Getting started with PEPA

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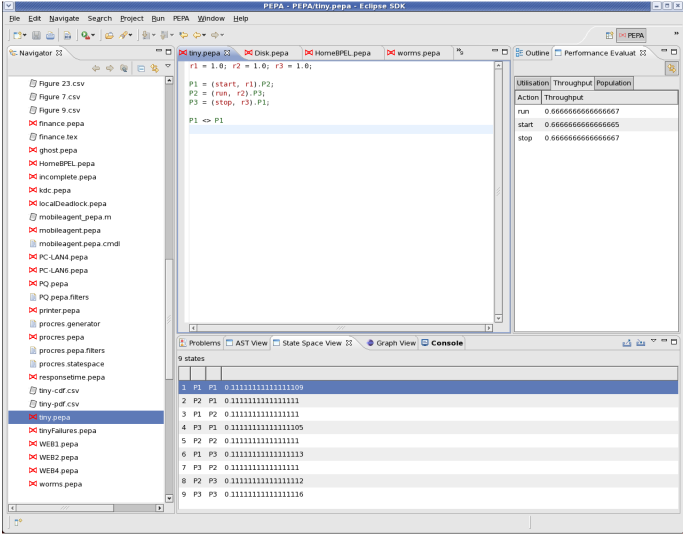
The PEPA Eclipse Plug-in processing the model



The PEPA website

`http://www.dcs.ed.ac.uk/pepa`

From the website the PEPA Eclipse Plug-in and some other tools are available for download.

There is also information about people involved in the PEPA project, projects undertaken and a collection of published papers.

Upgrading a PC LAN

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A node can transmit, only whilst it holds the token.

Upgrading a PC LAN

There are currently four PCs (or similar devices) connected to the LAN in a small office, but the company has recently recruited two new employees, each of whom will have a PC. Our task is to find out how the delay experienced by data packets at each PC will be affected if another two PCs are added.

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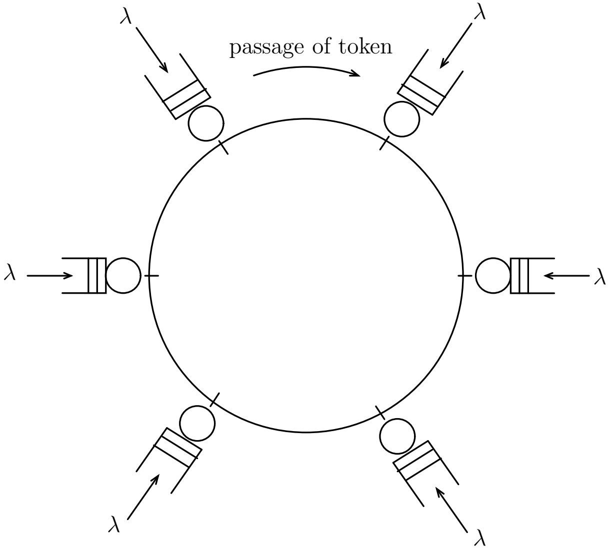
- The average rate that a PC generates data packets is λ.
- The mean duration of a data packet transmission is d ($d = 1/\mu$), and the mean time for the token to pass from one PC to the next is m ($m = 1/\omega$).
- Transmission is gated: each PC can transmit at most one data packet per visit of the token.


λ
λ
passage of token
λ
λ
λ
λ

Modelling the system: choosing components

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We will need another component to represent the medium. As remarked previously, the medium can be represented solely by the token.

Modelling the system: choosing actvities

The description of the PC is very simple in this case. It only has two activities which it can undertake:

- generate a data packet;
- transmit a data packet.

Moreover we are told that it can only hold one data packet at a time and so these activities must be undertaken sequentially.

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This suggests the following PEPA component for the *i*th PC:

$$
\begin{aligned}
PC_{i0} &\stackrel{\mathit{def}}{=} (\mathit{arrive}, \lambda).PC_{i1} \\
PC_{i1} &\stackrel{\mathit{def}}{=} (\mathit{transmit}_i, \mu).PC_{i0}
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This will need some refinement when we consider interaction with the token.

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- transmit a data packet if there is one to transmit and then walk on; or
- walk on at once if there is no data packet waiting.

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$$
\begin{aligned}
Token_i \stackrel{def}{=} & (walkon_{i+1}, \omega).Token_{i+1} + \\
& (transmit_i, \mu).(walk_{i+1}, \omega).Token_{i+1}
\end{aligned}
$$

Refining the components

In order to ensure that the token's choice is made dependent on the state of PC being visited, we add a walkon action to the PC when it is empty, and impose a cooperation between the PC and the Token for both walkon and transmit.

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\begin{aligned}
PC_{i0} &\stackrel{def}{=} (arrive, \lambda).PC_{i1} + (walkon_2, \omega).PC_{i0} \\
PC_{i1} &\stackrel{def}{=} (transmit_i, \mu).PC_{i0}
\end{aligned}
$$

Complete model: four PC case

$$PC_{10} \stackrel{def}{=} (arrive, \lambda).PC_{11} + (walkon_2, \omega).PC_{10}$$
$$PC_{11} \stackrel{def}{=} (transmit_1, \mu).PC_{10}$$

$$PC_{20} \stackrel{def}{=} (arrive, \lambda).PC_{21} + (walkon_3, \omega).PC_{20}$$
$$PC_{21} \stackrel{def}{=} (transmit_2, \mu).PC_{20}$$

$$PC_{30} \stackrel{def}{=} (arrive, \lambda).PC_{31} + (walkon_4, \omega).PC_{30}$$
$$PC_{31} \stackrel{def}{=} (transmit_3, \mu).PC_{30}$$

$$PC_{40} \stackrel{def}{=} (arrive, \lambda).PC_{41} + (walkon_1, \omega).PC_{40}$$
$$PC_{41} \stackrel{def}{=} (transmit_4, \mu).PC_{40}$$

$$
\begin{aligned}
Token_1 &\stackrel{def}{=} (walkon_2, \omega).Token_2 + (transmit_1, \mu).(walk_2, \omega).Token_2 \\
Token_2 &\stackrel{def}{=} (walkon_3, \omega).Token_3 + (transmit_2, \mu).(walk_3, \omega).Token_3 \\
Token_3 &\stackrel{def}{=} (walkon_4, \omega).Token_4 + (transmit_3, \mu).(walk_4, \omega).Token_4 \\
Token_4 &\stackrel{def}{=} (walkon_1, \omega).Token_1 + (transmit_4, \mu).(walk_1, \omega).Token_1
\end{aligned}
$$

$$
\begin{aligned}
LAN &\stackrel{def}{=} (PC_{10} \parallel PC_{20} \parallel PC_{30} \parallel PC_{40}) \underset{L}{\bowtie} Token_1 \\
&\quad \text{where } L = \{walkon_1, walkon_2, walkon_3, walkon_4, \\
&\qquad transmit_1, transmit_2, transmit_3, transmit_4\}.
\end{aligned}
$$

Here we have arbitrarily chosen a starting state in which all the PCs are empty and the Token is at PC1.