Chapter 2 The generalisation problem

We will now narrow our objectives and use the framework constructed in the previous chapter to formalise the problem of finding generalisations from experience. This problem is both the simplest and the most common illustration of non-deductive reasoning. Indeed, so central is it, that some philosophers even use the word induction to mean generalisation. Here is an archetypal piece of generalisation:

The sun rose yesterday
The sun rose today
___
The sun rises every day

Here is another:

This crow is black
That crow is black
___
Every crow is black

Some more complicated examples can be found in chapters 3, 4 and 5.

To follow the prescription of chapter 1, we must fill in several parameters.

First we must settle the hypothesis space. This has already been required to be some subset of the set of universal sentences. We will simply require that it is the set of universal sentences.

Next, we must settle the set of phenomena, and the circumstances of their occurrence, $\{e_i \Rightarrow f_i | i=1,n\}$ where $e_i = Ev(f_i)$ $(i=1,n)$ for a certain function Ev. Each phenomenon must be an experience. Let us say, as seems reasonable, that an experience is a fact experienced in certain circumstances, described by another set of facts. This is consistent with our decision to use => as indicating an experiment, if we regard an experience as an unintentional experiment. So f_i is a fact and e_i is a, supposedly finite, conjunction of facts. We assume that every fact can be described by a ground literal. This fits in well with the view that there are basic observational predicates and facts are what are observed. One might make two accusations of unnaturalness. First most facts stated in English are stated in a positive way, and this is usually possible. Thus consider the verbal exchange:

John: "The tap is on"

Mary: "No, it's off."

But here there is a piece of inbuilt knowledge, namely $\forall x\,(On(x) \equiv \neg Off(x))$. We wish to have a theory which includes the use of no knowledge, when one would want negations.

Secondly, it seems a little odd to allow function symbols other than constants. They do seem to arise occasionally in English in possessive phrases. "John's mother is beautiful" might be rendered as Beautiful(Mother(John)). Since they perhaps ought to be allowed in everyday use and since they certainly ought to be allowed for mathematical facts and are necessary if actions are regarded as functions (usually from situations to situations), we will allow them. At any rate the theory can be developed just as well for the general as the particular cases. It will, however, make a rather spectacular difference later on. If function symbols other than constants are allowed, then one version of the generalisation problem is unsolvable, while if they are not allowed, it is solvable.

Thus the phenomena are given by a set of facts, $f = \{f_i | i=1,n\}$ and a function **Ev** from f to finite conjunctions of facts. So far we have adopted the standard interpretation of => as an experiment, which justifies the selection of the hypothesis space. In any application alternative restrictions may be made on => which may induce other possible restrictions on any of the parameters: the hypothesis space, the set of possible phenomena, the method of explanation or even, perhaps, the niceness criterion.

We must remark that there are some problems whose solution we assume to have taken place. The set f should be a reasonable set for generalising from. **Ev** should choose the (or a superset of the) relevant or correct facts which describe where, when and/or why f_i took place. Such problems would be faced in formalizing the entire process of theory formulation. We do not attempt their formalization here, although one might suspect that if $Ev(e_i)$ is relevant to f_i then every term in f_i will contain a term occurring in $Ev(e_i)$. Perhaps the set of atoms in $Ev(e_i)$ and f_i would form a connected set under the relation of having a common subterm. This weak requirement will not be assumed, however.

We believe that an event is a fact. This allows a widening of the range of possible interpretations of =>. We make the notational convention that $\bar{e}_i$ is the clause $\{\bar{L} | L$ is one of the literals conjoined to form $e_i\}$.

No conditions are assumed for Th or Irr at the moment. We will not say how k ought to be split up into two parts, except that Irr may contain an account of the failure of certain experiments. That is, it may contain conjunctions of literals of the form $e \wedge \bar{f}$ where e is a conjunction of ground literals explaining some experimental set-up and $\bar{f}$ **is a literal which expresses** the fact that the expected outcome, f, did not occur. Irr will contain such "failures" when we do not wish to explain why the failure occurred, nor use the failure to explain the successes, but merely wish to find an explanation consistent with the failure.

