content of \( K_2 \) is determined along lines which are familiar from model-theoretic semantics. An *embedding* function determines a correspondence between the formal representation and some situation. That is, \( K_2 \) is true in a given model \( M \) if there is an embedding function \( f \) which maps the discourse markers in the universe of \( K_2 \) into the universe of \( M \), and if the objects in the range of \( f \) satisfy the conditions listed in \( K_2 \). More
specifically, it is true if there is a function \( f \) which maps \( x_0, x_1 \) and \( x_2 \) and \( x_3 \) into objects \( a, b, c \) and \( d \) such that the following hold in \( M \): \( a \) is Lee, \( b \) is a cat, \( a \) owns \( b \), \( c = b, d = a \), and \( c \) loves \( d \). It transpires, therefore, that the existential force associated with an indefinite NP is determined by the embedding conditions for the DRS in which the NP is contained.

The following definition gives a more formal account of the way in which a truth conditional interpretation is assigned to a DRS.

**Definition 1:**

Let \( K = \langle U.K, \text{Con.K} \rangle \) be a DRS where \( U.K \) is a set of discourse markers drawn from a nonempty set \( V \), and \( \text{Con.K} \) is a set of atomic conditions. Let \( M = \langle A, F \rangle \) be a model with universe \( A \) and interpretation function \( F \). Let \( f : V \rightarrow A \) be a partial function. Then \( f \) verifies \( K, f \models K \) iff \( U.K \subseteq \text{dom}(f) \) and \( f \models \text{Con.K} \). And \( f \models \text{Con.K} \) iff for each \( S \in \text{Con.K} \), \( f \models S \).

Assume for the time being that a basic condition \( S \) is always of the form \( R(x_1, \ldots, x_n) \), where \( R \) is an \( n \)-ary relation symbol, and \( x_1, \ldots, x_n \) are reference markers. Then we have the following:

**Definition 2:**

\( f \models R(x_1, \ldots, x_n) \) iff \( \langle f(x_1), \ldots, f(x_n) \rangle \in F(R) \)

Finally, we have:

**Definition 3:**

A DRS \( K \) is **true** iff there is some embedding function \( f \) such that \( f \models K \).

Universally quantified NPs receive a somewhat different analysis to indefinites in DR theory. This point can be briefly illustrated with the help of a 'donkey' sentence such as (2).

(2) Every farmer who owns a donkey beats it.

The problem in interpreting (2) is to provide a univocal treatment of a donkey which nevertheless accounts for the fact that it is perceived to have universal rather than existential force in this syntactic context.
Universal sentences (and conditionals) licence the introduction of two subDRSs, the first of which represents the antecedent (e.g. *man who owns a donkey*), and the second the consequent (e.g. *beats it*). Informally, the embedding conditions associated with a DRS like $K_3$ go as follows: the DRS is true just in case every embedding function that verifies the antecedent box can be extended to an embedding function that verifies the consequent. By virtue of this analysis, every discourse marker in the universe of the antecedent box, including the marker $\mathbf{x}$ which was introduced by *a donkey*, is universally quantified, and thus the correct truth conditions for (2) are obtained.

We can think of the split boxes in $K_3$ as being a new kind of condition, one in which $\Rightarrow$ is a two-place relation whose arguments are DRSs. We write such a condition as $\Rightarrow (K_1, K_2)$. Before giving it an interpretation, however, it is helpful to have some new notation (cf. Chierchia and Rooth (1984) and Zeevat (1984)).

**Definition 4:**

Let $X \subseteq V$ be a set of discourse markers, and let $f$ and $g$ be partial functions on $V$. Then $g$ is an $X$-extension of $f$, written $f \subseteq_X g$, iff $\text{dom}(g) = \text{dom}(f) \cup X$ and $f \subseteq g$.

That is, $g$ is an $X$-extension of $f$ iff $g$ assigns the same values as $f$ does to all the discourse markers in the domain of $f$, and moreover $g$ also assigns values to all the markers in $X$. The next definition gives Kamp’s interpretation of the universal/conditional arrow.

**Definition 5:**

$f \models \Rightarrow (K_1, K_2)$ iff

\[\forall g [f \subseteq_{U,K_1} g \land g \models \text{Con}.K_1 \Rightarrow \exists h [g \subseteq_{U,K_2} h \land h \models \text{Con}.K_2]]\]
Suppose, for example, that we evaluated $K_3$ relative to an $f$ whose domain is $\emptyset$, the empty set. Then for every $g$ which is a $\{x_0, x_1\}$-extension of $f$ and which verifies the antecedent subDRS of $K_3$, there must be a $\{x_2\}$-extension of $g$ which verifies the consequent subDRS.

3. VP ELLIPSIS

As we saw in the preceding section, Kamp's approach presupposes that when the DRS for a given sentence is constructed, a complete syntactic parse for that sentence has already been built. An alternative strategy would be to construct the DRS in a bottom-up fashion, so as to resemble the compositional construction of logical formulae in Montague semantics. What would the construction rules look like in this case?

