SPAs for performance modelling: Lecture 2 — Stochastic Process Algebras

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THE UNIVERSITY
of EDINBURGH

Outline

1 Process algebra and Markov processes

2 A semantics for PEPA — informally

3 A formal semantics for PEPA

4 The nature of synchronisation

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1 Process algebra and Markov processes

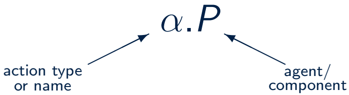
2 A semantics for PEPA — informally

3 A formal semantics for PEPA

4 The nature of synchronisation

Process Algebra

- Models consist of agents which engage in actions.



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action type or name

agent/ component

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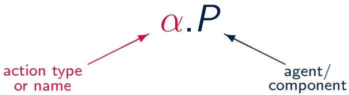

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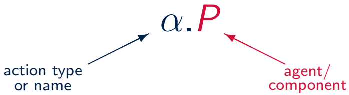

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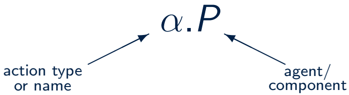

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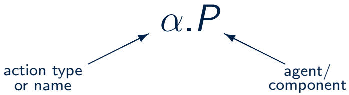

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Process algebra model

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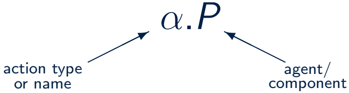

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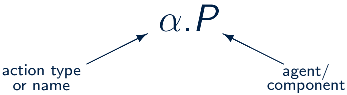

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Consider a web server which offers html pages for download:

$$Server \overset{def}{=} get.download.rel.Server$$

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Its clients might be web browsers, in a domain with a local cache of frequently requested pages. Thus any display request might result in an access to the server or in a page being loaded from the cache.

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A simple version of the Web can be considered to be the interaction of these components:

$$WEB \overset{def}{=} \big(Browser \parallel Browser\big) \mid Server$$

Qualitative Analysis

- The labelled transition system underlying a process algebra model can be used for functional verification e.g.: reachability analysis, specification matching and model checking.

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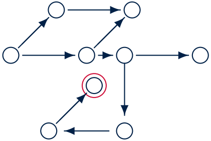
Will the system arrive
in a particular state?



Qualitative Analysis

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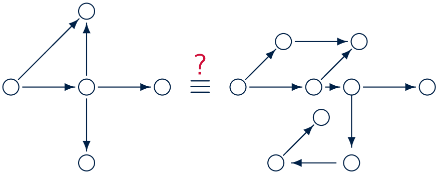
Does system behaviour match its specification?



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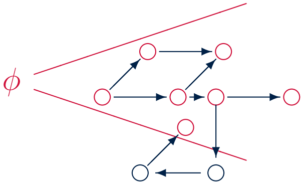
Does a given property ϕ hold within the system?



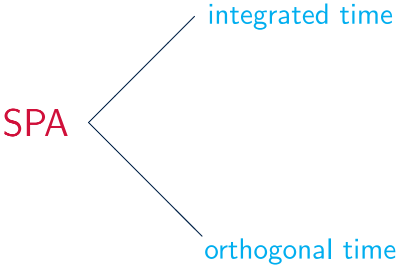
Stochastic process algebras

Process algebras where models are decorated with quantitative information used to generate a stochastic process are stochastic process algebras (SPA).