We will be particularly interested in the case when Th is empty, although we will also consider, in less detail, some other cases. However, much of the theory can, and will, be formulated in general.

There are some restrictions that must be placed on Th, Ev and f, if the phenomena are to admit any explanations.

<u>1</u> For every i, $e_i \rightarrow f_i$ must not be deducible from Th.

<u>2</u> Th $\wedge \bigwedge_{i=1}^{n} (e_i \wedge f_i)$ must be consistent.

We must now capture a suitable notion of generalisation in order to formalise the intended type of explanation. The word generalisation is used in many contexts both formally and informally. It is not clear that there is a common intuition behind these uses. For example $\forall x P(x)$ and $\exists x P(x)$ are, respectively called universal and existential generalisations of $P(a)$. Yet only the former could qualify as an inductive generalisation.

Generalisation from experience seems to occur in two stages. First a relevant part of experience is selected and then generalised by universal generalisation, that is, the replacement of constant terms by universally quantified variables. To reverse this, an experience is explained by a generalisation if it follows logically from an instance of it. It is this sense of generalisation for which we shall attempt a formal parallel.

In our case, an experience is typically an experiment described by $e_i \Rightarrow f_i$ for some i. We assert that the process of selecting a relevant part of it may be divided up into two parts; one of selection and one of rewording.

In a selection, some of the circumstances are regarded as irrelevant. That is, $e'_i \Rightarrow f'_i$ is a selection from $e_i \Rightarrow f_i$ if e' is obtained from e by removing some of its conjuncts.

In a rewording, the experience is redescribed in different terms, possibly using Th. This is how knowledge interacts with generalisation.

Formally, $e_i' \Rightarrow f_i'$ is a <u>rewording</u> of $e_i \Rightarrow f_i$ if $\vdash_{Th} e_i' \equiv e_i$ and $\vdash_{Th} f_i' \equiv f_i$, e_i' is a conjunction of ground literals and f_i' is a ground literal.

Finally, $e_i' \Rightarrow f_i'$ is a <u>part</u> of $e_i \Rightarrow f_i$ if it can be obtained from $e_i \Rightarrow f_i$ by a series of applications of selection and rewording to $e_i \Rightarrow f_i$. Notice that in this case, $\vdash_{Th} f_i' \equiv f_i$. Therefore $e_i' \rightarrow f_i'$ is true, as f_i is. However it may be no longer possible to justify the assertion that $e_i' \Rightarrow f_i'$ as some selection may have removed a relevant part of the circumstances.

The second stage is the replacement of constants by universally quantified variables. We say, therefore, that $\forall(e_i'' \Rightarrow f_i'')$ is a <u>generalisation from</u> $e_i \Rightarrow f_i$ iff for some σ and $e_i' \Rightarrow f_i'$, which is a part of $e_i \Rightarrow f_i$, $e_i''\sigma = e_i$ and $f_i''\sigma = f_i'$. ($\forall(e_i'' \Rightarrow f_i'')$ abbreviates $\forall x_1 \ldots x_n(e_i'' \Rightarrow f_i'')$, where $x_1 \ldots x_n$ are all the variables in $e_i'' \Rightarrow f_i''$.) Here, $\forall(e_i'' \Rightarrow f_i'')$ is said to be a <u>generalisation of</u> $e_i' \Rightarrow f_i'$.

To give a good meaning to $\forall(e_i'' \Rightarrow f_i'')$ would take a longer excursion than is practical. One would have to reread => as a counterfactual (Goodman, 1965, Tredwell, 1965). For example $\forall x$(Aristotle(x) => Speaksgreek(x)) would mean that anyone who was Aristotle could speak Greek.