Let us take as our model the Montagovian approach which is typically adopted in generalized phrase structure grammar (cf. Gazdar (1982), Gazdar et al. (1985), Klein and Sag (1985)). The grammar rules listed in (3) are pairs of phrase structure rules and translation rules:

\begin{enumerate}
\item $\langle S \rightarrow NP \ VP; NP'(VP') \rangle$
\item $\langle NP \rightarrow Det \ Nom; Det'(Nom') \rangle$
\item $\langle Nom \rightarrow N; N' \rangle$
\item $\langle VP \rightarrow V; V' \rangle$
\end{enumerate}

The translation rules dictate how the translation of the node on the lefthand side of the PS rule is built up from the translations of the nodes on the righthand side of the PS rule. So, for example, the translation rule in (3a) says that whenever a node $S$ in a tree expands as an NP followed by a VP, the translation of that $S$ constituent is obtained by combining the translation of the NP constituent (indicated as NP') as functor with the translation of the VP constituent (indicated as VP') as argument. The translations of NP and VP are derived in turn from the translations of their subconstituents by an inductive definition which takes as a basis the translations of lexical items into expressions of intensional logic. (4) gives a simple illustration, in which lambda expressions have been simplified where possible.

\begin{equation}
S, \forall x[boy'(x) \rightarrow run'(x)]
\end{equation}

\begin{center}
\begin{tikzpicture}
  \node (S) {$S, \forall x[\text{boy'(x) } \rightarrow \text{run'(x)}]$};
  \node (NP) [below left of=S] {$NP, \lambda P \forall x[\text{boy'(x) } \rightarrow P(x)]$};
  \node (VP) [below right of=S] {$VP, \text{run'}$};
  \node (Det) [below left of=NP] {$\text{Det, } \lambda Q \forall x[Q(x) \rightarrow P(x)]$};
  \node (Nom) [below right of=NP] {$\text{Nom, } \text{boy'}$};
  \node (V) [below right of=VP] {$V, \text{run'}$};
  \node (N) [below right of=Nom] {$\text{N, } \text{boy'}$};
  \node (X) [below right of=N] {\text{every}};
  \node (Y) [below right of=N] {\text{boy}};
  \node (Z) [below right of=V] {\text{runs}};
  \draw (S) -- (NP);\draw (S) -- (VP);
  \draw (NP) -- (Det);\draw (NP) -- (Nom);
  \draw (VP) -- (V);
  \draw (Det) -- (X);\draw (Det) -- (Y);
  \draw (Nom) -- (Z);
\end{tikzpicture}
\end{center}
Taking (4) as our model, we could try to formulate analogues of (3) which would associate a subDRS with each constituent admitted by a PS rule. The tree in (5) gives a rough illustration of the way this might work.

\[
S, \Rightarrow (\frac{x_0}{\text{boy}(x_0)}, \frac{\text{run}(x_0)}{\text{run}(x_0)}), Q)
\]

\[
NP, \Rightarrow (\frac{x_0}{\text{boy}(x_0)}, Q)
\]

\[
\text{Det}, \Rightarrow (P, Q)
\]

\[
\text{Nom}, \Rightarrow \text{every}
\]

\[
\text{boy}
\]

\[
\text{VP}
\]

Notice that a structure has been associated with the determiner every in which P and Q are place-holders for subDRS arguments of \(\Rightarrow\). Without going into details, I shall assume that there are operations which replace these place-holders with the appropriate subDRSs.

Even at this level of abstraction, a technical difficulty seems to arise. In order for the topmost DRS to receive the right truth conditions, it is essential that the same discourse marker (in this case \(x_0\)) occur as argument of both boy and run. However, it is not easy to see how to guarantee this result on a bottom-up translation process. That is, when some marker \(x_i\) is chosen to occupy the subject argument slot in the VP translation, this choice is independent of the discourse marker that is to occupy the argument slot in the Nom translation. It appears, then, that the DRSs associated with the Nom and VP in (5) do not represent the meanings of those constituents in a manner which is compatible with a compositional semantic analysis. The basis of a solution to this problem will be explored by examining the interpretation of VPs in greater detail.

The phenomenon of VP ellipsis in English has been subjected to close scrutiny in the literature, the central studies being Sag (1976) and Williams (1977). In the rest of this section, I shall investigate how DR theory might be extended to deal with some of the basic facts involving VP ellipsis. Despite the fact that many important and interesting issues will be ignored, the attempt to widen the coverage of DR theory in this way is interesting for its own sake, and will also give us some valuable clues as to the kind of representation that will have to be associated with VPs.
Consider a discourse like the following:

(6) Lee loves his cat. Gerry does too.

Without going into great detail about possessives, we might suppose that the DRS for the first sentence of (6) is something like this:

\[
\begin{array}{c|c|c}
  x_0 & x_1 & x_2 \\
  \hline
  \text{Lee}(x_0) & \text{cat}(x_1) & \text{of}(x_1, x_2) \\
  \text{love}(x_0, x_1) \\
  x_2 = x_0 \\
\end{array}
\]

The NP *his cat* has been analysed as equivalent to *a cat of him*. Consequently, it licenses the addition of two reference markers \(x_1\) and \(x_2\) to the universe. We add the conditions \(\text{cat}(x_1)\) and \(\text{of}(x_1, x_2)\) and, assuming that *his* is anaphoric to *Lee*, we also add the link \(x_2 = x_0\).

