SPAs for performance modelling: Lecture 10 — Modelling hybrid systems with Stochastic HYPE

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19th April 2013

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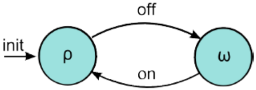
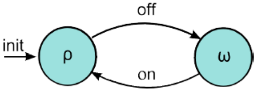
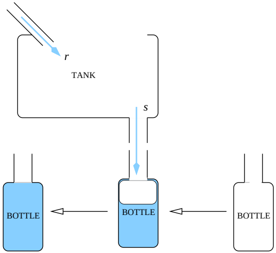
Hybrid Systems



Hybrid Systems





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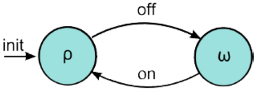

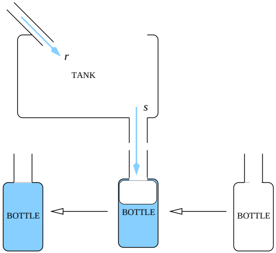

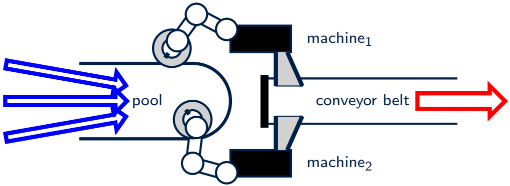

Outline

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We have also been motivated by incorporating more detailed representation of space within our process algebra models.

Other formal approaches to hybrid systems

Hybrid automata are a well-established approach to modelling hybrid systems which are supported by a number of tools and analysis techniques. Their drawbacks are that they are graphical rather than textual, and the approach is not generally compositional.

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- ACP^{srt}_{hs} — Bergstra and Middelburg
- HyPA — Cuijpers and Reniers
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These take a coarse-grained approach, with ODEs embedded within the syntax.

Introduction to Stochastic HYPE

- behaviours to be included
  - discrete behaviour: instantaneous events
  - continuous behaviour: ordinary differentials equations (ODEs)
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  - notions of equivalence
- the original definition of HYPE
  - only discrete and continuous behaviour
  - operational semantics define labelled transition system
  - mapping from labelled transition system to hybrid automaton

HYPE actions

We distinguish two types of actions in a system:

- events — instantaneous, discrete changes

$$\underline{a} \in \mathcal{E}$$

Each event is associated with an event condition: activation conditions and variable resets.

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- flows in HYPE ($W_j \subseteq \mathcal{V}$)

$$\begin{array}{ccccc} A_1 & \stackrel{def}{=} & \ldots & (\iota_1, r_1, I_1(\mathcal{W}_1)) & \ldots \\ \vdots & \vdots & & \vdots & \\ A_n & \stackrel{def}{=} & \ldots & (\iota_n, r_n, I_n(\mathcal{W}_n)) & \ldots \end{array}$$

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$$\text{and} \quad \frac{dV}{dt} = \sum \{ r_j . I_j(\mathcal{W}_j) \mid iv(\iota_j) = V, \dots \}$$

Stochastic HYPE

In addition to the (instantaneous) events and activities, we now also allow stochastic events.

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This is now generalised to events not triggered by system variable values but according to a random variable, which may depend on the value of system variables.

Stochastic HYPE model I

- Subcomponents

$$S ::= \underline{\mathrm{a}} : \alpha.C_s \mid \bar{a} : \alpha.C_s \mid S + S$$

where $\underline{\mathrm{a}} \in \mathcal{E}_d,\ \bar{a} \in \mathcal{E}_s,\ \mathcal{E}_d \cup \mathcal{E}_s = \mathcal{E},\ \alpha \in \mathcal{A}ct$

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Stochastic HYPE model II

- Uncontrolled System

$$\Sigma ::= C_s(\overrightarrow{V}) \mid C(\overrightarrow{V}) \mid \Sigma \underset{L}{\bowtie} \Sigma \quad L \subseteq \mathcal{E}$$

where $\overrightarrow{V}$ are system variables (cf. $\overrightarrow{X}$ of C or C_s).