SPA Languages

SPA

SPA Languages

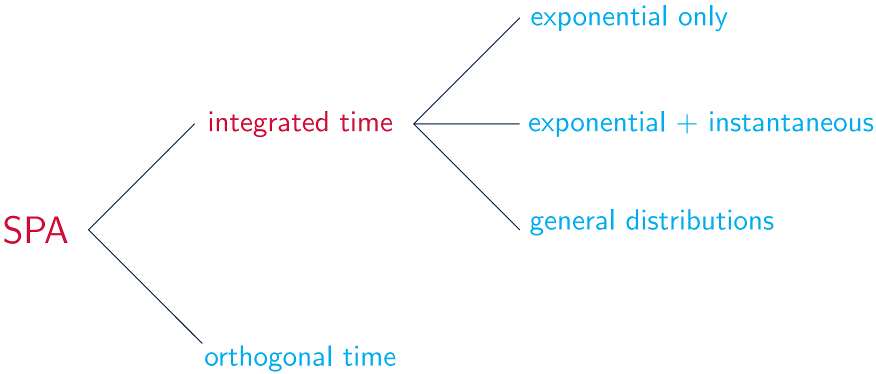


SPA

integrated time

orthogonal time

SPA Languages



SPA

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exponential only

exponential + instantaneous

general distributions

orthogonal time

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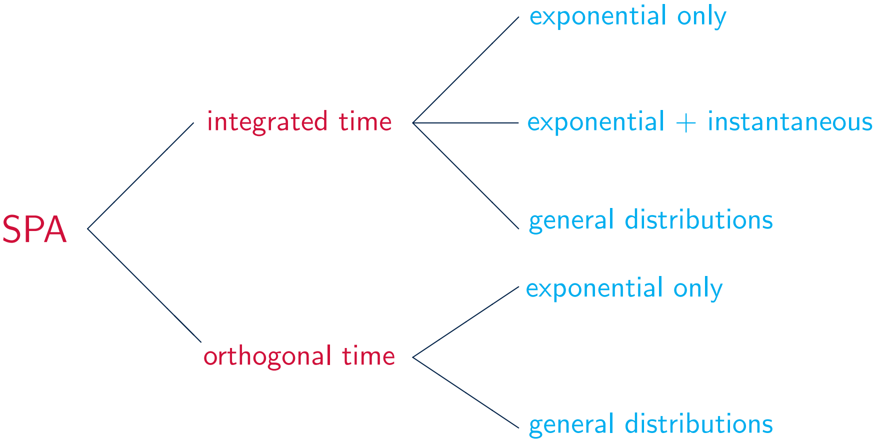

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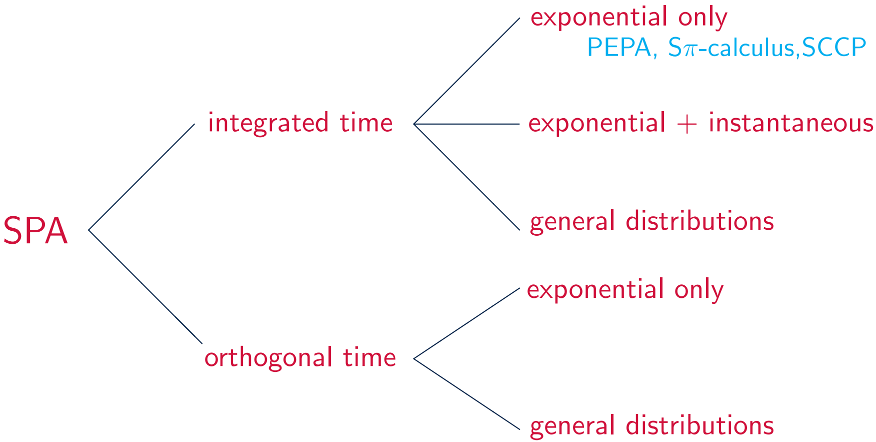

