CICADA Seminar

Continuous Interpretations of Process Algebra Models

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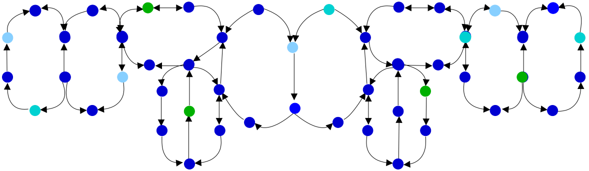
8th April 2008

Collective Dynamics

The behaviour of many systems can be interpreted as the result of the collective behaviour of a large number of interacting entities.



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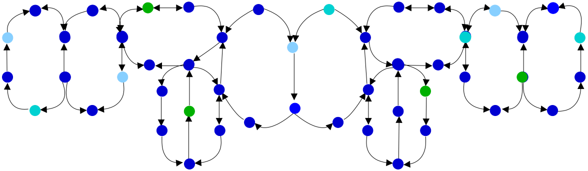

For such systems we are often as interested in the population level behaviour as we are in the behaviour of the individual entities.

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- Incorporate formal apparatus for reasoning about the behaviour of systems;
- Stochastic extensions, such as PEPA, enable quantified behaviour of the dynamics of systems.

In the CODA project we are developing stochastic process algebras and associated theory, tailored to the construction and evaluation of the collective dynamics of large systems of interacting entities.

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- Linking process algebra and continuous mathematical models for dynamic evaluation represents a paradigm shift in how such systems are studied.
- The prospect of formally-based quantified evaluation of dynamic behaviour could have significant impact in application domains such as:
  - Large scale software systems;
  - Biochemical signalling pathways;
  - Epidemiological systems.

Outline

1. Continuous Approximation of PEPA models
   - Stochastic Process Algebra
   - Continuous Approximation
   - Numerical illustration
2. A Process Algebra for Hybrid Systems
   - HYPE definition
   - Semantics – operational, hybrid, equivalence
3. Conclusions

Outline

Process Algebra

- Models consist of agents which engage in actions.

$$\underbrace{\alpha}_{\text{action type or name}}.\underbrace{P}_{\text{agent/ component}}$$

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As the size of the state space becomes large it becomes infeasible to carry out numerical solution and extremely time-consuming to conduct stochastic simulation.

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Use ordinary differential equations to represent the evolution of those variables over time.

New mathematical structures: differential equations

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Appropriate for models in which there are large numbers of components of the same type, i.e. models of populations.

Differential equations from PEPA models

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Consider the change in a small time δt:

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N(\mathcal{C}_{i_j}, t+\delta t) \;-\; N(\mathcal{C}_{i_j}, t) = \\
& - \underbrace{\sum_{(\alpha,r)\in Ex(\mathcal{C}_{i_j})} r \times \min_{\mathcal{C}_{k_l}\in Ex(\alpha,r)} (N(\mathcal{C}_{k_l}, t))\, \delta t}_{\text{exit activities}} \\
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Differential equations from PEPA models

Let $N(\mathcal{C}_{i_j}, t)$ denote the number of $\mathcal{C}_{i_j}$ type components at time t.

Dividing by δt and taking the limit, $\delta t \longrightarrow 0$:

$$\frac{dN(C_{i_j}, t)}{dt} = - \sum_{(\alpha,r) \in Ex(\mathcal{C}_{i_j})} r \times \min_{\mathcal{C}_{k_l} \in Ex(\alpha,r)} (N(C_{k_l}, t)) + \sum_{(\alpha,r) \in En(\mathcal{C}_{i_j})} r \times \min_{\mathcal{C}_{k_l} \in Ex(\alpha,r)} (N(C_{k_l}, t))$$

Activity matrix

Derivation of the system of ODEs representing the PEPA model then proceeds via an activity matrix which records the influence of each activity on each component type/derivative.

The matrix has one row for each component type and one column for each activity type.

One ODE is generated corresponding to each row of the matrix, taking into account the negative entries in the non-zero columns as these are the components for which this is an exit activity.

Stochastic Process Algebra

- Models are constructed from components which engage in activities.

$$(\underbrace{\alpha}_{\text{action type or name}}, \underbrace{r}_{\text{activity rate}}).\underbrace{P}_{\text{component/ derivative}}$$

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The language may be used to generate a CTMC.