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- Controllers only have events:

$$M ::= \underline{\mathrm{a}}.M \mid 0 \mid M + M \quad \underline{\mathrm{a}} \in \mathcal{E}, L \subseteq \mathcal{E}$$

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$$ConSys ::= \Sigma \underset{L}{\bowtie} \underline{\mathrm{init}}.Con \quad L \subseteq \mathcal{E}.$$

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Stochastic HYPE model III

uncontrolled system — controllers/sequencers

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subcomponents are parameterised by variables

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Stochastic HYPE model III

uncontrolled system controllers/sequencers

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$ec(\overline{\mathrm{a}}_j) = (f(\mathcal{V}), \mathcal{V}' = f'(\mathcal{V}))$ with $f : \mathbb{R}^{|\mathcal{V}|} \to [0, \infty)$ stochastic

Stochastic HYPE model III

uncontrolled system

controllers/sequencers

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influence names are mapped to variables

$$iv(\iota_j) \in \mathcal{V}$$

Stochastic HYPE model IV

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controller grammar

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$$M ::= \mathrm{a}.M \mid 0 \mid M + M$$

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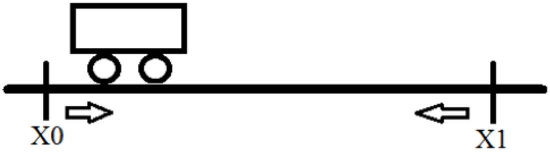
Outline

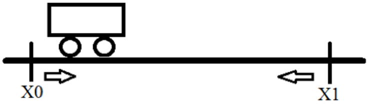
Simple Example: shuttle bus

We consider the simple example of an idealised shuttle bus which serves two stops, $X0$ and $X1$.



When the shuttle bus arrives at one stop, it will stop for a while, say 5 minutes, and then move to the other stop. Thus, there are two flows influencing the shuttle bus. The first is the time flow and the other influences the position of the shuttle bus.

Shuttle bus example



We represent the two flows by two subcomponents below:

$$\begin{aligned}
\mathit{Movement} &= \underline{\mathrm{init}} : (x, 0, \mathit{const}).\mathit{Movement} \\
&+ \overline{\mathrm{toX0}} : (x, -s, \mathit{const}).\mathit{Movement} \\
&+ \underline{\mathrm{toX1}} : (x, s, \mathit{const}).\mathit{Movement} \\
&+ \underline{\mathrm{stop}} : (x, 0, \mathit{const}).\mathit{Movement} \\
\mathit{Time} &= \underline{\mathrm{init}} : (t, 1, \mathit{const}).\mathit{Time}
\end{aligned}$$

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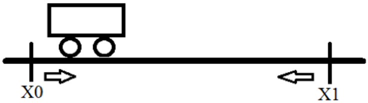

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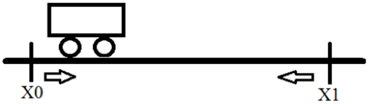
$$

In *Movement*, there are three distinct activities:

- $(x, 0, const)$ — stopped at a station;
- $(x, -s, const)$ — travelling from $X1$ to $X0$; and
- $(x, s, const)$ — travelling from $X0$ to $X1$.

Shuttle bus example: uncontrolled system



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$$Sys \stackrel{def}{=} Movement \underset{init}{\bowtie} Time$$

Note that no causal or temporal constraints on the events have been imposed yet (hence "uncontrolled").

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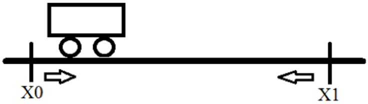

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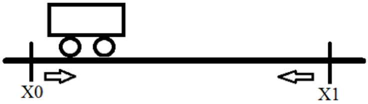
For instance, we need to specify that the shuttle bus can only move to $X1$ when it has previously moved to $X0$ and stopped for 5 minutes.