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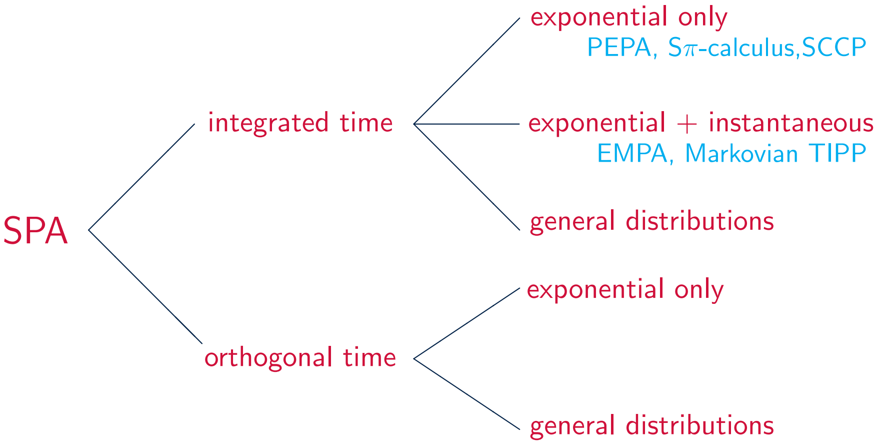

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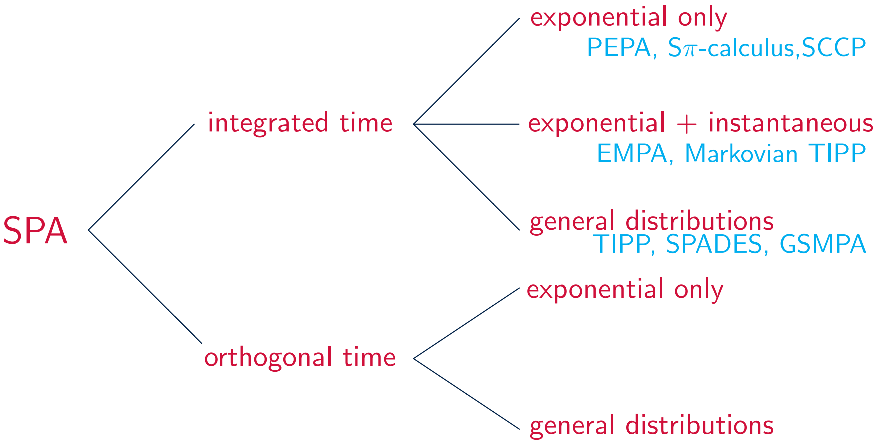

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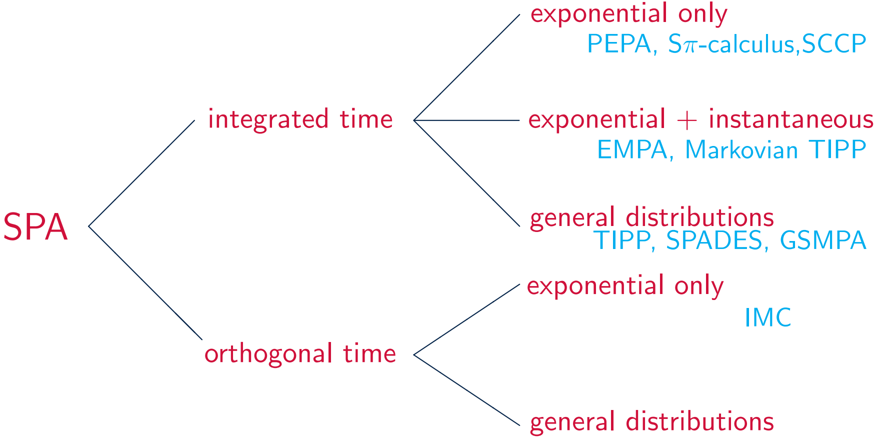

