SPA for quantitative analysis: Lecture 7 — Modelling Hybrid Systems

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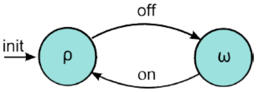
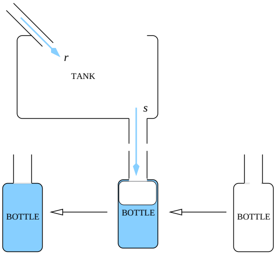
Hybrid Systems





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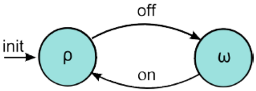

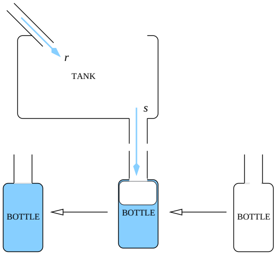

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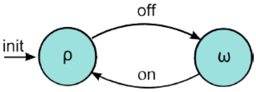

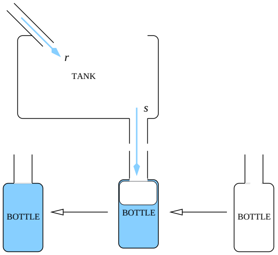

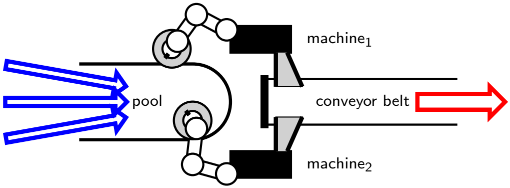

Outline

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We have also been motivated by incorporating more detailed representation of space within our process algebra models.

Other formal approaches to hybrid systems

Hybrid automata are a well-established approach to modelling hybrid systems which are supported by a number of tools and analysis techniques. Their drawbacks are that they are graphical rather than textual, and the approach is not generally compositional.

There have also been a number of other process algebras for hybrid systems:

- ACP^{srt}_{hs} — Bergstra and Middelburg
- HyPA — Cuijpers and Reniers
- hybrid χ — van Beek *et al*
- ϕ-calculus — Rounds and Song

These take a coarse-grained approach, with ODEs embedded within the syntax.

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Introduction to Stochastic HYPE

- behaviours to be included
  - discrete behaviour: instantaneous events
  - continuous behaviour: ordinary differentials equations (ODEs)
  - stochastic behaviour: exponentially-distributed durations
- process algebra approach
  - formal languages for expressing concurrency
  - compositional semantics
  - notions of equivalence
- the original definition of HYPE
  - only discrete and continuous behaviour
  - operational semantics define labelled transition system
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HYPE actions

We distinguish two types of actions in a system:

- events — instantaneous, discrete changes

$$\underline{a} \in \mathcal{E}$$

  Each event is associated with an event condition: activation conditions and variable resets.

- activities — influences on a continuous aspect of system behaviour, also termed flows

$$\alpha \in \mathcal{A} \qquad \alpha(\vec{X}) = (\iota, r, I(\vec{X}))$$

  where

  - $\vec{X}$ are formal parameters,
  - ι is the influence name and r is its rate,
  - $I(\vec{X})$ is the influence type, i.e. $[\![I(\vec{X})]\!] = f(\vec{X})$.

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Language considerations: ODEs versus flows

- notation: $\mathcal{V}$, a set of continuous variables
- monolithic ODEs in existing hybrid process algebras

$$A \stackrel{def}{=} \ldots \quad [\frac{dV}{dt} = f(\mathcal{V})] \quad \ldots$$

- flows in HYPE ($W_j \subseteq \mathcal{V}$)

$$\begin{array}{lllll} A_1 & \stackrel{def}{=} & \ldots & (\iota_1, r_1, I_1(\mathcal{W}_1)) & \ldots \\ \vdots & \vdots & & \vdots & \\ A_n & \stackrel{def}{=} & \ldots & (\iota_n, r_n, I_n(\mathcal{W}_n)) & \ldots \end{array}$$

and $\quad \frac{dV}{dt} = \sum\{r_j.I_j(\mathcal{W}_j) \mid iv(\iota_j) = V, \ldots\}$

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Stochastic HYPE

In addition to the (instantaneous) events and activities, we now also allow stochastic events.

$$\bar{a} \in \mathcal{E}$$

Previously in HYPE we allowed non-urgent transitions to be specified with the event condition $\perp$.