Let us illustrate the above by an informal example in which => means mo more than material implication and therefore causes no trouble. Suppose we observe that 'some crow in Stonehaven is black' and that 'some other crow in Dunoon is also black'. We may decide that the place of observation is irrelevant and that the essential observations are that both crows are black. Now if we know, (from Th) that "craw" is a Scottish word meaning "crow" we may reword the observations, so noting that two craws were black. From this we conclude by a stage of "generalisation of" that all craws are black. We can even, by a final rewording see that all crows are black.

At this point, there occurs an important transition. Rather than dealing with assertions of the form e => f (where e is a conjunction of ground literals, and f is a ground literal) we consider the corresponding clause $\bar{e} \cup \{f\}$. The above definition of generalisation will be mirrored by an analogous one for clauses which will allow a mathematical theory to be developed using reasonably well-known ideas. This theory makes no use of => and consequently some rough justice is dealt to => in the transition.

As a matter of fact, we could have continued to use =>. But we feel that there would be a mismatch between our informal sign, =>, and the increasingly formal nature of the rest of the work. A more formal treatment, perhaps using modal logic, would result in a better-knit theory.

The clauses $C_i = \bar{e}_i \cup \{f_i\}$ are particularly important. We set $H_0 = \{C_i | i=1,n\}$. There are some conventions. A clause, C, abbreviates a formula which is a disjunction of all its members. A set of clauses, H, abbreviates a conjunction of all its members. $\forall C$ is the universal closure of the formula C abbreviates. $\forall H$ is the universal closure of the formula H abbreviates.

A clause, C, is a <u>selection</u> from D iff $C \subseteq D$. If D corresponds to $e_i' \Rightarrow f_i'$ which is a part of some $e_i \Rightarrow f_i$, and C contains f_i' this corresponds to the above definition of selection. We allow C not to contain f_i since this will make rather smoother going, formally. It will not alter the class of chosen hypotheses since C must then be inconsistent with $\bigwedge_{i=1}^{n}(e_i \wedge f_i)$ and so will not be chosen by any of our induction methods. For the same reason the reader will see that neither will any rewording or generalisation of C or indeed any clause reached by continued application of the generalisation operations to C be chosen.

The clauses C and D are said to be <u>rewordings relative to Th</u>, iff $\vdash_{Th} \forall x_1 \ldots\ldots x_n (C \equiv D)$ where $x_1 \ldots\ldots x_n$ are the variables in C or in D. This is equivalent to $\vdash_{Th} C \equiv D$.

Even when C and D are ground, this need not correspond to the previous definition of a rewording. This is the only place where real injustice is done to =>. However, when Th is empty, $\vdash_{Th} C \equiv D$ iff $C = D$ and then the two definitions correspond properly. The reader may however verify, at the appropriate points in chapter five that for the various types of Th investigated in any detail, the two definitions do in fact correspond.

The clause C is a <u>generalisation of</u> D iff, for some substitution, σ, $C\sigma = D$. This does not quite correspond to the previous definition. For when D corresponds to $e_i' \Rightarrow f_i'$, which is a part of some $e_i \Rightarrow f_i$, then it may be that $C = \overline{e_i''} \cup C'$ where C' is not a singleton. However the correspondence could be straightened out by weakening the demand, implicit in the above definition of a "generalisation from", that f_i'' be a literal. This would seem to be harmless.

There seem to be several possible definitions of "generalisation from". A weak one, which seems to be a special case of the corresponding definition is:

C is a <u>generalisation from</u> C_i relative to Th, iff there are clauses E and F and a substitution σ such that $C\sigma = E$, $E \subseteq F$, F is ground and $\vdash_{Th} F \equiv C_i$.

We may certainly drop the restriction that F is ground, for suppose F has the variables $x_1 \ldots\ldots x_n$ and let $\mu = \{a()/x_1, \ldots\ldots, a()/x_n\}$. Then we have $C\sigma\mu = E\mu$, $E\mu \subseteq F\mu$, $F\mu$ is ground and $\vdash_{Th} F\mu \equiv C_i$ since $C_i\mu = C_i$, (C_i is ground). Therefore this definition is a special case of the more general definition:

C is a <u>generalisation from</u> D <u>relative to Th</u> iff for some E and σ, $C\sigma \subseteq E$ and $\vdash_{Th} E \equiv D$.