When we come to process the second sentence, we need to say something about the structure of the ellipsed VP *does*. Consider first a VP like (7) in which *do* takes a complement:

(7) does like the cat

Following Gazdar, Pullum and Sag (1982), I shall assign it the following structure:

(8) \[
\begin{array}{c}
  \text{VP[FIN, 3s]} \\
  \text{V} \\
  \text{VP[BSE]} \\
  \text{does} \\
  \text{like the cat} \\
\end{array}
\]

That is, it is analysed as a finite, third person singular VP consisting of a head V *does* and a base-form VP complement.
The ellipsed VP will be assigned exactly the same structure. The only difference is that we add the feature specification [+ NULL] to the complement VP, and let such VPs expand as the empty string:

(9) VP[FIN, 3s]
    V
    VP[BSE, + NULL]
    does e

Let's assume that this complement is to be interpreted as a VP anaphor, and will itself license the introduction of discourse marker, say $P$. Ignoring the word *too*, we might therefore extend $K_4$ as follows:

$K_5$:

<table>
<thead>
<tr>
<th></th>
<th>$x_0$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lee($x_0$)</td>
<td>cat($x_1$)</td>
<td>of($x_1$, $x_2$)</td>
<td>love($x_0$, $x_1$)</td>
<td>$x_2 = x_0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Gerry($x_3$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>do($x_3$, $P$)</td>
</tr>
</tbody>
</table>

Now we are faced with the question: what can $P$ be linked to? If we pursue the analogy with nominal anaphora, we are led to the conclusion that there should be an additional marker, say $Q$, which has already been introduced into the universe of the DRS, and whose value is constrained by the conditions associated with the VP of the preceding sentence. However, there is a problem in implementing this idea which is brought out in the attempted construction $K_6$. Here, we have grouped the conditions associated with *loves his cat* into a subDRS $Q$: 
Let us suppose that all the reference markers in $Q$ are assigned values by whatever embedding function is used to verify the superordinate DRS $K_6$. Then, in particular, $x_0$ will be assigned the value Lee. But this conflicts with the intention that $Q$ should group together conditions that apply equally to Lee and Gerry. One way of dealing with this difficulty would be to invoke some kind of relettering operation which would make $P$ equivalent to the result of replacing the relevant occurrences of $x_0$ in $Q$ by $x_3$. An alternative would be to modify the subDRS $Q$ so that it was interpreted as expressing a property that could be predicated of different subjects. There are two reasons why the second option is preferable. First, the deployment and analysis of properties has been much studied in formal semantics (see Chierchia (1984) for an interesting recent investigation), and other things being equal, it is sensible to build on established foundations. Second, Kamp (1983) introduces just the mechanism we require in the context of providing an account of definite noun phrases, so we already have independent motivation for such an extension to the theory.

Kamp suggests that, when a DRS is to be construed as a predicate, we add to its universe a distinguished reference marker "which plays the role of the individual to which the predicate is applied" (1983: 52). I will indicate the distinguished marker by enclosing it in square brackets. Thus, $Q$ above would be modified to as follows:
Let us call a structure like this a *predicate-DRS*, and call *P, Q, ... discourse markers of the predicate type*, or predicate markers, for short. Moreover, we use the notation \([x]K\) to represent a DRS which has a distinguished marker *x*. According to Kamp,

an object *a* satisfies \([a \text{ predicate-DRS } [x]K]\) if it is possible to extend the correspondence \((x, a)\), between the object and the distinguished marker, to a proper embedding of the entire DRS. (1983: 53)

Thus, an object *a* satisfies *Q* if there are objects *b* and *c* such that *a* loves *b*, *b* is a cat, *a* is 'of' *c* and *c* is identical to *a*. Reverting temporarily to the notation of lambda-calculus, *Q* is equivalent under its intended interpretation to the following:

\[
\lambda x_4 \exists x_1 \exists x_2 [\text{love}(x_4, x_1) \& \text{cat}(x_1) \& \text{of}(x_1, x_2) \& x_2 = x_4]
\]

This seems to be what we need. If *P* is linked to *Q*, then \(do(x_3, P)\) will be true under an embedding \(f\) if \(f(x_3)\) satisfies *Q*.

Given this new apparatus, the DRS *K₆* can be revised along the lines shown in *K₇* below.

The conditions (i) and (ii) in the subDRS *Q* in *K₇* represent alternative, mutually exclusive ways in which we could link *x₃* to a previous discourse marker. If we take option (i), then *Q* expresses the property of being a *x₄* such that *x₄* loves *x₄*’s cat. Consequently, the discourse receives the so-called ‘sloppy identity’ reading, according to which Lee loves Lee’s cat and Gerry loves Gerry’s cat. Adopting condition (ii) instead gives rise to the ‘strict identity’ reading, where *Q* expresses the property of being a *x₄* such that *x₄* loves *x₄*’s (i.e. Lee’s) cat.