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Stochastic HYPE model I

- Subcomponents

$$S ::= \underline{\mathrm{a}} : \alpha.C_s \mid \bar{a} : \alpha.C_s \mid S + S$$

where $\underline{\mathrm{a}} \in \mathcal{E}_d,\ \bar{a} \in \mathcal{E}_s,\ \mathcal{E}_d \cup \mathcal{E}_s = \mathcal{E},\ \alpha \in \mathcal{A}ct$

  - subcomponent names: $C_s(\overrightarrow{X}) = S$

- Components

$$P ::= C_s(\overrightarrow{X}) \mid C(\overrightarrow{X}) \mid P \underset{L}{\bowtie} P \quad L \subseteq \mathcal{E}$$

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Stochastic HYPE model II

- Uncontrolled System

$$\Sigma ::= C_s(\overrightarrow{V}) \mid C(\overrightarrow{V}) \mid \Sigma \underset{L}{\bowtie} \Sigma \quad L \subseteq \mathcal{E}$$

where $\overrightarrow{V}$ are system variables (cf. $\overrightarrow{X}$ of C or C_s).

- Controllers only have events:

$$M ::= \underline{\mathrm{a}}.M \mid 0 \mid M + M \quad \underline{\mathrm{a}} \in \mathcal{E}, L \subseteq \mathcal{E}$$

$$Con ::= M \mid Con \underset{L}{\bowtie} Con.$$

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$$ConSys ::= \Sigma \underset{L}{\bowtie} \underline{\mathrm{init}}.Con \quad L \subseteq \mathcal{E}.$$

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Stochastic HYPE model III

uncontrolled system

controllers/sequencers

$$(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})) \quad \underset{*}{\bowtie} \quad \underline{\mathrm{init}}.(Con_1 \underset{L_2}{\bowtie} \cdots \underset{L_m}{\bowtie} Con_m)$$

well-defined subcomponent

$$C(\mathcal{V}) \stackrel{def}{=} \sum_j \mathrm{a}_j : \alpha_j \,.\, C(\mathcal{V}) + \underline{\mathrm{init}} : \alpha \,.\, C(\mathcal{V})$$

events have event conditions: guards/durations and resets

$ec(\underline{\mathrm{a}}_j) = (g(\mathcal{V}), \mathcal{V}' = g'(\mathcal{V}))$ with $g : \mathbb{R}^{|\mathcal{V}|} \to \{true, false\}$ discrete

$ec(\overline{\mathrm{a}}_j) = (f(\mathcal{V}), \mathcal{V}' = f'(\mathcal{V}))$ with $f : \mathbb{R}^{|\mathcal{V}|} \to [0, \infty)$ stochastic

Stochastic HYPE model III

uncontrolled system

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$$\underset{*}{\bowtie}$$

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uncontrolled system — controllers/sequencers

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subcomponents are parameterised by variables

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$$ec(\underline{\mathrm{a}}_j) = (g(\mathcal{V}), \mathcal{V}' = g'(\mathcal{V})) \text{ with } g : \mathbb{R}^{|\mathcal{V}|} \to \{true, false\} \quad \text{discrete}$$