PEPA MODEL $\xrightarrow{\text{SOS rules}}$ LABELLED TRANSITION SYSTEM $\xrightarrow[\text{diagram}]{\text{state transition}}$ CTMC **Q**

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A simple example: processors and resources

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\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, \top).Proc_1 \\
Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{def}{=} (task1, r_1).Res_1 \\
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CTMC interpretation

Processors (P)	Resources (R)	States (2^{P+R})
1	1	4
2	1	8
2	2	16
3	2	32
3	3	64
4	3	128
4	4	256
5	4	512
5	5	1024
6	5	2048
6	6	4096
7	6	8192
7	7	16384
8	7	32768
8	8	65536
9	8	131072
9	9	262144
10	9	524288
10	10	1048576

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ODE interpretation

$$
\begin{aligned}
\frac{\mathrm{d}Proc_0}{\mathrm{d}t} &= -r_1 \min(Proc_0, Res_0) \\
&\quad + r_2\, Proc_1 \\
\frac{\mathrm{d}Proc_1}{\mathrm{d}t} &= r_1 \min(Proc_0, Res_0) \\
&\quad - r_2\, Proc_1 \\
\frac{\mathrm{d}Res_0}{\mathrm{d}t} &= -r_1 \min(Proc_0, Res_0) \\
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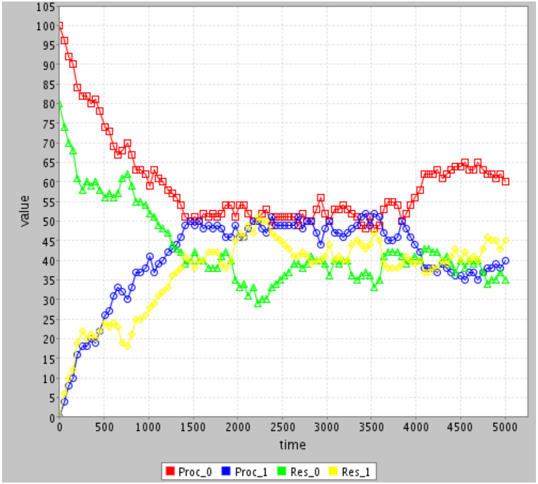
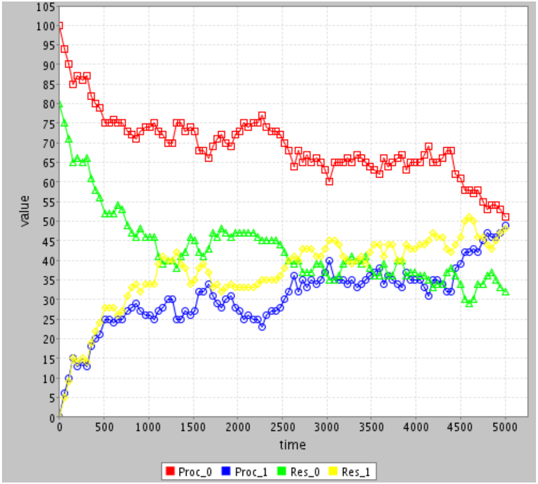
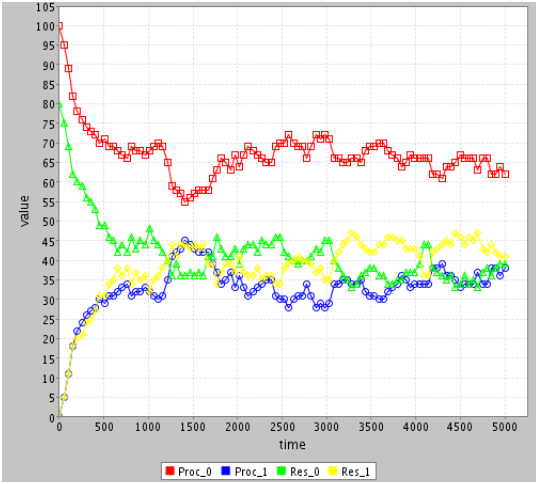
Processors and resources (simulation run A)



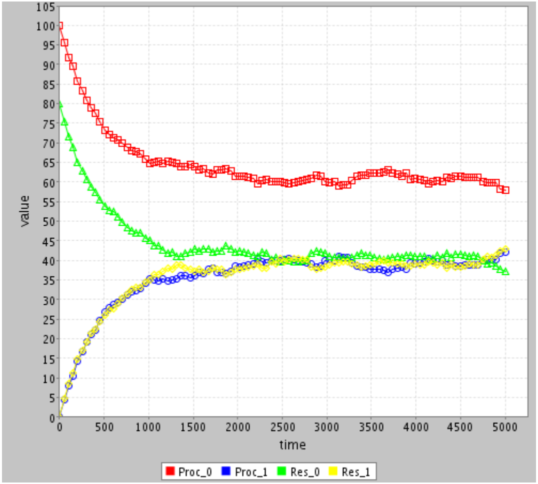
Processors and resources (simulation run B)