Shuttle bus example: controller



$$Con_{movement} = \underline{\mathrm{stop}}.\underline{\mathrm{toX1}}.\underline{\mathrm{stop}}.\overline{\mathrm{toX0}}.Con_{movement}$$

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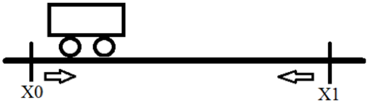

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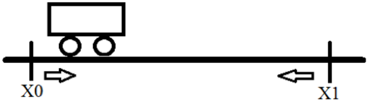
The controlled system is constructed from synchronization of the controller and the uncontrolled system:

$$ShuttleBusCtrl = Sys \underset{M}{\bowtie} \underline{\text{init}}.Con_{movement}$$

with $M = \{\underline{\text{init}}, \overline{\text{toX0}}, \underline{\text{toX1}}, \underline{\text{stop}}\}$.

Influences and event conditions



We need to link each influence with an actual variable, e.g.

$$iv(x) = Pos, \quad iv(t) = T$$

where Pos captures the position of the shuttle bus, T, the current time. In this case we define the influence types as $const = 1$.

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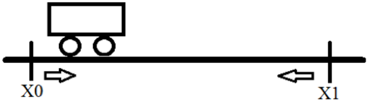

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We also define the event conditions ec to trigger each event:

$$\begin{aligned}
ec(\underline{\text{init}}) &= (true, Pos' = X_0 \wedge T' = 0) \\
ec(\underline{\text{stop}}) &= (Pos \leq X_0 \vee Pos \geq X_1, ArrivalTime' = Time) \\
ec(\underline{\text{toX1}}) &= (Pos \leq X_0 \wedge T - ArrivalTime == five_min, true) \\
ec(\overline{\text{toX0}}) &= (f = e^{-5(T - ArrivalTime)}, true)
\end{aligned}$$

Outline

Semantics for HYPE

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To get an executable interpretation of a model we map to a form of hybrid automaton:
Transition-Driven Stochastic Hybrid Automata (TDSHA),
which are themselves given a semantics in terms of
Piecewise Deterministic Markov Processes (PDMP).

Operational semantics

Prefix with influence:

$$\frac{}{\langle \mathrm{a}\!:\!(\iota, r, I).E, \sigma \rangle \xrightarrow{a} \langle E, \sigma[\iota \mapsto (r, I)] \rangle}$$

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$$\frac{\langle E, \sigma \rangle \xrightarrow{a} \langle E', \sigma' \rangle}{\langle E + F, \sigma \rangle \xrightarrow{a} \langle E', \sigma' \rangle} \qquad \frac{\langle F, \sigma \rangle \xrightarrow{a} \langle F', \sigma' \rangle}{\langle E + F, \sigma \rangle \xrightarrow{a} \langle F', \sigma' \rangle}$$

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- Γ is defined for all well-defined stochastic HYPE models
  - syntactic restrictions on influences and events

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  - pairs of modes and union of variables
  - combining transitions
    (with conditions on resets and initial values)

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- class of stochastic processes
- continuous trajectories over subsets of $\mathbb{R}^{|\mathbf{X}|}$
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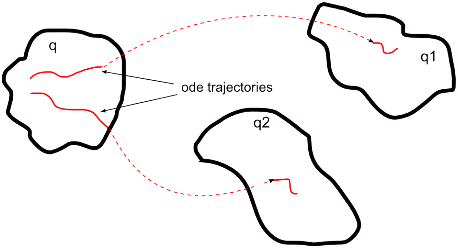

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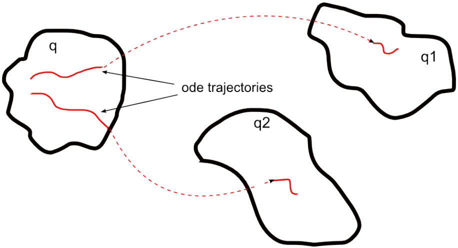

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- we have defined an algorithm to check when a stochastic HYPE model is well-behaved.