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Stochastic HYPE model III

uncontrolled system controllers/sequencers

$$\big(C_1(\mathcal{V}) \underset{*}{\bowtie} \cdots \underset{*}{\bowtie} C_n(\mathcal{V})\big) \quad \underset{*}{\bowtie} \quad \underline{\mathrm{init}}.\big(Con_1 \underset{L_2}{\bowtie} \cdots \underset{L_m}{\bowtie} Con_m\big)$$

well-defined subcomponent

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influences are defined by a triple

$$\alpha_j = (\iota_j, r_j, I_j(\mathcal{V}))$$

influence names are mapped to variables

$$iv(\iota_j) \in \mathcal{V}$$

Stochastic HYPE model IV

uncontrolled system — controllers/sequencers

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controller grammar

$$M ::= \mathrm{a}.M \mid 0 \mid M + M$$

$$Con ::= M \mid Con \underset{*}{\bowtie} Con$$

Stochastic HYPE model IV

uncontrolled system controllers/sequencers

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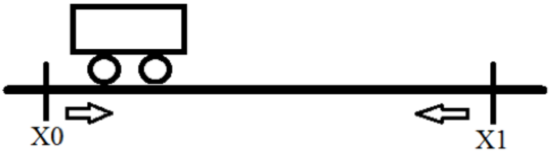
Outline

Simple Example: shuttle bus

We consider the simple example of an idealised shuttle bus which serves two stops, $X0$ and $X1$.



When the shuttle bus arrives at one stop, it will stop for a while, say 5 minutes, and then move to the other stop. Thus, there are two flows influencing the shuttle bus. The first is the time flow and the other influences the position of the shuttle bus.

Shuttle bus example

We represent the two flows by two subcomponents below:

$$\begin{aligned}
\mathit{Movement} &= \underline{\mathrm{init}} : (x, 0, \mathit{const}).\mathit{Movement} \\
&+ \overline{\mathrm{toX0}} : (x, -s, \mathit{const}).\mathit{Movement} \\
&+ \underline{\mathrm{toX1}} : (x, s, \mathit{const}).\mathit{Movement} \\
&+ \underline{\mathrm{stop}} : (x, 0, \mathit{const}).\mathit{Movement} \\
\mathit{Time} &= \underline{\mathrm{init}} : (t, 1, \mathit{const}).\mathit{Time}
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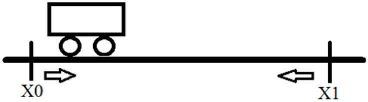
\end{aligned}
$$

In *Movement*, there are three distinct activities:

- $(x, 0, \mathit{const})$ — stopped at a station;
- $(x, -s, \mathit{const})$ — travelling from $X1$ to $X0$; and
- (x, s, const) — travelling from $X0$ to $X1$.

Shuttle bus example: uncontrolled system



The uncontrolled system is constructed by the combination of subsystems:

$$Sys \stackrel{def}{=} Movement \underset{init}{\bowtie} Time$$

Note that no causal or temporal constraints on the events have been imposed yet (hence "uncontrolled").

For instance, we need to specify that the shuttle bus can only move to $X1$ when it has previously moved to $X0$ and stopped for 5 minutes.

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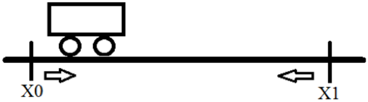

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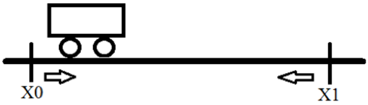

$$Con_{movement} = \underline{\text{stop}}.\underline{\text{toX1}}.\underline{\text{stop}}.\overline{\text{toX0}}.Con_{movement}$$

Controllers consist of only event prefixes, but these may be affected by the state of the system through event conditions.

The controlled system is constructed from synchronization of the controller and the uncontrolled system:

$$ShuttleBusCtrl = Sys \underset{M}{\bowtie} \underline{\text{init}}.Con_{movement}$$

with $M = \{\underline{\text{init}}, \overline{\text{toX0}}, \underline{\text{toX1}}, \underline{\text{stop}}\}$.

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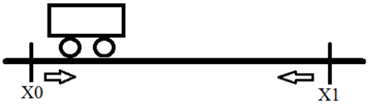

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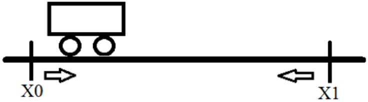
Influences and event conditions



We need to link each influence with an actual variable, e.g.

$$iv(x) = Pos, \quad iv(t) = T$$

where Pos captures the position of the shuttle bus, T, the current time. In this case we define the influence types as $const = 1$.