Before concluding this section, I wish to briefly consider an alternative analysis of VP ellipsis in DR theory. According to this alternative, there
VP Ellipsis in DR Theory

is no postulation of an anaphoric link between predicate markers. Instead, the conditions for a null VP are obtained by 'copying' the conditions associated with some previous VP in the discourse, accompanied by an appropriate relettering of the subject-position discourse marker. To make things clearer, let us look at an example like (11).

(11) Sam found a cat. Kit did too.

Suppose that K8 represents the first sentence.
According to the alternative proposal, we can extend this DR to the second sentence of (11) in the following manner. First we add a condition (and discourse marker) for Kit, say Kit(x_2). Next, we copy over both the VP conditions of the first sentence, replacing all occurrences of \( x_0 \) by \( x_2 \):

\[
\begin{array}{|c|c|c|}
\hline
x_0 & x_1 & x_2 \\
\hline
\text{Sam}(x_0) & \text{cat}(x_1) & \text{find}(x_0, x_1) \\
\text{Kit}(x_2) & \text{cat}(x_1) & \text{find}(x_2, x_1) \\
\hline
\end{array}
\]

But we now encounter a severe difficulty. By copying the VP conditions of the first sentence over to the second one, we have to use the same discourse marker to represent a cat in the ellipsed VP as we used in the full VP. According to the definition of truth, this means that Sam and Kit found the same cat. In other words, we are rendering (11) equivalent to (12):

(12) Sam found a cat. Kit found it too.

And this of course is not at all what we want. Moreover, once we have adopted a copying analysis of VP ellipsis, it is difficult to see how this problem could be circumvented without radically changing the analysis of indefinites in DR theory.

Superficially, it might seem that the anaphoric approach that I have proposed would founder in the same manner. But first impressions are deceptive. The semantics that we independently require for predicate-DRSs says, in effect, that a structure of the form

\[
\begin{array}{|c|c|}
\hline
[x_0] & x_1 \\
\hline
\text{cat}(x_1) & \text{find}(x_0, x_1) \\
\hline
\end{array}
\]
VP Ellipsis in DR Theory

denotes a function $\varphi$ from individuals to sets of embedding functions such that for any individual $a$, a function $f$ belongs to $\varphi(a)$ only if it verifies $K_{10}$ when it assigns $a$ as the value of $x_0$. (This will be elaborated more formally in the next section.) Since the values assigned to $x_1$ depend on the set of embedding functions associated with a given $a$, we can find a different cat for each $a$ that satisfies $K_{10}$.

4. THE SEMANTICS OF PREDICATE-DRSs

I have not yet made clear how predicate markers are to be assigned values under an embedding function. The first step in rectifying this is to give a new definition of DRSs.

**Definition 6:**

(i) The set $V$ of discourse markers $= Ind \cup Pred$, where $Ind$, $Pred$ are disjoint, nonempty sets. $Ind$ is the set of individual markers and $Pred$ is the set of predicate markers.

(ii) A DRS $K$ is a pair $\langle U.K, Con.K \rangle$, where each element of $U.K$ belongs to $V$, and each condition in $Con.K$ is either an atomic sentence or else an expression of the form $P: [x]K'$, where $P$ is a predicate marker and $[x]K'$ is a predicate-DRS with distinguished marker $x$.

This new kind of condition we have allowed, $P: [x]K'$, says that $P$ is a predicate marker whose value is constrained to be the interpretation of $[x]K'$. In order to spell this out in more detail, we have to be more specific about the semantic value which is to be assigned to predicate-DRSs.

As the name suggests, a predicate-DRS provides us with a means of representing a complex one-place predicate in DR theory. In standard first order logic, a one-place predicate denotes a function from individuals to sentence-denotations, i.e. truth values. The syntactic counterpart to a (non-atomic) sentence in DR theory is a DRS, and so the task of finding an appropriate denotation for a predicate-DRS involves finding an appropriate denotation for an ordinary DRS. The simplest solution would be to identify it with the set of embedding functions which make the DRS true. But this fails to take into account the particular role of discourse markers in the theory. Instead, I shall say that the denotation of a DRS $K$ is a function which, relative to a partial embedding function $f$, yields the set of partial functions which extend $f$ to $U.K$ and which verify $K$.5
Definition 7:
Let $f$ be an embedding function, and $K$ a DRS. Then $\llbracket K \rrbracket^f = \{g: f \subseteq \text{U.K} \land g \models \text{Con}_{[x]K}\}$. Moreover, $K$ is true relative to $f$ iff $\exists g \in \llbracket K \rrbracket^f$.

If we pursue this line of analysis, then the denotation of a predicate-DRS can be construed roughly as a function from individuals to DRS denotations, i.e. sets of embedding functions.

Since predicate markers will be assigned the same kind of values as predicate-DRSs, the notion of an embedding function has to be extended to take this into account. Since, in addition, an arbitrary DRS containing a predicate marker can always be made into a predicate-DRS which is contained within a larger DRS, the definition must be inductive.