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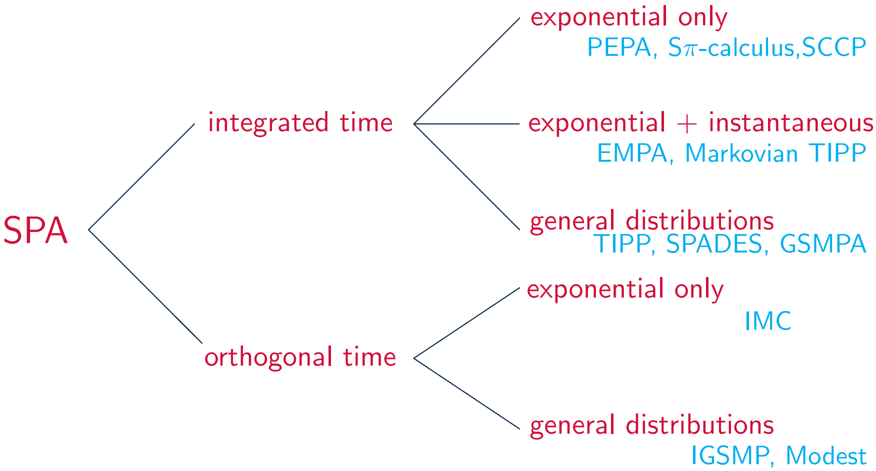

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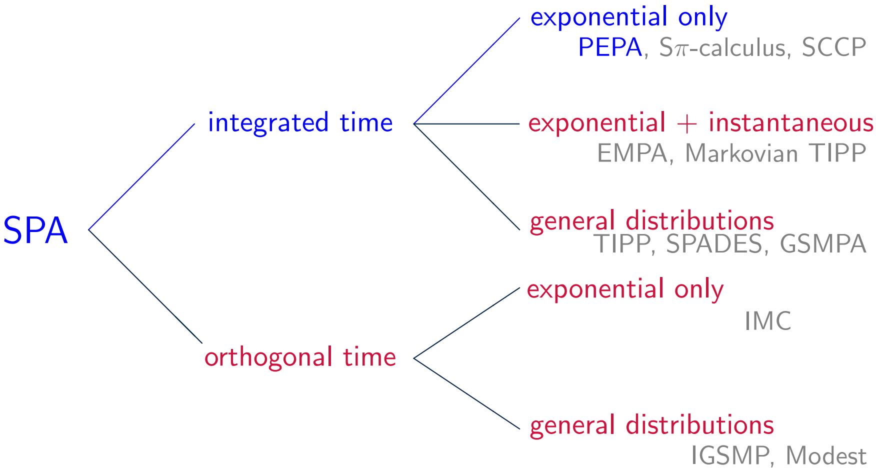

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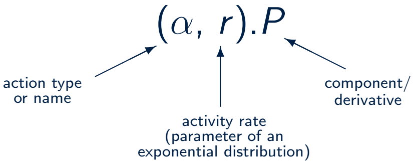
- From the process algebra side the Markov chain had a profound influence on the design of the language and in particular on the interactions between components.

- From the Markov chain perspective the process algebra structure has been exploited to find aspects of independence even between interacting components.

Performance Evaluation Process Algebra

- Models are constructed from components which engage in activities.



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action type or name

activity rate (parameter of an exponential distribution)

component/ derivative

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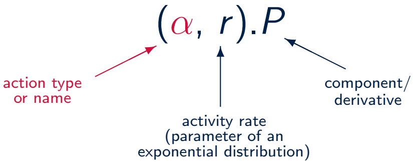

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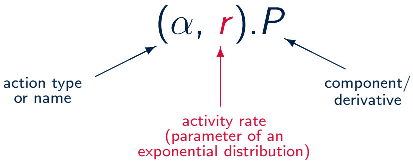

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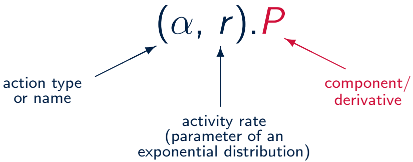

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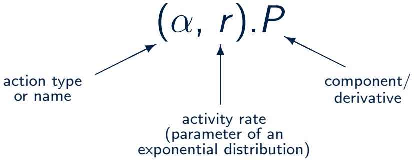

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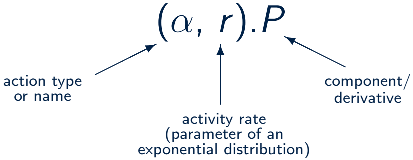