We also define the event conditions ec to trigger each event:

$$
\begin{aligned}
ec(\underline{\mathrm{init}}) &= (true, Pos' = X_0 \wedge T' = 0) \\
ec(\underline{\mathrm{stop}}) &= (Pos \leq X_0 \vee Pos \geq X_1, ArrivalTime' = Time) \\
ec(\underline{\mathrm{toX1}}) &= (Pos \leq X_0 \wedge T - ArrivalTime == five\_min, true) \\
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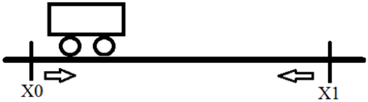

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Semantics for HYPE

HYPE is given a structured operational semantics, in terms of system configurations where (broadly speaking) a configuration is a set of influences currently at play in the system.

This gives us a semantics which allows us to reason about models and compare them in terms of bisimulation equivalence.

It does not give us a means to execute models as the implementation details of influence definitions and event conditions are not captured.

To get an executable interpretation of a model we map to a form of hybrid automaton:
Transition-Driven Stochastic Hybrid Automata (TDSHA),
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Operational semantics

Prefix with influence:

$$\frac{}{\langle \mathrm{a}\!:\!(\iota, r, I).E, \sigma \rangle \xrightarrow{a} \langle E, \sigma[\iota \mapsto (r, I)] \rangle}$$

Prefix without influence:

$$\frac{}{\langle \mathrm{a}.E, \sigma \rangle \xrightarrow{a} \langle E, \sigma \rangle}$$

Choice:

$$\frac{\langle E, \sigma \rangle \xrightarrow{a} \langle E', \sigma' \rangle}{\langle E + F, \sigma \rangle \xrightarrow{a} \langle E', \sigma' \rangle} \qquad \frac{\langle F, \sigma \rangle \xrightarrow{a} \langle F', \sigma' \rangle}{\langle E + F, \sigma \rangle \xrightarrow{a} \langle F', \sigma' \rangle}$$

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Operational semantics (continued)

Parallel without synchronisation:

$$\frac{\langle E, \sigma \rangle \xrightarrow{a} \langle E', \sigma' \rangle}{\langle E \underset{M}{\bowtie} F, \sigma \rangle \xrightarrow{a} \langle E' \underset{M}{\bowtie} F, \sigma' \rangle} \qquad a \notin M$$

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Operational semantics (continued)

- updating function: $\sigma[\iota \mapsto (r, I)]$

$$\sigma[\iota \mapsto (r, I)](x) = \begin{cases} (r, I) & \text{if } x = \iota \\ \sigma(x) & \text{otherwise} \end{cases}$$

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Transition-driven stochastic hybrid automata

- TDSHA: transition-driven stochastic hybrid automata $\subseteq$ PDMP: piecewise deterministic Markov processes
- set of modes, Q and set of continuous variables, $\mathbf{X}$
- instantaneous transitions
  - source mode, target mode, event name
  - guard: activation condition over variables
  - reset: function determining new values of variables
  - priority/weight: to resolve non-determinism
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  - priority/weight: to resolve non-determinism
- stochastic transitions
  - source mode, target mode, event name
  - rate: function defining speed of transition
  - guard: activation condition over variables
  - reset: function determining new values of variables

Transition-driven stochastic hybrid automata (continued)

- continuous transitions (flows)
  - source mode
  - vector specifying variables involved
  - Lipschitz continuous function
- continuous behaviour in a mode
  - consider all continuous transitions in that mode
  - trajectory is given by solution of $d\mathbf{X}/dt = \sum s \cdot f(\mathbf{X})$
- instantaneous behaviour: fire when guard becomes true
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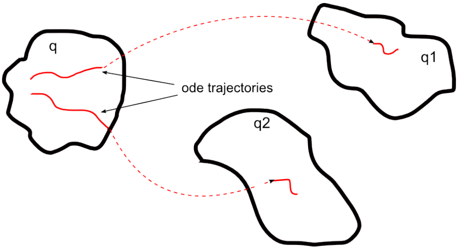
Piecewise deterministic Markov processes