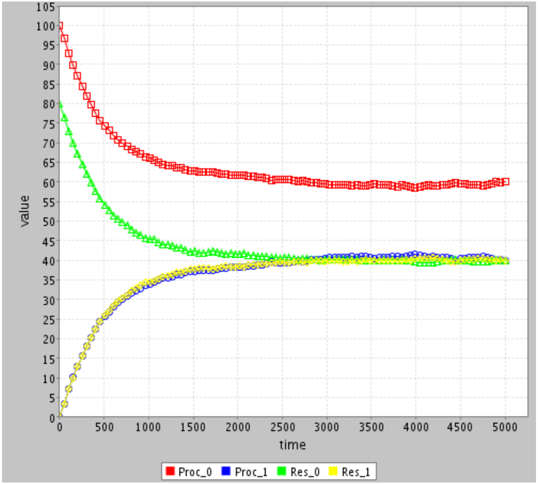
Processors and resources (simulation run C)



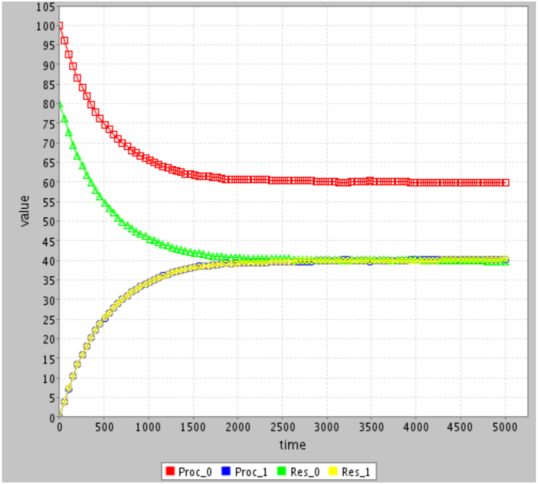
Processors and resources (average of 10 runs)



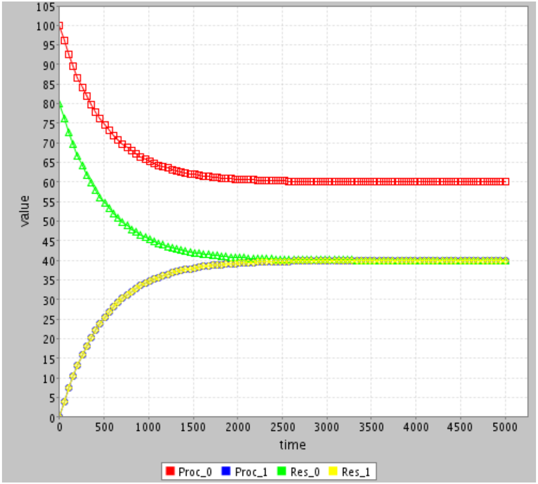
Processors and resources (average of 100 runs)



Processors and resources (average of 1000 runs)



Processors and resources (ODE solution)



Outline

1 Continuous Approximation of PEPA models
- Stochastic Process Algebra
- Continuous Approximation
- Numerical illustration

2 A Process Algebra for Hybrid Systems
- HYPE definition
- Semantics – operational, hybrid, equivalence

3 Conclusions

Introduction

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In part our motivation was consideration of PEPA models in which only some components could be legitimately subjected to continuous approximation.

For this and other reasons we were keen to develop an approach in which ODEs describing continuous behaviour could be derived in a fairly natural way from the process algebra model which captured both the discrete events and continuous changes.

Other formal approaches to hybrid systems

Hybrid automata are a well-established approach to modelling hybrid systems which are supported by a number of tools and analysis techniques. Their drawbacks are that they are graphical rather than textual, and the approach is not very compositional.

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There have also been a number of other process algebras for hybrid systems:

- ACP^{srt}_{hs} — Bergstra and Middelburg
- HyPA — Cuijpers and Reniers
- hybrid χ — van Beek *et al*
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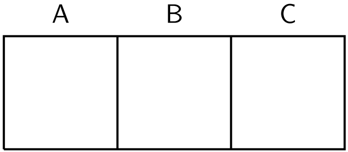
These take a coarse-grained approach, often with ODEs embedded within the syntax.