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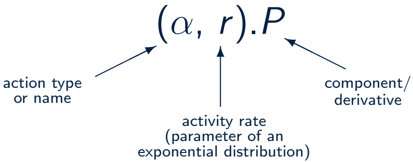

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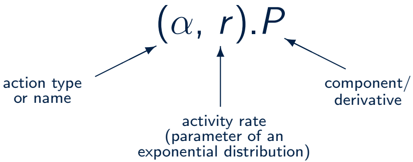

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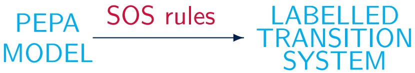

PEPA MODEL

SOS rules

LABELLED TRANSITION SYSTEM

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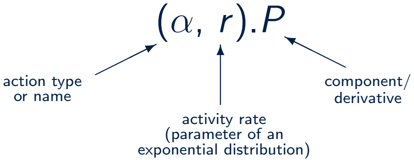

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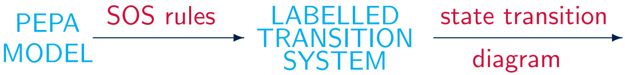

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state transition

diagram

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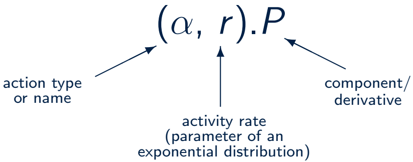

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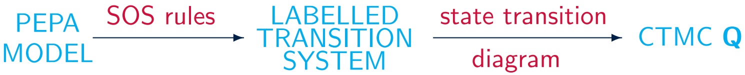

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HIDING:	P/L	abstraction $\alpha \in L \Rightarrow \alpha \rightarrow \tau$

Example: Browsers, server and download

$$
\begin{aligned}
Server &\stackrel{\mathit{def}}{=} (get, \top).(download, \mu).(rel, \top).Server \\
Browser &\stackrel{\mathit{def}}{=} (display, p\lambda).(get, g).(download, \top).(rel, r).Browser \\
&\quad + (display, (1-p)\lambda).(cache, m).Browser \\
WEB &\stackrel{\mathit{def}}{=} \big(Browser \parallel Browser\big) \underset{L}{\bowtie} Server
\end{aligned}
$$

where $L = \{get, download, rel\}$

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PEPA activities and rates

When enabled an activity, $a = (\alpha, \lambda)$, will delay for a period determined by its associated distribution function, i.e. the probability that the activity a happens within a period of time of length t is $F_a(t) = 1 - e^{-\lambda t}$.

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- When the first timer finishes that activity takes place—the activity is said to complete or succeed.
- This means that the activity is considered to “happen”: an external observer will witness the event of activity of type α.
- An activity may be preempted, or aborted, if another one completes first.

PEPA and Markov processes

In a PEPA model if we define the stochastic process $X(t)$, such that $X(t) = C_i$ indicates that the system behaves as component C_i at time t, then $X(t)$ is a Markov process which can be characterised by a matrix, $\boldsymbol{Q}$.

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This distribution is found by solving the global balance equation

$$\boldsymbol{\pi}\boldsymbol{Q} = \mathbf{0}$$

subject to the normalisation condition

$$\sum \boldsymbol{\pi}(C_i) = 1.$$

PEPA and time

All PEPA models are time-homogeneous since all activities are time-homogeneous: the rate and type of activities enabled by a component are independent of time.

PEPA and irreducibility and positive-recurrence

The other conditions, irreducibility and positive-recurrent states, are easily expressed in terms of the derivation graph of the PEPA model.

We only consider PEPA models with a finite number of states so if the model is irreducible then all states must be positive-recurrent i.e. the derivation graph is strongly connected.

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In terms of the PEPA model this means that all behaviours of the system must be recurrent; in particular, for every choice, whichever path is chosen it must eventually return to the point where the choice can be made again, possibly with a different outcome.

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In the rules, the derivation below the line can be inferred from the premise above the line.