- class of stochastic processes
- continuous trajectories over subsets of $\mathbb{R}^{|\mathbf{X}|}$
- instantaneous jumps at boundaries of regions
- stochastic jumps when guards are true



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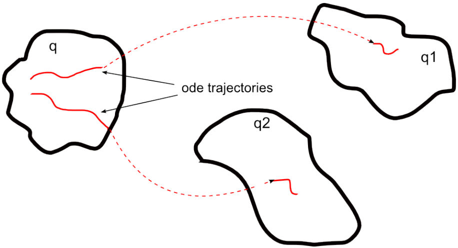

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Two equivalent semantics

- compositional mapping to TDSHA
  - define TDSHA for each subcomponent (no event conditions)
  - define TDSHA for each sequential controller
  - use TDSHA product to compose into TDSHA of whole model
- mapping from LTS to TDSHA
  - event labelled transition system over configurations
  - configuration: $\langle Sys \underset{*}{\bowtie} Con, \sigma \rangle$
  - state: σ : influence $\mapsto$ (influence strength, influence type)
  - configurations are mapped to modes
  - states giving ODEs which become continuous transitions

$$\left(\frac{dV}{dt}\right)_{\sigma} = \sum \{ r \cdot [\![I(\overrightarrow{W})]\!] \mid iv(\iota) = V, \sigma(\iota) = (r, I(\overrightarrow{W})) \}$$

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Well-behaved stochastic HYPE models

- PDMP definition only allow jumps to interiors of regions
- finite sequences of instantaneous events in TDSHA can be combined and mapped to a jump to an interior
- avoid instantaneous Zeno behaviour: infinite sequences of instantaneous events occurring at a time point
- finite sequence of instantaneous events is delimited by stochastic event or period of continuous evolution
- we have defined an algorithm to check when a stochastic HYPE model is well-behaved.

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Outline

Equivalence semantics for stochastic HYPE

- stochastic system bisimulation with respect to $\equiv$ over states (models that only differ in their controlled systems)

given an equivalence relation $B \subseteq \mathcal{C} \times \mathcal{C}$

then for all $(P, Q) \in B$, $\sigma \equiv \tau$, $C \in (\mathcal{F}/B)/\equiv$,

1. for all $\underline{a} \in \mathcal{E}_d$, whenever $\langle P, \sigma\rangle \xrightarrow{\underline{a}} \langle P', \sigma'\rangle \in C$, $\exists\ \langle Q', \tau'\rangle \in C$ with $\langle Q, \tau\rangle \xrightarrow{\underline{a}} \langle Q', \tau'\rangle$ and whenever $\langle Q, \tau\rangle \xrightarrow{\underline{a}} \langle Q', \tau'\rangle \in C$, $\exists\ \langle P', \sigma'\rangle \in C$ with $\langle P, \sigma\rangle \xrightarrow{\underline{a}} \langle P', \sigma'\rangle$.
2. for all $\overline{a} \in \mathcal{E}_s$, $r(\langle P, \sigma\rangle, \overline{a}, C) = r(\langle Q, \tau\rangle, \overline{a}, C)$.

- notation: $P \sim^{\equiv} Q$
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Properties of the Bisimulation

- $\sim^{\equiv}$ is a congruence
- This ensures that if P and Q are uncontrolled systems, and $P \sim^{\equiv} Q$, then if they are placed under the same controller then the controlled systems $P \bowtie_{*} C \sim^{\equiv} Q \bowtie_{*} C$.
- If $P \sim^{\equiv} Q$ are controlled systems, in bisimilar configurations the corresponding set of ODEs will be the same.