There is a much stronger general definition: C is a <u>generalisation from</u> D <u>relative to Th</u> iff there are $D_j (j=1,m)$ such that $C = D_1$, $D = D_m$ and $D_j \subseteq D_{j+1}$ or $D_j\sigma = D_{j+1}$ for some σ or $\vdash_{Th} D_j \equiv D_{j+1}$ (for $j=1,m-1$).

This turns out to be equivalent to the weaker definition. We show this by induction on m. Suppose m=1. Then as $C\varepsilon \subseteq C$ and $\vdash_{Th} C \equiv C$, the condition is satisfied. Suppose j>1, then by induction there is a σ and an E such that $D_2\sigma \subseteq E$ and $\vdash_{Th} E \equiv D$. Suppose $C\mu = D_2$ for some μ. Then $C\mu\sigma \subseteq E$ and so the condition is satisfied for C and D. Suppose $C \subseteq D_2$, then $C\sigma \subseteq D_2\sigma \subseteq E$ and the condition is satisfied in this case too. Suppose, finally, that $\vdash_{Th} C \equiv D_2$. Then $\vdash_{Th} C\sigma \equiv D_2\sigma$ and so $\vdash_{Th} C\sigma \cup E \equiv D_2\sigma \cup E$. But since $D_2\sigma \subseteq E$, $\vdash_{Th} C\sigma \cup E \equiv E$. Now, as $\vdash_{Th} E \equiv D$, $\vdash_{Th} C\sigma \cup E \equiv D$ and we see in this last case that the condition is satisfied.

We are thereby justified in adopting the simpler condition as our definition. It is worth introducing some extra symbolism. By $C \leq D$ (Th) (read C generalises D relative to Th), we mean that C and D satisfy the above condition.

We can now forget the notions of selection and rewording and the distinction between "generalisation from" and "generalisation of". All these notions served only to help establish our notion of relative generalisation.

As an example, let us formalise the crows. We can take $f = \{\text{Black(crow1)}, \text{Black(crow2)}\}$, Ev is given by:

Ev(Black(crow1)) = Crow(crow1) $\wedge$ Place(crow1,Stonehaven)

Ev(Black(crow2)) = Crow(crow2) $\wedge$ Place(crow2,Dunoon).

We may suppose that Th includes the statement:

$\forall x(\text{Crow}(x) \equiv \text{Craw}(x))$.

Then $\{\neg \text{Craw}(x), \text{Black}(x)\} \leq \{\neg \text{Crow}(\text{crow1}),$
$\text{Place}(\text{crow1}, \text{Stonehaven}), \text{Black}(\text{crow1})\}$ (Th).

When Th is empty, then $\vdash_{Th} C \equiv D$ iff either C = D or both C and D are tautologies. Then $C \leq D$ ($\emptyset$) iff there is a σ such that $C\sigma \subseteq D$ or D is a tautology. For if D is a tautology then $\vdash_{Th} C\sigma \cup D \equiv D$, no matter what σ is. Let us write $C \leq D$ (read C generalises D) iff there is a σ such that $C\sigma \subseteq D$. (In the literature, this relation is called subsumption.) Therefore $C \leq D$ is not identical to $C \leq D$ ($\emptyset$). Practically, however, there is no difference since assumption 1 (that no f_i follows from $Ev(f_i)$) ensures that no C_i is a tautology.

Rather than assume that one generalisation can consistently explain all the phenomena, we look for a set of generalisations which do. That is we expect to find a set, H, of clauses such that for every C_i there is a C in H so that $C \leq C_i$ (Th). This allows for the possibility that several distinct classes of phenomena have been bunched together in f by mistake. We adopt the following notation: $H_1 \leq H_2$ (Th) (read H_1 generalises H_2 relative to Th) iff for every C_2 in H_2 there is a C_1 in H_1 such that $C_1 \leq C_2$ (Th).