Heater example I



- three adjacent rooms

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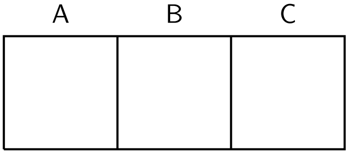

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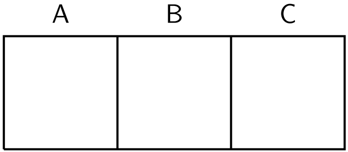

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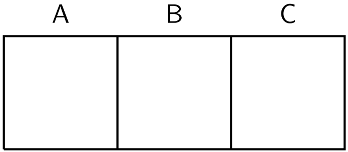

- three adjacent rooms
- fan heaters can be placed in each room
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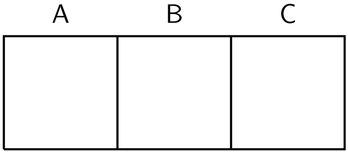

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  - no affect on non-adjacent room

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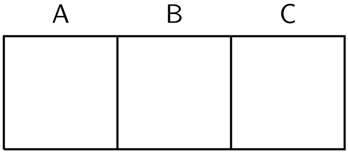

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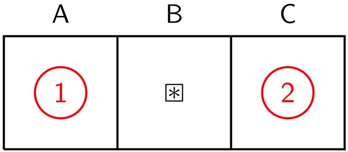

- three adjacent rooms
- fan heaters can be placed in each room
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- How does the temperature in Room B change if there is one heater in Room A and one in Room C?

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$$\alpha \in \mathcal{A} \qquad \alpha(\vec{X}) = (\iota, r, I(\vec{X}))$$

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HYPE definition (cont.)

- subcomponents

$$S ::= \underline{\mathsf{a}} : \alpha . C_s \mid S + S \qquad \underline{\mathsf{a}} \in \mathcal{E}, \alpha \in \mathcal{A}$$

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$$P ::= C(\vec{X}) \mid P \underset{L}{\bowtie} P \qquad L \subseteq \mathcal{E}$$

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- uncontrolled system

$$\Sigma ::= C(\vec{V}) \mid \Sigma \underset{L}{\bowtie} \Sigma \qquad L \subseteq \mathcal{E}$$

Heater example II

- room: $Room_x(T) \stackrel{def}{=} \underline{\mathrm{init}}:(t_{0,x}, -1, linear(T)).Room_x(T)$

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- fan i in Room x affecting Room y:

$$\begin{aligned} Fan_{i,x,y} &\stackrel{def}{=} \underline{\text{init}} : (t_{i,y}, 0, const).Fan_{i,x,y} + \\ &\quad \underline{\text{off}}_i : (t_{i,y}, 0, const).Fan_{i,x,y} + \\ &\quad \underline{\text{on}}_i : (t_{i,y}, r_i, const_{\psi(x,y)}).Fan_{i,x,y} \\ \psi(x,y) &= \begin{cases} in & \text{if } x = y \\ adj & \text{if } x \text{ and } y \text{ are adjacent} \\ far & \text{otherwise} \end{cases} \end{aligned}$$

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- uncontrolled system:

$$Sys \stackrel{def}{=} (Fan_{1,A,B} \underset{\{\underline{\text{init}}\}}{\bowtie} Fan_{2,C,B}) \underset{\{\underline{\text{init}}\}}{\bowtie} Room_B(T_B)$$

HYPE definition (cont.)

We assume, additionally, that the system may be subject to a controller which can impose orderings on events:

$$M ::= \underline{\mathrm{a}}.M \mid 0 \mid M + M \qquad \underline{\mathrm{a}} \in \mathcal{E}$$

$$Con ::= M \mid Con \underset{L}{\bowtie} Con \qquad L \subseteq \mathcal{E}$$

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When the controller is composed with the uncontrolled system we obtain a controlled system:

$$ConSys ::= \Sigma \underset{L}{\bowtie} \underline{\mathrm{init}}.Con \qquad L \subseteq \mathcal{E}$$

Heater example II

- controller:

$$Con \stackrel{def}{=} Con_1 \underset{\emptyset}{\bowtie} Con_2 \qquad Con_i \stackrel{def}{=} \underline{\mathrm{on}}_i.\underline{\mathrm{off}}_i.Con_i$$

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- controller:

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- controlled system:

$$MF \stackrel{def}{=} Sys \underset{K}{\bowtie} \underline{\mathrm{init}}.Con \qquad K = \{\underline{\mathrm{init}}, \underline{\mathrm{on}}_1, \underline{\mathrm{off}}_1, \underline{\mathrm{on}}_2, \underline{\mathrm{off}}_2\}$$

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- variable: $\mathcal{V} = \{T_B\}$ with $\mathrm{iv}(t_{i,B}) = T_B$

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  $\mathrm{ec}(\underline{\mathsf{init}}) = (true, (T_B' = T_0))$, $\mathrm{ec}(\underline{\mathsf{off}}_i) = (\bot, (T_B' = T_B))$,
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  $\mathrm{ec}(\underline{\mathsf{on}}_i) = (\perp, true)$
- influence definitions:
  $[\![const_{in}]\!] = 1$, $[\![const_{adj}]\!] = 0.5$, $[\![const_{far}]\!] = [\![const]\!] = 0$,
  $[\![linear(X)]\!] = X$

HYPE definition (cont.)