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Equivalence semantics for TDSHA

- TDSHA labelled bisimulation

  given a measurable relation $B \subseteq (Q_1 \times \mathbb{R}^{n_1}) \times (Q_2 \times \mathbb{R}^{n_2})$

  then for all $((q_1, \mathbf{x}_1), (q_2, \mathbf{x}_2)) \in B$

  - $\mathrm{out}_1(\mathbf{x}_1) = \mathrm{out}_2(\mathbf{x}_2)$
  - exit rates of q_1 and q_2 must be equal
  - disjunction of guards must evaluate to the same for $\mathbf{x}_1$ and $\mathbf{x}_2$
  - disjunction of guards must become true at the same time
  - for all $\underline{\mathrm{a}} \in \mathcal{E}_d$, one step priorities must match
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Results

- $\sim^{\equiv}$ is a congruence (under certain conditions on $\equiv$)
- if $Con_1 \sim^{\equiv} Con_2$ then $Sys \bowtie_{*} \underline{\mathrm{init}}.Con_1 \sim^{\equiv} Sys \bowtie_{*} \underline{\mathrm{init}}.Con_2$
- *additively equivalent*: $\sigma \doteq \tau$ iff for all $V \in \mathcal{V}$ and $f(\mathcal{W})$

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$$\sum \{\!| \, r \mid iv(\iota) = V, \sigma(\iota) = (r, I(W)), f(\mathcal{W}) = [\![I(W)]\!] \, |\!\}$$

- $P_1 \sim^{\doteq} P_2$ implies $\mathcal{T}(P_1) \sim^{\ell}_{T} \mathcal{T}(P_2)$

Results

- $\sim^{\equiv}$ is a congruence (under certain conditions on $\equiv$)
- if $Con_1 \sim^{\equiv} Con_2$ then $Sys \bowtie_* \underline{\mathrm{init}}.Con_1 \sim^{\equiv} Sys \bowtie_* \underline{\mathrm{init}}.Con_2$
- *additively equivalent*: $\sigma \doteq \tau$ iff for all $V \in \mathcal{V}$ and $f(\mathcal{W})$

$$\mathrm{sum}(\sigma, V, f(\mathcal{W})) = \mathrm{sum}(\tau, V, f(\mathcal{W}))$$

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Outline

Applications of stochastic HYPE

- biological systems
  - Repressilator: 3 gene system with inhibition
  - circadian clock of *Ostreococcus tauri*
- human-constructed systems
  - planetary orbiter
  - railway crossing (train gate)
  - opportunistic networks
- combined systems
  - Zebranet: MSc dissertation of Cheng Feng

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ZebraNet modelling

- animal-based opportunistic network
  - collect data from zebra with low human intervention
  - data sent from zebra to zebra, both wearing collars
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  - high latency is tolerated but lack of delivery is not
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- syntactic extension to allow definition of parameterised subcomponents and automated expansion
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  - two-dimensional model of zebra movement
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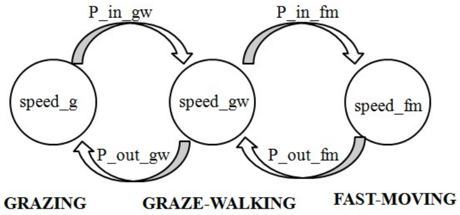
Mobility model of zebras

Zebras have three distinct movement patterns:
grazing, grazing-walking, and fast-moving.



Movement is also influenced by the proximity of watering holes and the state of thirstiness of the zebra.

The HYPE model captures all these influences on the (x, y)-position of the zebra.
Additional variables capture the speed and mode of movement, the thirstiness and the distance from the watering hole.

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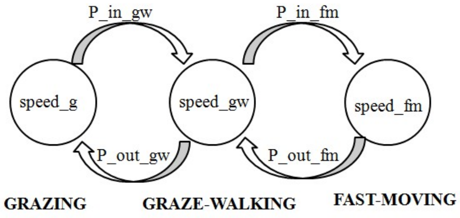

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Mobility model of zebras I

The variables recording the state of the zebra with respect to its movement include its x and y coordinates, its speed and direction of travel, its thirstiness, the nearest water source and its current mode of travel.