Structured Operational Semantics

PEPA is defined using a Plotkin-style structured operational semantics (a “small step” semantics).

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In the rules, the derivation below the line can be inferred from the premise above the line.

Note that in this semantics the rate information is only treated as an additional label.

Structured Operational Semantics: Prefix and Choice

Prefix

$$\frac{}{(\alpha, r).E \xrightarrow{(\alpha,r)} E}$$

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Choice

$$\frac{E \xrightarrow{(\alpha,r)} E'}{E + F \xrightarrow{(\alpha,r)} E'}$$

$$\frac{F \xrightarrow{(\alpha,r)} F'}{E + F \xrightarrow{(\alpha,r)} F'}$$

Structured Operational Semantics: Cooperation ($\alpha \notin L$)

Cooperation

$$\frac{E \xrightarrow{(\alpha,r)} E'}{E \underset{L}{\bowtie} F \xrightarrow{(\alpha,r)} E' \underset{L}{\bowtie} F} \; (\alpha \notin L)$$

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Structured Operational Semantics: Cooperation ($\alpha \in L$)

Cooperation $$\frac{E \xrightarrow{(\alpha,r_1)} E' \qquad F \xrightarrow{(\alpha,r_2)} F'}{E \underset{L}{\bowtie} F \xrightarrow{(\alpha,R)} E' \underset{L}{\bowtie} F'} \ (\alpha \in L)$$

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where $$R = \frac{r_1}{r_\alpha(E)} \frac{r_2}{r_\alpha(F)} min(r_\alpha(E), r_\alpha(F))$$

Apparent Rate

$$
\begin{aligned}
r_\alpha((\beta, r).P) &= \begin{cases} r & \beta = \alpha \\ 0 & \beta \neq \alpha \end{cases} \\
r_\alpha(P + Q) &= r_\alpha(P) + r_\alpha(Q) \\
r_\alpha(A) &= r_\alpha(P) \qquad \text{where } A \overset{def}{=} P \\
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$$

Structured Operational Semantics: Hiding

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$$\frac{E \xrightarrow{(\alpha,r)} E'}{E/L \xrightarrow{(\tau,r)} E'/L}\ (\alpha \in L)$$

Structured Operational Semantics: Constants

Constant

$$\frac{E \xrightarrow{(\alpha,r)} E'}{A \xrightarrow{(\alpha,r)} E'} \; (A \overset{\mathit{def}}{=} E)$$

Properties of the definition (1)

PEPA has no “nil” (a deadlocked process).

This is because the PEPA language is intended for modelling non-stop processes (such as Web servers, operating systems, or manufacturing processes) rather than for modelling terminating processes (a compilation, a sorting routine, and so forth).

Creating a deadlocked process

When we are interested in transient behaviour we use the deadlocked process *Stop* to signal a component which performs no further actions.

$$Stop \overset{def}{=} \Big(((a, r).Stop) \underset{\{a,b\}}{\bowtie} ((b, r).Stop)\Big)/\{\, a, b \,\}$$

Properties of the definition (2)

Cooperation in PEPA is multi-way. Two, three, four or more partners may cooperate, and they all need to synchronise for the activity to happen.

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This is used to have "witnesses" to events (known as stochastic probes).

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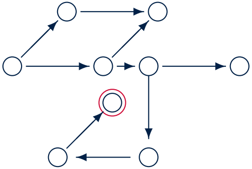
- As we have seen a continuous time Markov chain (CTMC) is generated from a PEPA model via its structured operational semantics.
- Linear algebra is used to solve the model in terms of equilibrium behaviour.
- The resulting probability distribution is seldom the ultimate goal of performance analysis; a modeller derives performance measures from this distribution via a reward structure.

Integrated analysis

Qualitative verification can now be complemented by quantitative verification.