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- If $P \sim^{\equiv} Q$ are controlled systems, in bisimilar configurations the corresponding set of ODEs will be the same.

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Outline

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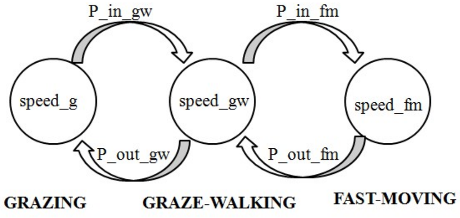
Mobility model of zebras

Zebras have three distinct movement patterns:
grazing, grazing-walking, and fast-moving.



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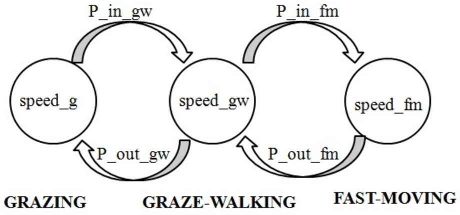

Movement is also influenced by the proximity of watering holes and the state of thirstiness of the zebra.

The HYPE model captures all these influences on the (x, y)-position of the zebra.
Additional variables capture the speed and mode of movement, the thirstiness and the distance from the watering hole.

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The variables recording the state of the zebra with respect to its movement include its x and y coordinates, its speed and direction of travel, its thirstiness, the nearest water source and its current mode of travel.

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For example, the flow influencing the x-position of a zebra is represented by the subcomponent:

$$\begin{aligned} ZXmove = {} & \underline{\mathrm{init}} : (zebra\_x\#, 0, const).ZXmove + \\ & \underline{\mathrm{move\_off}} : (zebra\_x\#, 0, const).ZXmove + \\ & \underline{\mathrm{move\_on}} : (zebra\_x\#, 1, cos(D2R(angle\#)) * z\_speed\#).ZXmove \end{aligned}$$

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Many more events and variables are used to give a faithful representation of zebra movement: fine-grained compositionality is used extensively.

Mobility model of zebras II

Then, if we combine the two subcomponents of a zebra's movement, we get the component which represents the mobility model of zebras:

$$Comp_{mobility\_model} \stackrel{def}{=} ZXmove \underset{*}{\bowtie} ZebraYmove$$

Controller for the mobility model

Separate controllers are defined to impose appropriate constraints on each aspect affecting movement.

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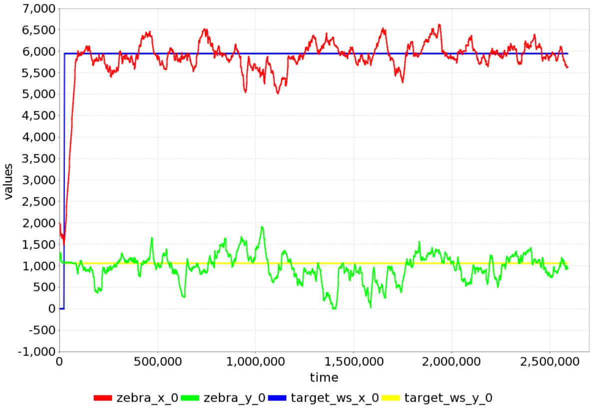
These are then combined to give the controller for the mobility model:

$$Con_{mobility\_model} = Con_{move}[N] \underset{\emptyset}{\bowtie} Con_{speed\_change\_on}[N] \underset{\emptyset}{\bowtie} Con_{new\_day}[N]$$

where N represents the number of zebras in the model.

The trajectory of a zebra's position in one month



Solid lines show the position of the watering hole.