For example, the flow influencing the x-position of a zebra is represented by the subcomponent:

$$\begin{aligned} ZXmove = \underline{\mathrm{init}} &: (zebra\_x\#, 0, const).ZXmove + \\ \underline{\mathrm{move\_off}} &: (zebra\_x\#, 0, const).ZXmove + \\ \underline{\mathrm{move\_on}} &: (zebra\_x\#, 1, cos(D2R(angle\#)) * z\_speed\#).ZXmove \end{aligned}$$

Many more events and variables are used to give a faithful representation of zebra movement: fine-grained compositionality is used extensively.

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Mobility model of zebras II

Then, if we combine the two subcomponents of a zebra's movement, we get the component which represents the mobility model of zebras:

$$Comp_{mobility\_model} \stackrel{def}{=} ZXmove \underset{*}{\bowtie} ZebraYmove$$

Controller for the mobility model

Separate controllers are defined to impose appropriate constraints on each aspect affecting movement.

These are then combined to give the controller for the mobility model:

$$\begin{aligned} Con_{mobility\_model} &= Con_{move}[N] \underset{\emptyset}{\bowtie} Con_{speed\_change\_on}[N] \\ &\qquad \underset{\emptyset}{\bowtie} Con_{new\_day}[N] \end{aligned}$$

where N represents the number of zebras in the model.

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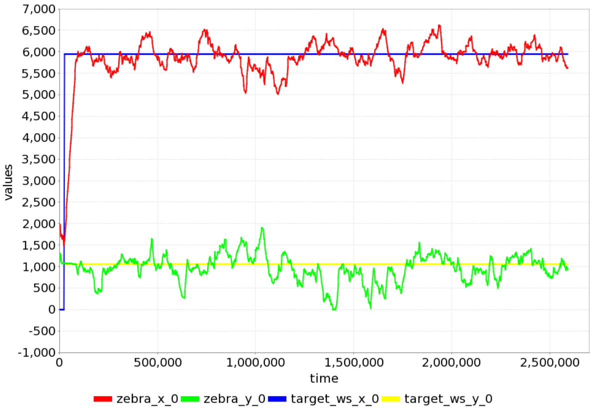
The trajectory of a zebra's position in one month



Solid lines show the position of the watering hole.

Other aspects

The other elements of zebra behaviour are modelled in a similar compositional style:

- data sharing between zebras and with the ferry
- battery consumption

The complete model is then the composition of the uncontrolled systems and controllers for each element:

$$\begin{aligned} Zebra &= Comp_{mobility\_model} \underset{*}{\bowtie} Comp_{data\_model} \underset{*}{\bowtie} Comp_{energy\_model} \\ Con_{zebra} &= Con_{mobility\_model} \underset{\emptyset}{\bowtie} Con_{data\_model} \underset{\emptyset}{\bowtie} Con_{energy\_model} \end{aligned}$$

A number of zebras are then combined with the data ferry and time:

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Results

The resulting model is 440 lines of HYPE definitions, compared with 5941 lines of code in C in the original ZNetSim model.

Moreover it was developed in less than three weeks.

Unfortunately the model suffers from flow and event explosion, meaning that computationally it is extremely expensive to simulate in the SimHyA tool.

The parameterised nature of the model means that we can represent arbitrary numbers of zebras but currently the simulation is limited to 6 zebras.