Definition 8:
Let $M = \langle A, F \rangle$ be a model with universe $A$ and valuation $F$. The set $G$ of embedding functions into $M$ is defined as follows:
(i) $G^0 = \{f: f$ is a partial function from $\text{Ind}$ into $A\}$
(ii) $G^n = \{f: f$ is a partial function which maps $\text{Ind}$ into $A$ and $\text{Pred}$ into $\{\varphi: A \mapsto \text{pow}(G^{n-1})\}\}$
(iii) $G = \bigcup G^n$

Suppose that we determine the denotation of $[x]K$ relative to some function $f$. Then every $g$ in the value space of $\llbracket [x]K \rrbracket^f$ must be a suitable extension of $f$. By way of example, consider a DRS like $K_{11}$:

$K_{11}$:

<table>
<thead>
<tr>
<th>$x_0$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lee($x_0$)</strong></td>
<td>$Q$:</td>
</tr>
<tr>
<td>$[x_4]$</td>
<td>$x_2$</td>
</tr>
<tr>
<td>cat($x_2$)</td>
<td>of($x_2$, $x_3$)</td>
</tr>
<tr>
<td>$x_3 = x_0$</td>
<td></td>
</tr>
</tbody>
</table>
Let $f$ be a function that verifies $K_{11}$, and let it assign to the predicate-DRS the value $\varphi$. For any $a \in A$, $\varphi(a)$ must be a set of embedding functions which extend $f$ to the universe $\{x_2, x_3\}$ and which verify the conditions in the predicate-DRS when $a$ is the value of $x_4$. But it is not adequate to require that every $g \in \varphi(a)$ be a $\{x_2, x_3\}$-extension of $f$. For $f$ has to assign a value to $Q$, and the value of $Q$ is set identical to the predicate-DRS. If $f(Q) = \varphi$ and every $g \in \varphi$ includes $f$, then $f$ will be contained in its own function space and will not be well-founded. Notice, however, that this problem can be avoided if we impose the syntactic condition that a predicate marker such as $Q$ cannot occur anywhere within the predicate-DRS that determines its value. We can then say that the embedding functions in $f(Q)(a)$ only have to extend $f$ with respect to individual reference markers.

We use $f|X$ to denote the restriction of $f$ to the set $X$.

**Definition 9:**

If $f$ is a partial function, then $f|X$ is that partial function $g \subseteq f$ such that $\text{dom}(g) = \text{dom}(f) \cap X$.

We now modify our definition of $X$-extension to take this restriction into account.

**Definition 10:**

Let $X \subseteq V$ be a set of discourse markers, and let $f$ and $g$ be partial functions on $V$. Then $f \subseteq X \setminus \text{Ind}$ $g$ iff $\text{dom}(g) = \text{dom}(f \setminus \text{Ind}) \cup X$ and $f \setminus \text{Ind} \subseteq g \setminus \text{Ind}$.

The interpretation of a predicate-DRS is defined as follows:

**Definition 11:**

Let $f$ be an embedding function, let $[x]K$ be a predicate-DRS with distinguished variable $x$, and let $G$ be the set of embedding functions. Then $\| [x]K \|^{f}$ is that function $\varphi : A \mapsto \text{pow}(G)$ such that for any $a \in A$, $\varphi(a) = \{g : f \subseteq_{U,[x]K} \text{Ind} g \& g[a/x] = \text{Con.}[x]K\}$

Expressed in English, the extension of predicate DRS $[x]K$, relative to $f$, is a function $\varphi$ with domain $A$ that assigns to each argument $a$ the set of embedding functions $g$ which are $U,[x]K \setminus \text{Ind}$-extensions of $f$ and which verify all the conditions in $[x]K$ when they assign $a$ as the value of $x$.

Finally, we spell out the way in which conditions of the form ‘$P : [x]K$’ and ‘$P(x_i)$’ are interpreted.

**Definition 12:**

If $P$ is a predicate marker and $[x]K$ is predicate-DRS, then $f \models P : [x]K$ iff $f(P) = \| [x]K \|^{f}$. 
Definition 13:
If $P$ is a predicate marker and $x_i$ is a reference marker, then $f = P(x_i)$ iff $\exists g \in f(P(f(x_i)))$.

5. QUANTIFICATION

At this point, it is pertinent to see how our proposal meshes in with another important use of subDRSs, namely in the representation of universal quantification. Consider, for example, the standard DR analysis of (13).

(13) Every boy who owns a cat loves it.

\[ K_{12}: \]
\[
\begin{array}{c|c|c}
  x_0 & x_1 & x_2 \\
\hline
  \text{boy}(x_0) & \text{cat}(x_1) & \text{own}(x_0, x_1) \\
\end{array}
\Rightarrow
\begin{array}{c}
  \text{loves}(x_0, x_2) \\
  x_2 = x_1 \\
\end{array}
\]

Suppose that we wished to give a compositional interpretation to $K_{12}$. This would require assigning an interpretation to each subDRS, and interpreting $\Rightarrow$ as a binary relation on DRSs. If the subDRSs are to be taken as expressing properties, we arrive at an analysis which is strikingly reminiscent of the work on generalized quantifiers which treats determiners as binary relations on sets. On such an approach, every denotes the relation $D$ such that for any sets $A, B$, $D(A, B)$ is true if $A \subseteq B$. This analogy can be made more explicit if we modify $K_{12}$ to incorporate predicate-DRSs as follows.