$H_1 \leq H_2$ (read as H_1 generalises H_2) iff for every C_2 in H_2 there is a C_1 in H_1 such that $C_1 \leq C_2$.

We may now define the notion of explanation which will be used.

Recall that $C_i = \bar{e}_i \cup \{f_i\}$ for $i=1,n$ and that $H_0 = \{C_i | i=1,n\}$.

$\forall H$ explains $\{e_i \Rightarrow f_i | i=1,n\}$ iff

1) For some C in H, $C \leq C_i$ (Th), given any i between 1 and n.
2) $\forall H \wedge Th \wedge Irr \wedge \bigwedge_{i=1}^{n} (e_i \wedge f_i)$ is consistent.

We do not need a condition corresponding to **E2** of the previous chapter, since it has already been assumed. Condition **E4** is also redundant since we have shown that $\forall H$ is lawlike. This discussion of **E4** only applies when => represents an experiment.

We can write the conditions that $\forall H$ explains every phenomenon, as

1) $H \leq H_0$ (Th)
2) $H \wedge Th \wedge Irr \wedge \bigwedge_{i=1}^{n} (e_i \wedge f_i)$ is consistent.

Only one thing is now left unspecified, the niceness relation, $\prec$. Several different niceness relations will be considered most of which are constructed from quite simple syntactic measures. However most attention will be paid to one in particular, $\prec$cpg. Others will be considered, particularly when we wish to show in chapter 4 that an unsolvability result is largely independent of the choice of $\prec$; some philosophical discussion in chapter 5 will also use a different niceness relation from $\prec$cpg.

In order to 'mix' quasi-orderings, (that is, reflexive and transitive binary relations) we define the <u>lexicographic product</u> $\precsim \circ \precsim'$ of two quasi-orderings $\precsim$ and $\precsim'$, thus

$H_1 \precsim \circ \precsim' H_2$ iff either $H_1 \precsim H_2$ and $H_2 \not\precsim H_1$ or else $H_1 \precsim H_2$, $H_2 \precsim H_1$ and $H_1 \precsim' H_2$.

Thus to decide which is nicer, the lexicographic product, $\precsim \circ \precsim'$ first consults $\precsim$ and if that gives no definite decision, tries $\precsim'$. We also say that $\precsim \circ \precsim'$ is a <u>lexicographic refinement</u> of $\precsim$.

<u>Lemma 1</u> The lexicographic product $\precsim \circ \precsim'$ is itself a quasi-ordering. If $\precsim$ and $\precsim'$ are linear, so is $\precsim'$. The lexicographic product is associative and idempotent.

<u>Proof</u> Reflexivity is obvious. Suppose $H_1 \precsim \circ \precsim' H_2$ and $H_2 \precsim \circ \precsim' H_3$. If $H_1 \precsim H_2$ and $H_2 \not\precsim H_1$ then $H_1 \precsim H_3$ and $H_3 \not\precsim H_1$, since $\precsim$ is transitive. It follows, in this case, that $H_1 \precsim \circ \precsim' H_2$. Suppose $H_1 \precsim H_2$, $H_2 \precsim H_1$, $H_2 \precsim H_3$ and $H_3 \not\precsim H_2$. Then $H_3 \not\precsim H_1$ and so here, too, $H_1 \precsim \circ \precsim' H_3$. The only other case is when $H_1 \precsim H_2$, $H_2 \precsim H_1$, $H_1 \precsim' H_2$, $H_2 \precsim H_3$, $H_3 \precsim H_2$ and $H_2 \precsim' H_3$. In this case, as $\precsim'$ is transitive, $H_1 \precsim H_3$, $H_3 \precsim H_1$ $\precsim' H_3$ and therefore $H_1 \precsim \circ \precsim' H_3$.