HYPE model

$$(\mathit{ConSys}, \mathcal{V}, \mathcal{X}, \mathit{IN}, \mathit{IT}, \mathcal{E}, \mathcal{A}, \mathrm{ec}, \mathrm{iv}, \mathit{EC}, \mathit{ID})$$

- event conditions

$$\mathrm{ec} : \mathcal{E} \rightarrow \mathit{EC}$$

EC consists of activation conditions and resets associated with events

HYPE definition (cont.)

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- influences and variables

$$\mathrm{iv} : \mathit{IN} \rightarrow \mathcal{V}$$

  each influence is associated with one variable

Well-defined HYPE model

We impose a number of (mild) conditions on the formation of a HYPE model in order for it to be considered well-defined:

- for each subcomponent $\mathcal{C}_s(\vec{X}) \stackrel{def}{=} S$, only $\mathcal{C}_s(\vec{X})$ can appear in S, $\underline{a}$ can only appear once and each ι must also appear in a prefix with $\underline{init}$.

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The heater example is well-defined.

Operational semantics

We define the state of the system to be the set of currently acting influences: $\sigma : IN \rightarrow (\mathbb{R}^{+} \times IT)$ and for convenience, we write it as a list of triples $(\iota, r, I(\vec{X}))$

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A configuration is then a controlled system together with its state: $\langle ConSys, \sigma \rangle$

We define the semantics of the system as a set of operational rules which tell us how states evolve through actions, and give rise to a labelled transition system: $(\mathcal{F}, \mathcal{E}, \rightarrow \subseteq \mathcal{F} \times \mathcal{E} \times \mathcal{F})$

Auxilliary functions

We require two auxilliary functions to define the rules:

- updating function: $\sigma[\iota \mapsto (r, I)]$

$$\sigma[\iota \mapsto (r, I)](x) = \begin{cases} (r, I) & \text{if } x = \iota \\ \sigma(x) & \text{otherwise} \end{cases}$$

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- change identifying function: $\Gamma : \mathcal{S} \times \mathcal{S} \times \mathcal{S} \to \mathcal{S}$

$$(\Gamma(\sigma, \tau, \tau'))(\iota) = \begin{cases} \tau(\iota) & \text{if } \sigma(\iota) = \tau'(\iota), \\ \tau'(\iota) & \text{if } \sigma(\iota) = \tau(\iota), \\ \text{undefined} & \text{otherwise} \end{cases}$$

Operational semantics (cont.)

Prefix with influence:

$$\frac{}{\langle \underline{a}:(\iota, r, I).E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma[\iota \mapsto (r, I)]\rangle}$$

Prefix without influence:

$$\frac{}{\langle \underline{a}.E, \sigma\rangle \xrightarrow{\underline{a}} \langle E, \sigma\rangle}$$

Choice:

$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle} \qquad \frac{\langle F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}{\langle E + F, \sigma\rangle \xrightarrow{\underline{a}} \langle F', \sigma'\rangle}$$

Constant:

$$\frac{\langle E, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}{\langle A, \sigma\rangle \xrightarrow{\underline{a}} \langle E', \sigma'\rangle}(A \stackrel{def}{=} E)$$

Operational semantics (cont.)

Parallel without synchronisation:

$$\frac{\langle E, \sigma \rangle \xrightarrow{\underline{a}} \langle E', \sigma' \rangle}{\langle E \underset{K}{\bowtie} F, \sigma \rangle \xrightarrow{\underline{a}} \langle E' \underset{K}{\bowtie} F, \sigma' \rangle} \quad \underline{a} \notin K$$

$$\frac{\langle F, \sigma \rangle \xrightarrow{\underline{a}} \langle F', \sigma' \rangle}{\langle E \underset{K}{\bowtie} F, \sigma \rangle \xrightarrow{\underline{a}} \langle E \underset{K}{\bowtie} F', \sigma' \rangle} \quad \underline{a} \notin K$$