\[ K_{13}: \]
\[
\begin{array}{c|c|c}
  [x_0] & x_1 & [x_2] \\
\hline
  \text{boy}(x_0) & \text{cat}(x_1) & \text{loves}(x_0, x_2) \\
  \text{own}(x_0, x_1) & & x_3 = x_1 \\
\end{array}
\Rightarrow
\]

-
In the light of the preceding section, we might define the interpretation of $\Rightarrow$ along the following lines:

\begin{equation}
(14) \text{Let } [x_0]K, [x_1]K' \text{ be predicate-DRSs. Then}
\end{equation}

\begin{equation*}
f : \Rightarrow ([x_0]K, [x_1]K') \iff \text{for all } a \in A, \text{ for all } g, \text{ if}
\end{equation*}

\begin{equation*}
g \in \mathcal{L}[x_0]K \mathcal{L}[x_1]K'(a) \text{ then there is an } h \text{ such that } h \in \mathcal{L}[x_1]K' \mathcal{L}(a).
\end{equation*}

Suppose that for a given individual $a$, there is some function $g$ in $\mathcal{L}[x_0]K \mathcal{L}(a)$. Then by (14) and Definition 11, there must be a corresponding function $h$ such that $h$ is an extension of $g$ to the universe of $[x_1]K'$ and such that $h[a/x_1]$ verifies $[x_1]K'$. To see that this gives the same results in the case of $K_{13}$ as Kamp's definition, consider the following model:

\begin{equation}
(15) \begin{align*}
F(&\text{boy}) = \{ \text{Lee} \} \\
F(&\text{cat}) = \{ \text{Tom, Felix} \} \\
F(&\text{own}) = \{ \langle \text{Lee, Tom} \rangle, \langle \text{Lee, Felix} \rangle \} \\
F(&\text{love}) = \{ \langle \text{Lee, Tom} \rangle, \langle \text{Tom, Felix} \rangle \}
\end{align*}
\end{equation}

The relevant reading of (13) is one on which every boy loves every cat that he owns. Thus we would expect it to be false in this model, since there is one cat owned by Lee which he fails to love, namely Felix. To see how things work out, let `$[x_0]Nom$' represent the first DRS argument of $\Rightarrow$ in $K_{14}$, and let `$[x_2]VP$' represent the second argument. Then their extensions are given in (16a) and (16b), respectively.

\begin{equation}
(16) \begin{align*}
a. \mathcal{L}[x_0]Nom & = \{ \langle \text{Lee, } \{ g, g' \} \rangle, \\
& \langle \text{Tom, } \emptyset \rangle, \\
& \langle \text{Felix, } \emptyset \rangle \} \\
b. \mathcal{L}[x_2]VP & = \{ \langle \text{Lee, } h \rangle \}, \\
& \langle \text{Tom, } \emptyset \rangle, \\
& \langle \text{Felix, } \emptyset \rangle \}
\end{align*}
\end{equation}

\begin{equation*}
\mathcal{L}[x_2]VP' = \{ \langle \text{Lee, } \emptyset \rangle, \\
& \langle \text{Tom, } \{ h' \} \rangle \\
& \langle \text{Felix, } \emptyset \rangle \}
\end{equation*}

where

\begin{equation*}
f = \emptyset \\
g = \{ \langle x_0, \text{Lee} \rangle, \langle x_1, \text{Tom} \rangle \} \\
g' = \{ \langle x_0, \text{Lee} \rangle, \langle x_1, \text{Felix} \rangle \} \\
h = \{ \langle x_0, \text{Lee} \rangle, \langle x_1, \text{Tom} \rangle, \langle x_2, \text{Lee} \rangle, \langle x_3, \text{Tom} \rangle \} \\
h' = \{ \langle x_0, \text{Lee} \rangle, \langle x_1, \text{Felix} \rangle, \langle x_2, \text{Tom} \rangle, \langle x_3, \text{Felix} \rangle \}
\end{equation*}
According to the interpretation of $\Rightarrow$ given in (14) above, $K_{13}$ is true under $f$ just in case whenever a function $g \in \llbracket x_0 \rrbracket \text{Nom} \llbracket (a) \rrbracket$, for any $a \in A$, there is also a function $h \in \llbracket x_2 \rrbracket \text{VP} \llbracket (a) \rrbracket$. But this fails. For although the function $g$ in $\llbracket x_0 \rrbracket \text{Nom} \llbracket (\text{Lee}) \rrbracket$ has a corresponding function $h$ in $\llbracket x_2 \rrbracket \text{VP} \llbracket (\text{Lee}) \rrbracket$, there is no function in $\llbracket x_2 \rrbracket \text{VP} \llbracket (\text{Lee}) \rrbracket$ which corresponds to $g'$ in $\llbracket x_0 \rrbracket \text{Nom} \llbracket (\text{Lee}) \rrbracket$. In fact, $\llbracket x_2 \rrbracket \text{VP} \llbracket (\text{Lee}) \rrbracket = \emptyset$.

Our general strategy in constructing a DRS will be to associate a subDRS with each constituent in a sentence. Each subDRS will be labelled by a predicate marker, and it is convenient for expository reasons to let category labels such as 'Nom' and 'VP' be aliases for $P_1$, $P_2$, etc. In this way, we can make explicit what the syntactic origin of each subDRS is. $K_{14}$ illustrates a DRS for (17).