Suppose $\precsim$ and $\precsim'$ are linear. Suppose that $H_1 \precsim \circ \precsim' H_2$ is false. Then firstly $H_1 \not\precsim H_2$ or $H_2 \precsim H_1$, and secondly either $H_1 \not\precsim H_2$ or $H_2 \not\precsim H_1$ or $H_1 \precsim' H_2$. Therefore, as $\precsim$ is linear $H_2 \precsim H_1$. If $H_1 \not\precsim H_2$ then $H_2 \precsim \circ \precsim' H_1$. If $H_1 \precsim H_2$ then $H_1 \not\precsim' H_2$. Therefore, as $\precsim'$ is linear, $H_2 \precsim' H_1$. So in this case, too, $H_2 \precsim \circ \precsim' H_1$ and we see that $\precsim \circ \precsim'$ is linear.

In proving associativity, the fact that $H_1 \not\precsim_1 H_2$ implies the falsity of $H_1 \precsim_1 \circ \precsim_2 H_2$, is useful. Suppose $H_1 \not\precsim H_2$. Then $H_1 \precsim \circ (\precsim' \circ \precsim'') H_2$ cannot hold, nor can $H_1 \precsim \circ \precsim' H_2$ and so neither can $H_1 (\precsim \circ \precsim') \circ \precsim'' H_2$. Suppose $H_1 \precsim H_2$ and $H_2 \not\precsim H_1$. Then $H_1 \precsim \circ (\precsim' \circ \precsim'') H_2$. Also $H_1 \precsim \circ \precsim' H_2$ holds and $H_2 \precsim \circ \precsim' H_1$ does not hold. Therefore $H_1 (\precsim \circ \precsim') \circ \precsim'' H_2$ holds.

Suppose $H_1 \precsim H_2$ and $H_2 \precsim H_1$. Then $H_1 \precsim \circ (\precsim' \circ \precsim'') H_2$ holds iff $H_1 \precsim' \circ \precsim'' H_2$ holds. Similarly $H_1 \precsim \circ \precsim H_2$ holds iff $H_1 \precsim' H_2$ holds and $H_2 \precsim \circ \precsim' H_1$ holds iff $H_2 \precsim' H_1$ holds. Therefore $H_1 (\precsim \circ \precsim') \circ \precsim''$ holds iff $H_1 \precsim' \circ \precsim'' H_2$ holds iff $H_1 \precsim \circ (\precsim' \circ \precsim'') H_2$ holds. In all cases we see that $H_1 (\precsim \circ \precsim') \circ \precsim'' H_2$ holds iff $H_1 \precsim \circ (\precsim' \circ \precsim'') H_2$ holds, which concludes the proof of associativity.

$H_1 \precsim \circ \precsim H_2$ holds iff $H_1 \precsim H_2$ and either $H_2 \not\precsim H_1$ or $H_1 \precsim H_2$. This condition is evidently equivalent to requiring that $H_1 \precsim H_2$, which proves idempotency and concludes the proof.

Here are a few quasi-orderings which give possible measures of niceness.

1 Complexity $H_1 \precsim_c H_2$ iff $||H_1|| \leq ||H_2||$.

2 Power Let Power(C) = $||\{C_i \in H_0 \mid C \leq C_i \ (Th)\}||$,

Power(H) = $\sum_{C \in H}$ Power(C),

$H_1 \precsim_p H_2$ iff Power(H_1) $\geq$ Power(H_2).

This ordering is defined in terms of H_0 and so of Ev and f.

3 Generality $H_1 \preceq_g H_2$ iff $H_2 \leq H_1$ (Th).

4 Literals $H_1 \preceq_l H_2$ iff $|| \bigcup H_1 || \leq || \bigcup H_2 ||$.

5 Literal occurrences $H_1 \preceq_{l'} H_2$ iff $\sum_{C \in H_1} ||C|| \leq \sum_{C \in H_2} ||C||$.

6 Symbols $H_1 \preceq_s H_2$ iff the number of symbols in H_1 is less than or equal to that in H_2.

7 Symbol occurrences $H_1 \preceq_{s'} H_2$ iff the number of symbol occurrences in H_1 is less than or equal to that in H_2.