Parallel with synchronisation:

$$\frac{\langle E, \sigma \rangle \xrightarrow{\underline{a}} \langle E', \tau \rangle \quad \langle F, \sigma \rangle \xrightarrow{\underline{a}} \langle F', \tau' \rangle}{\langle E \underset{K}{\bowtie} F, \sigma \rangle \xrightarrow{\underline{a}} \langle E' \underset{K}{\bowtie} F', \Gamma(\sigma, \tau, \tau') \rangle}$$

$$\underline{a} \in K, \ \Gamma \text{ defined}$$

Heater example III

- transition derivation

$$\frac{\langle Fan_{1,A,B}, \tau \rangle \xrightarrow{\text{init}} \langle Fan_{1,A,B}, \tau_1 \rangle \qquad \langle Fan_{2,C,B}, \tau \rangle \xrightarrow{\text{init}} \langle Fan_{2,C,B}, \tau_2 \rangle}{\langle Fan_{1,A,B} \underset{\underline{\text{init}}}{\bowtie} Fan_{2,C,B}, \tau \rangle \xrightarrow{\text{init}} \langle Fan_{1,A,B} \underset{\underline{\text{init}}}{\bowtie} F_{2,C,B}, \tau_3 \rangle}$$

$$\begin{aligned} \tau &= \{t_{0,B} \mapsto *, t_{1,B} \mapsto *, t_{2,B} \mapsto *\} \\ \tau_1 &= \tau[t_{1,B} \mapsto (0, c)] = \{t_{0,B} \mapsto *, t_{1,B} \mapsto (0, c), t_{2,B} \mapsto *\} \\ \tau_2 &= \tau[t_{2,B} \mapsto (0, c)] = \{t_{0,B} \mapsto *, t_{1,B} \mapsto *, t_{2,B} \mapsto (0, c)\} \\ \tau_3 &= \Gamma(\tau, \tau_1, \tau_2) = \{t_{0,B} \mapsto *, t_{1,B} \mapsto (0, c), t_{2,B} \mapsto (0, c)\} \end{aligned}$$

Heater example III (cont.)

- labelled transition system

$$
\begin{tikzcd}
& \langle Sys \underset{K}{\bowtie} \underline{\mathrm{init}}.Con, \_ \rangle \arrow[d, "\underline{\mathrm{init}}"'] & \\
& \langle Sys \underset{K}{\bowtie} (Con_1 \parallel Con_2), \sigma_0 \rangle \arrow[dl, shift right, "\underline{\mathrm{on}}_1"'] \arrow[dr, shift left, "\underline{\mathrm{on}}_2"] & \\
\langle Sys \underset{K}{\bowtie} (\underline{\mathrm{off}}_1.Con_1 \parallel Con_2), \sigma_1 \rangle \arrow[ur, shift right, "\underline{\mathrm{off}}_1"'] \arrow[dr, shift right, "\underline{\mathrm{on}}_2"'] & & \langle Sys \underset{K}{\bowtie} (Con_1 \parallel \underline{\mathrm{off}}_2.Con_2), \sigma_2 \rangle \arrow[ul, shift left, "\underline{\mathrm{off}}_2"] \arrow[dl, shift left, "\underline{\mathrm{on}}_1"] \\
& \langle Sys \underset{K}{\bowtie} (\underline{\mathrm{off}}_1.Con_1 \parallel \underline{\mathrm{off}}_2.Con_2), \sigma_3 \rangle \arrow[ul, shift right, "\underline{\mathrm{off}}_2"'] \arrow[ur, shift left, "\underline{\mathrm{off}}_1"] &
\end{tikzcd}
$$

Heater example III (cont.)

- for MF, there are four states

$$\sigma_0 = \{t_{0,B} \mapsto (-1, linear(T_B)), t_{1,B} \mapsto (0, const), t_{2,B} \mapsto (0, const)\}$$

$$\sigma_1 = \{t_{0,B} \mapsto (-1, linear(T_B)), t_{1,B} \mapsto (r_1, const_{adj}), t_{2,B} \mapsto (0, const)\}$$

$$\sigma_2 = \{t_{0,B} \mapsto (-1, linear(T_B)), t_{1,B} \mapsto (0, const), t_{2,B} \mapsto (r_2, const_{adj})\}$$

$$\sigma_3 = \{t_{0,B} \mapsto (-1, linear(T_B)), t_{1,B} \mapsto (r_1, const_{adj}), t_{2,B} \mapsto (r_2, const_{adj})\}$$