(17) Every boy loves his cat.

$K_{14}$:

\[
\Rightarrow (\text{Nom, VP})
\]

\textbf{Nom:}

\[
[x_0] \\
\text{boy}(x_0)
\]

\textbf{VP:}

\[
[x_2] x_1 x_3 \\
\text{cat}(x_1) \\
\text{of}(x_1, x_3) \\
\text{love}(x_2, x_1) \\
x_3 = x_2
\]

Since $\Rightarrow$ takes predicate markers as its arguments on this approach, we need to slightly modify its interpretation rule. This ensures that the embedding conditions for (17) are exactly the same as if we had directly used the subDRSs as arguments of the determiner.

\textit{Definition 14:}

\[ f \models \Rightarrow (P_1, P_2) \text{ iff for every } a \in A, \text{ for every } g, \text{ if } g \in f(P_1)(a) \]

\[ \text{then there is an } h \text{ such that } h \in g(P_2)(a). \]
On Kamp's approach, there are two differences between the treatment of *every* and *a*. The former triggers the introduction of two subDRSs, while the latter does not. And the latter triggers the introduction of a discourse marker, while the former does not. However, if subDRSs are associated with subconstituents in a sentence in the manner I have suggested, the first difference disappears. That is, suppose we are dealing with a syntactic structure of the following form.

\[
(18)
S
\]

\[
\begin{array}{c}
NP \\
\text{Det} \\
\text{Nom} \\
\end{array}
\]

\[
\text{VP}
\]

In order to obtain some generality in the DRS construction rules, I shall suppose that the subDRS associated with the Det constituent always involves a condition which contains a connective and two predicate-markers which are subsequently linked to the predicate-DRSs for Nom and VP. As we saw, this condition is \((P_1, P_2)\) in the case of the determiner *every*. The corresponding condition for *a* will be of the form \(\land (P_1(x), P_2(x))\), where \(\land\) is a connective corresponding to conjunction, and \(x\) is a new reference marker whose introduction into the appropriate universe is licensed by the indefinite. The DRS \(K_{15}\) illustrates the resulting structure:

\[(19)\] A boy loves his cat.

\[
K_{15}:
\]

\[
\begin{array}{c}
x_4 \\
\land (\text{Nom}(x_4), \text{VP}(x_4)) \\
& [x_0] \\
& \text{boy}(x_0) \\
\end{array}
\]

\[
\begin{array}{c}
& [x_2] x_1 x_3 \\
& \text{cat}(x_1) \\
& \text{off}(x_1, x_3) \\
& \text{love}(x_2, x_1) \\
& x_3 = x_2
\end{array}
\]
The connective $\land$ has an interpretation which is analogous to the one proposed for the conditional connective $\rightarrow$. The main difference is we wish to admit conditions like $P(x_1)$ as arguments of the relation. In the circumstances, it seems reasonable to first extend our notion of denotation so as to encompass arbitrary conditions within a DRS as well as a DRS itself.

**Definition 15:**

Let $S$ be a condition. Then $\ll S \gg = \{ g : \text{dom}(g) = \text{dom}(f) \text{ and } g \models S \}$

**Definition 16:**

Let $S$ and $T$ be conditions. Then $f \models \land(S, T)$ iff there is a $g \in \ll S \gg$ and there is an $h \in \ll T \gg$.

It should be fairly easy to formulate an inference rule of $\land$-elimination which would render $K_{15}$ equivalent to a more familiar DRS of the following sort:

$$K_{16}:$$

```
x_1  x_3  x_4

boy(x_4)  
cat(x_1)  
of(x_1, x_3)  
love(x_4, x_1)  

x_3 = x_4
```
than coreferring terms seems to disappear. Consider, for example, a ‘standard’ DRS for (17), namely $K_{17}$:

$$K_{17}:
\begin{array}{c}
\text{boy}(x_0) \\
\hline
x_0 \\
\end{array} \\
= \\
\begin{array}{c}
\text{cat}(x_1) \\
\text{of}(x_1, x_2) \\
\text{loves}(x_0, x_1) \\
x_2 = x_0
\end{array}$$

The pronominal *his*, which we might expect to be interpreted as a bound variable, receives a representation which is uniform with that accorded to other pronouns. That is, it licenses the introduction of a new discourse marker $x_2$, together with a condition that links $x_2$ to an accessible marker that was introduced at a prior stage in the construction. In this particular case, the relevant marker, $x_0$, occurs in the subDRS that represents the nominal sister of the determiner *every*, but it is difficult to relate this fact to the presence of a c-command binding relation between quantifier and pronoun.

By contrast, on the approach we are adopting, an analogue of the binding interpretation emerges rather naturally. Consider again the DRS $K_{15}$, which is our version of $K_{16}$. What seems to be relevant is that the reference marker $x_3$ that corresponds to *his* is linked to the distinguished marker $[x_2]$ which co-occurs in the universe of VP. This suggests the following idea:

(20) A reference marker $x_i$ occurring in the universe of a predicate-DRS $[x_j]K$ is *bound* just in case $[x_j]K$ contains the condition $x_i = x_j$.