As remarked above, we will mostly be interested in $\preceq_{cpg}$ which is $\preceq_c \circ \preceq_p \circ \preceq_g$.

This choice may be partially justified. The niceness criterion is $(\preceq_c \circ \preceq_p) \circ \preceq_g$ and corresponds to preferring the least general of the simplest, given by $\preceq_{cp} = \preceq_c \circ \preceq_p$, hypotheses. There are three conflicting properties guiding the choice of hypotheses current in the literature. One would want a hypothesis to be justifiable and the less general it is the more it is likely to be justifiable. However it is quite clear that the least general hypothesis which explains the phenomena is H_0, which is even weaker than the statement of the phenomena themselves. Another factor is the desire that the hypothesis be as general as possible in order to say something interesting. However there is no most general hypothesis since one can keep on adding irrelevant clauses to H. The first factor would, we suppose, be emphasized by Carnapians and the second by Popperians. But we see that the first leads to no interesting hypotheses whatever and the second does not seem to lead anywhere. The solution we adopt, as urged by Goodman (1961) is to place simplicity, the third factor, in first place.

So we only debate whether to follow the counsel of safety or strength among hypotheses of equal simplicity. We will discuss this in more detail in chapter 5 when we have a little more mathematical equipment.

As can be seen, in choosing $\mathscr{S}_{cpg}$ we have decided on the safest of the simplest hypotheses. Essentially this is an ad hoc decision. We can scarcely decide between Carnap and Popper, and some choice had to be made. We chose the Carnapian one, since it allows us to prove some theorems to the effect that the hypotheses formed in this way will be acceptable to certain Carnapians in a precise technical sense (Hintikka and Hilpinen, 1966, Hilpinen, 1968).

We should now give some justification of the simplicity measure, $\mathscr{S}_{cp}$. This is based on both a loose analogy with a measure for propositional formulae and also, simply, mathematical convenience. Propositional formulae in conjunctive normal form are often compared by preferring first those with fewer conjuncts and of those with the same number of conjuncts that one is preferred which has the fewest number of literals. This analogy would favour $\mathscr{S}_c \circ \mathscr{S}_{1'}$, rather than $\mathscr{S}_c \circ \mathscr{S}_p$. However power has the property that $H_1 \leq H_2$ implies $H_1 \ \mathscr{S}_p \ H_2$, which is not shared by $\mathscr{S}_{1'}$, although for propositional formulae both $\mathscr{S}_{1'}$ and $\mathscr{S}_p$ have this property. Thus the choice of $\mathscr{S}_{cp}$ is dictated by rather ad hoc factors. A fuller study would try the effect of other obvious measures of simplicity in more detail and use any relevant philosophical work on simplicity.

We have now specified or indicated the possible values of every parameter and can give the formal problem of finding a nicest explanatory generalisation from experience.

One is given $k = Th \wedge Irr$, Ev and f subject to the restrictions:

1 $k \wedge \bigwedge_{i=1}^{n} (e_i \wedge f_i)$ is consistent.

2 For no i does $e_i \rightarrow f_i$ follow from Th.

One is required to find a set of clauses, H, such that

P1 $H \leq H_0$ (Th).

P2 $\forall H \wedge Th \wedge Irr \wedge \bigwedge_{i=1}^{n} (e_i \wedge f_i)$ is consistent.

P3 Of all the sets of clauses satisfying P1 and P2, H is minimal with respect to $\prec$.

We will be particularly interested in the case where $\prec$ is $\prec_{cpg}$. When Th is empty, the problem may be reformulated by replacing P1 by:

P1' $H \leq H_0$.

The next chapter develops the relevant formal properties of relative generalisation. In order to make it formally self-contained, several definitions are repeated. This chapter has been concerned with giving a fairly rational explanation for the choices of the definitions.