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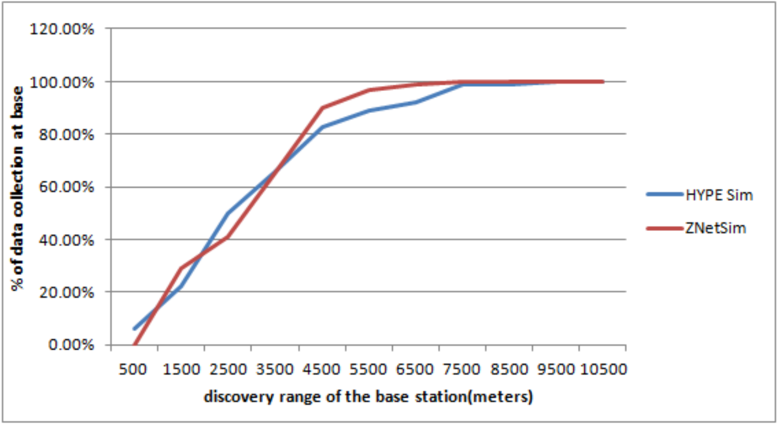
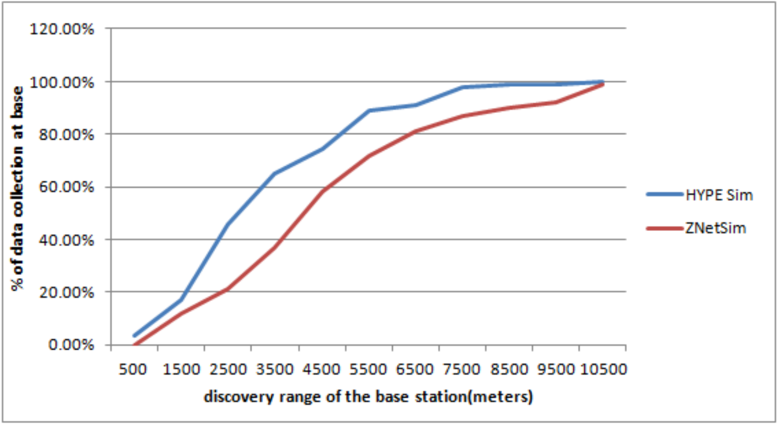
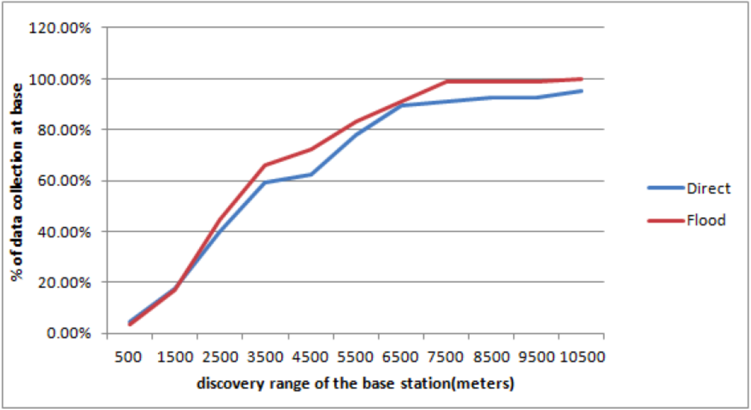


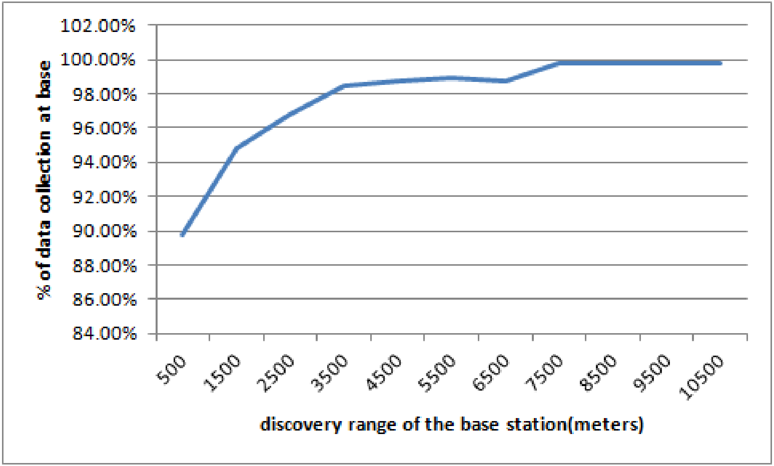
Comparison of success rate under constrained storage and bandwidth



Data collected by protocol



Data collected with different ranges for the mobile base station



References

- L. Bortolussi, V. Galpin and J. Hillston, *HYPE with stochastic events*, in the Proceedings of Intl. Workshop on Quantitative Analysis of Programming Languages, QAPL 2011, pp. 120–133, 2011.
- C. Feng, *Modelling Opportunistic Networks with HYPE*, MSc dissertation, School of Informatics, University of Edinburgh, September 2012.