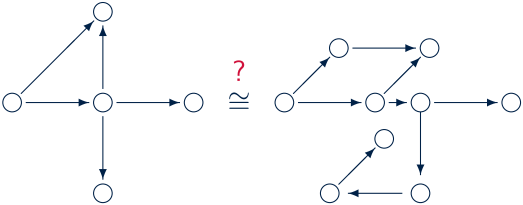
Integrated analysis: Reachability analysis

How long will it take for the system to arrive in a particular state?



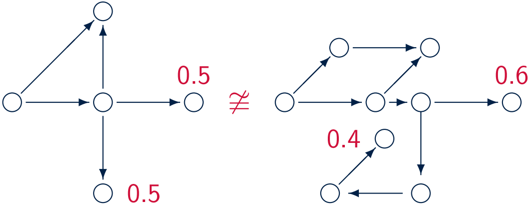
Integrated analysis: Specification matching

With what probability does system behaviour match its specification?



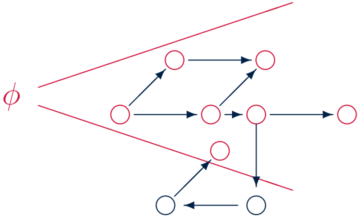
Integrated analysis: Specification matching

Does the "*frequency profile*" of the system match that of the specification?



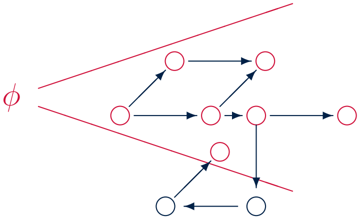
Integrated analysis: Model checking

Does a given property ϕ hold within the system with a given probability?



Integrated analysis: Model checking

For a given starting state
how long is it until
a given property ϕ holds?



The Importance of Being Exponential

$$\begin{tikzcd}
 & (\alpha, r).\mathit{Stop} \parallel (\beta, s).\mathit{Stop} \arrow[dl, "{(\alpha, r)}"'] \arrow[dr, "{(\beta, s)}"] & \\
\mathit{Stop} \parallel (\beta, s).\mathit{Stop} \arrow[dr, "{(\beta, s)}"'] & & (\alpha, r).\mathit{Stop} \parallel \mathit{Stop} \arrow[dl, "{(\alpha, r)}"] \\
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The memoryless property of the negative exponential distribution means that residual times do not need to be recorded.

The exponential distribution and the expansion law

We retain the expansion law of classical process algebra:

$$\begin{aligned}&(\alpha, r).\mathit{Stop} \parallel (\beta, s).\mathit{Stop} = \\ &\qquad (\alpha, r).(\beta, s).(\mathit{Stop} \parallel \mathit{Stop}) + (\beta, s).(\alpha, r).(\mathit{Stop} \parallel \mathit{Stop})\end{aligned}$$

only if the negative exponential distribution is used.

Outline

1. Process algebra and Markov processes
2. A semantics for PEPA — informally
3. A formal semantics for PEPA
4. The nature of synchronisation

Parallel Composition

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- In classical process algebra is it often associated with communication.

- When the activities of the process algebra have a duration the definition of parallel composition becomes more complex.

- The issue of what it means for two timed activities to synchronise is a vexed one....

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CCS-style

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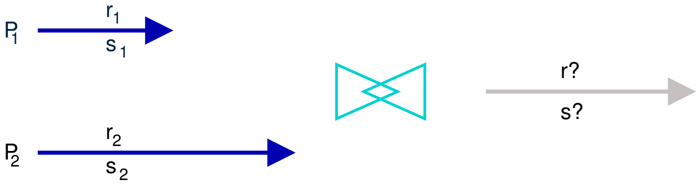
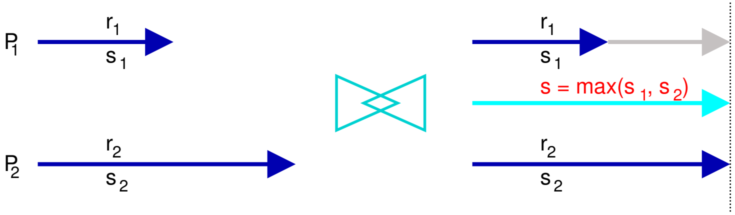
Most stochastic process algebras adopt CSP-style synchronisation.