Hybrid semantics

- extract ODEs from each state σ in the labelled transition system

$$CS_\sigma = \left\{ \text{ODE for variable } V \;\middle|\; V \in \mathcal{V} \right\} \text{ where}$$

$$\frac{dV}{dt} = \sum \left\{ r[\![I(\vec{W})]\!] \mid \mathrm{iv}(\iota) = V \text{ and } \sigma(\iota) = (r, I(\vec{W})) \right\}$$

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  - for any influence name associated with V
  - determine from σ its rate and influence type
  - multiply its rate and influence function together
  - sum these over all associated influence names

Heater example IV

- state σ_0 occurs when both fans are off

$$MF_{\sigma_0} = \left\{ \frac{dT_B}{dt} = -T_B \right\}$$

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$$MF_{\sigma_3} = \left\{ \frac{dT_B}{dt} = -T_B + 0.5(r_1 + r_2) \right\}$$

Hybrid automata

- $(V, E, \mathbf{X}, \mathcal{E}, \mathit{flow}, \mathit{init}, \mathit{inv}, \mathit{event}, \mathit{jump}, \mathit{reset}, \mathit{urgent})$

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    - invariants: $\mathit{inv}(v)$
- (control) switches: $e \in E$
    - events: $\mathit{event}(e) \in \mathcal{E}$
    - predicate on $\mathbf{X}$: $\mathit{jump}(e)$
    - predicate on $\mathbf{X} \cup \mathbf{X}'$: $\mathit{reset}(e)$
    - boolean: $\mathit{urgent}(e)$

HYPE model to hybrid automaton

- modes V: set of reachable configurations

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- $\mathit{inv}(v) = \mathit{true}$
- let e be an edge associated with $\underline{a}$ and let $ec(\underline{a}) = (\mathit{act}_{\underline{a}}, \mathit{res}_{\underline{a}})$
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    else $\mathit{jump}(e) = \mathit{true}$ and $\mathit{urgent}(e) = \mathit{false}$

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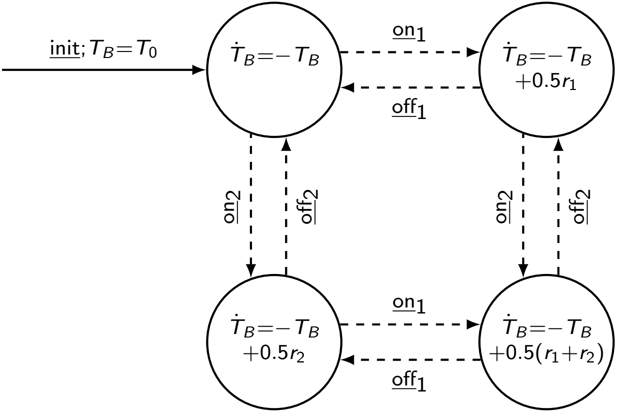
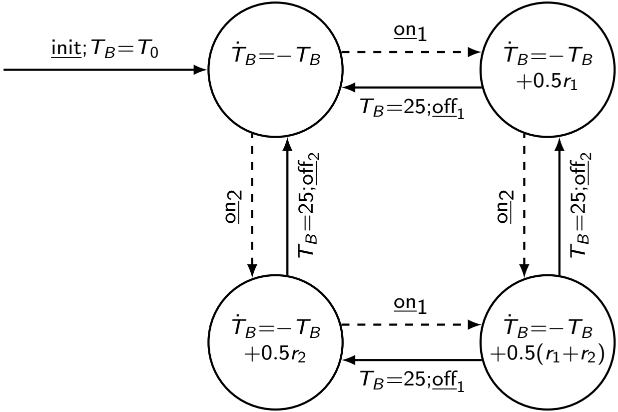
- $inv(v) = true$
- let e be an edge associated with $\underline{a}$ and let $ec(\underline{a}) = (act_{\underline{a}}, res_{\underline{a}})$
  - $event(e) = \underline{a}$ and $reset(e) = res_{\underline{a}}$
  - if $act_{\underline{a}} \neq \perp$ then $jump(e) = act_{\underline{a}}$ and $urgent(e) = true$
    else $jump(e) = true$ and $urgent(e) = false$
- $init(v) = \begin{cases} res_{\underline{init}} & \text{if } v = \langle P, \sigma \rangle \text{ with primes removed} \\ false & otherwise \end{cases}$

Heater example V: Hybrid automaton



Heater example VI: Hybrid automaton

- max temp of $25^\circ C$: $\mathrm{ec}(\underline{\mathsf{off}}_i) = ((T_B = 25), \mathit{true})$