Other aspects

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The complete model is then the composition of the uncontrolled systems and controllers for each element:

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A number of zebras are then combined with the data ferry and time:

$$\begin{aligned} \mathit{Sys} &= \mathit{Zebra}[N] \underset{*}{\bowtie} \mathit{Ferry} \underset{*}{\bowtie} \mathit{Time} \\ \mathit{Con} &= \mathit{Con}_{\mathit{zebra}} \underset{\emptyset}{\bowtie} \mathit{Con}_{\mathit{ferry}} \underset{\emptyset}{\bowtie} \mathit{Con}_{\mathit{time}} \\ \mathit{ZebraNetCtrl} &= \mathit{Sys} \underset{*}{\bowtie} \underline{\mathrm{init}}.\mathit{Con} \end{aligned}$$

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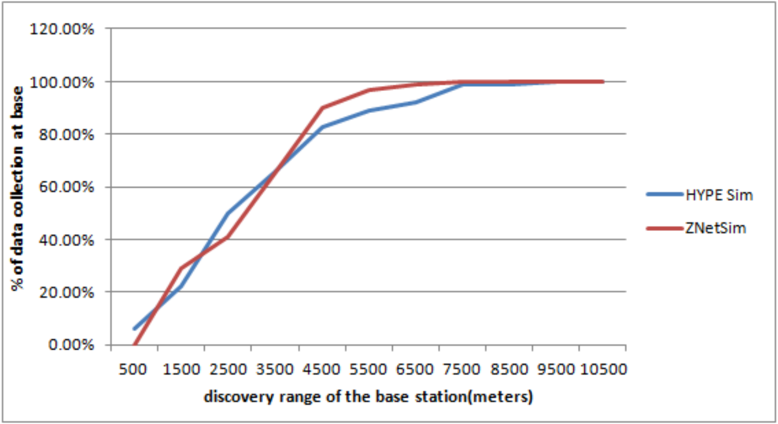
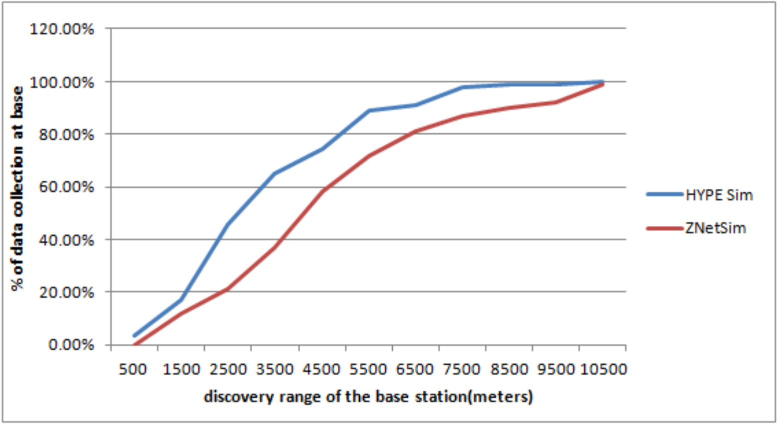
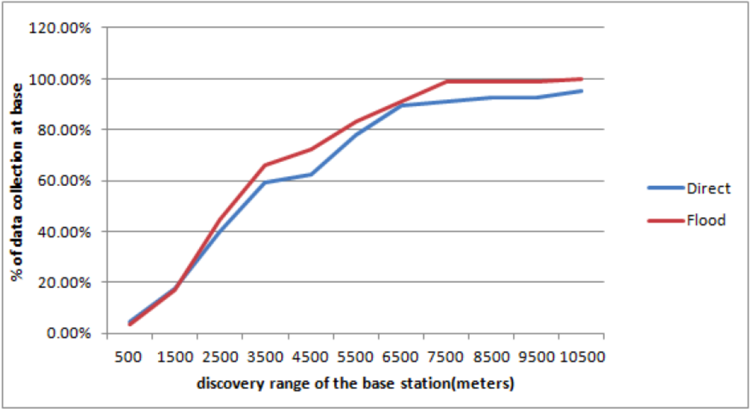
Comparison of success rate under infinite storage and bandwidth



Comparison of success rate under constrained storage and bandwidth



Data collected by protocol



Data collected with different ranges for the mobile base station