Such a definition may be easier to motivate if we recall the standard account of quantifier binding in Montague grammar. In order for a variable $x_i$ occurring in an expression $\beta$ to be bound by a term $\alpha$, we must employ a quantification rule with index $i$. The accompanying semantic rule yields $\alpha' (\lambda x_i \beta')$, i.e. it applies the translation of $\alpha$ to the result of abstracting over $x_i$ in the translation of $\beta$. In effect, the pronouns bound by $\alpha$ are all those which translate as $x_i$ and which are seman-
tically bound by $\lambda x_1$. Thus example (19), *A boy loves his cat*, would have a Montague translation roughly along the lines of (21), where the variable $x_0$ corresponds to a pronoun bound by *a boy*.

(21)  $(a'(\text{boy'}))(\lambda x_0[\text{love' } [x_0\text{'s-cat'}](x_0)]$

A further point of interest is that although $K_{15}$ assigns the pronominal *his* a bound reading, there is also a coreference reading which is truth-conditionally equivalent. To get this, we simply replace the condition $x_3 = x_2$ in VP by $x_3 = x_4$. A consequence of this fact is that indefinite NPs should be like definites in giving rise to the strict identity reading in VP ellipsis. That this prediction is indeed correct can be seen more clearly in an example like (22).\(^{10}\)

(22)  *A boy stroked his cat and my friend did too.*

By contrast, where an anaphoric pronoun can only receive a bound interpretation, only the sloppy interpretation is available:

(23)  *Every boy stroked his cat and my friend did too.*

The correlation between the presence of both a bound reading and a coreference reading for a pronoun with the presence of both a sloppy and strict identity interpretation in an associated VP ellipsis has long been established in the semantics literature.\(^{11}\) It is striking that once predicate-DRSs are introduced, this correlation falls out in such a simple way.

At first sight, the generalized quantifier approach to NP interpretations seemed disturbingly remote from the approach adopted in DR theory. What I have tried to show is that we can have our cake and eat it too: assigning NPs to essentially the same semantic type is not incompatible with drawing a distinction between those NPs which trigger the introduction of a discourse marker, and hence give rise to discourse anaphora, and those NPs which do not.

6. SUMMARY

The main goal of this paper has been to lay the groundwork for an approach to DRS construction which would be compatible with existing approaches to semantic translation in extended Montague grammar. As a subgoal of this enterprise, I sketched a possible analysis of VP ellipsis in DR theory, on the assumption that this would give us some insight into an appropriate notion of VP interpretation. Starting from the assumption that the relation
between the ellipsed VP and its antecedent should be treated as special case of anaphora within DR theory, I introduced the construct ‘predicate DRS’ to serve as the representation of a VP and defined a model theoretic semantics for it. I then showed that this allowed us to give an account of the sloppy/strict ambiguity analogous to one using lambda abstracts.

I suggested that predicate DRSs could also be deployed in the analysis of quantification, that is, by treating them as arguments in the representation of the determiners *every* and *a*. Such a relational representation brought the DR account of determiners into line with the large body of work on generalized quantifiers, while retaining the advantages of Kamp’s analysis of donkey sentences. A further advantage was that we could establish a link between the sloppy/strict distinction and the bound anaphora/discourse anaphora distinction, as proposed by Reinhart. Indefinite NPs were assimilated to the pattern of generalized quantifiers, yet also triggered the introduction of a discourse marker in the usual fashion. This captured the fact that, like proper names, but unlike universal NPs, indefinites also give rise to the strict/sloppy distinction in VP ellipsis.

NOTES

3. After completing the main substance of this paper, I became aware that a study of VP ellipsis within DR theory has also been carried out by van Eijck (1985). The points of agreement between the two treatments probably outweigh the differences.
4. Roberts (1984) briefly proposes such an analysis, which she attributes to Hans Kamp.
5. This is very similar to the proposal in Zeevat (1984), according to which the denotation \( \mathcal{K} \) of a DRS K with universe U.K. is a pair \( \langle U.K, G \rangle \), where G is the set of (total) embedding functions which verify K. Given the denotation I have suggested, Zeevat’s structure is definable as \( \langle \{ x: \forall f \in \mathcal{K}^{\mathbb{U}}[x \in \text{dom}(f)] \}, \{ g: \exists f \in \mathcal{K}^{\mathbb{U}}[f \subseteq g] \rangle \) (where \( \mathbb{U} \) is the totally undefined embedding function).
6. See, for example, Barwise and Cooper (1981), and van Benthem (1983).
7. It should be briefly mentioned at this point that although we have treated \( \Rightarrow \) as a relation on predicate-DRSs, minor modifications suffice to also interpret it as a relation on ordinary DRSs.
8. Partee has consistently emphasized the importance of this issue, and one of the earliest discussions is Partee (1970).
9. This point has also been made by van Eijck (1985) in the same connection as the present
one; viz. accounting for the sloppy/strict ambiguity in a DRS approach to VP ellipsis.

10. The difficulty with a present tense example like *A boy strokes his cat* is that it invites a generic interpretation, something which I do not wish to consider at present.


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