Equivalence semantics

- system bisimulation: relation B if for all $(P, Q) \in B$ whenever
  1. $\langle P, \sigma \rangle \xrightarrow{\mathsf{a}} \langle P', \sigma' \rangle$, there exists $\langle Q', \sigma' \rangle$ with $\langle Q, \sigma \rangle \xrightarrow{\mathsf{a}} \langle Q', \sigma' \rangle$ and $(P', Q') \in B$.
  2. $\langle Q, \sigma \rangle \xrightarrow{\mathsf{a}} \langle Q', \sigma' \rangle$, there exists $\langle P', \sigma' \rangle$ with $\langle P, \sigma \rangle \xrightarrow{\mathsf{a}} \langle P', \sigma' \rangle$ and $(P', Q') \in B$.

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- *Theorem 3*: if $P \sim_s Q$ then $P_\sigma = Q_\sigma$ for all σ, assuming well-defined systems

Heater example VII

- Consider two fans in Room C and none in Room A

$$Sys' \stackrel{def}{=} (Fan_{1,C,B} \underset{\{\underline{\text{init}}\}}{\bowtie} Fan_{2,C,B}) \underset{\{\underline{\text{init}}\}}{\bowtie} Room_B(T_B)$$

$$MF' \stackrel{def}{=} Sys' \underset{K}{\bowtie} \underline{\text{init}}.Con \qquad K = \{\underline{\text{init}}, \underline{\text{on}}_1, \underline{\text{off}}_1, \underline{\text{on}}_2, \underline{\text{off}}_2\}$$

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- Sys and Sys' have the same prefixes

$$\bigcup_{i=1,2} \{\underline{\mathrm{init}}:(t_{i,y}, 0, const), \underline{\mathrm{on}}_i:(t_{i,y}, r_i, const_{adj}), \underline{\mathrm{off}}_i:(t_{i,y}, 0, const)\}$$

$$\cup\ \{\underline{\mathrm{init}}:(t_{0,x}, -1, linear(T))\}$$

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- by Theorem 2, $MF \sim_s MF'$
- by Theorem 3, MF and MF' have the same ODEs

Outline

Conclusions and further work

- Many interesting and important systems can be regarded as examples of collective dynamics and emergent behaviour.
- Process algebras, such as PEPA and HYPE, are well-suited to modelling the behaviour of such systems in terms of the individuals and their interactions.
- Continuous approximation of PEPA models allows an alternative mathematical analysis of the average behaviour of discrete event systems.
- HYPE is offering a fine-grained, flow-based process algebra approach to modelling hybrid systems.

Heater example VIII

- system with temperature limit in ACP^{srt}_{hs}
- $\theta \equiv (T_B^{\bullet} = {}^{\bullet}T_B) \quad \psi \equiv (T_B = 25)$

$$
\begin{aligned}
\text{Start} &\stackrel{def}{=} (T_B = T_0) \wedge\!\!\blacktriangle\ \text{Off12} \\
\text{Off12} &\stackrel{def}{=} (\dot{T}_B = -T_B) \curvearrowright\!\!\blacktriangledown\ \sigma^*_{\text{rel}}(\theta \ulcorner\!\!\blacktriangledown\ (on_1 \cdot \text{On1} + on_2 \cdot \text{On2})) \\
\text{On1} &\stackrel{def}{=} (T_B \leq 25 \wedge \dot{T}_B = -T_B + 0.5r_1) \\
&\quad \curvearrowright\!\!\blacktriangledown\ \sigma^*_{\text{rel}}((\theta \ulcorner\!\!\blacktriangledown\ on_2 \cdot \text{On12}) + (\psi :\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\ off_1 \cdot \text{Off12}))) \\
\text{On2} &\stackrel{def}{=} (T_B \leq 25 \wedge \dot{T}_B = -T_B + 0.5r_2) \\
&\quad \curvearrowright\!\!\blacktriangledown\ \sigma^*_{\text{rel}}((\theta \ulcorner\!\!\blacktriangledown\ on_1 \cdot \text{On12}) + (\psi :\longrightarrow (\theta \ulcorner\!\!\blacktriangledown\ off_2 \cdot \text{Off12}))) \\
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Further Work

The relationship between the CTMC and the ODEs generated by the continuous approximation of a PEPA model has already been established (Kurtz's Theorem), but there is more to do. For example, analysing the approximation error for a given number of copies of components.

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- Defining alternative forms of equivalence;
- Comparisons with other process algebra for hybrid systems;
- Implementing the mapping to hybrid automata;
- Further case studies.

Thanks!

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Acknowledgements: collaborators
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More information:
http://www.dcs.ed.ac.uk/pepa