Timed Synchronisation

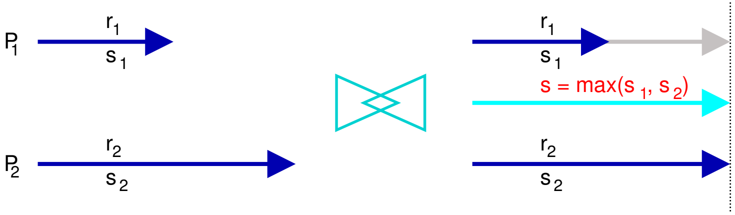


Timed Synchronisation



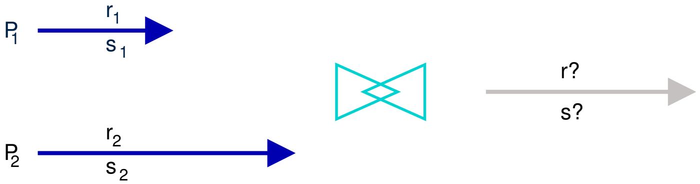
Barrier Synchronisation

Timed Synchronisation



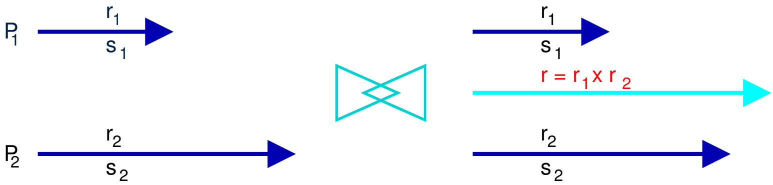
s is no longer exponentially distributed

Timed Synchronisation



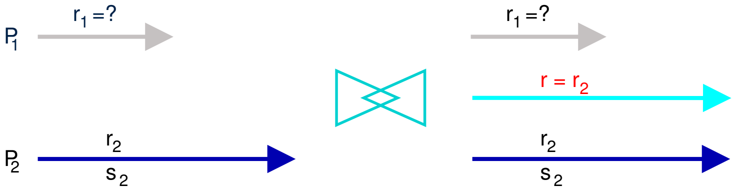
algebraic considerations limit choices

Timed Synchronisation



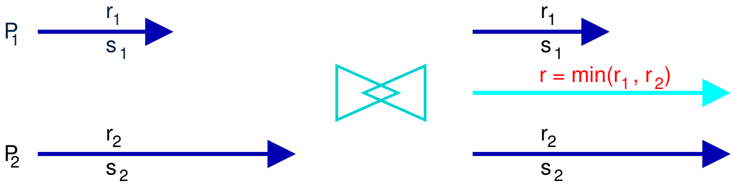
TIPP: new rate is product of individual rates

Timed Synchronisation



EMPA: one participant is passive

Timed Synchronisation



bounded capacity: new rate is the minimum of the rates

Bounded capacity

Within the cooperation framework, PEPA assumes bounded capacity: that is, a component cannot be made to perform an activity faster by cooperation, so the rate of a shared activity is the minimum of the apparent rates of the activity in the cooperating components.

Apparent Rate

$$
\begin{array}{rcll}
r_\alpha((\beta, r).P) & = & \begin{cases} r & \beta = \alpha \\ 0 & \beta \neq \alpha \end{cases} \\[2ex]
r_\alpha(P + Q) & = & r_\alpha(P) + r_\alpha(Q) \\[2ex]
r_\alpha(A) & = & r_\alpha(P) & \text{where } A \overset{def}{=} P \\[2ex]
r_\alpha(P \underset{L}{\bowtie} Q) & = & \begin{cases} r_\alpha(P) + r_\alpha(Q) & \alpha \notin L \\ min(r_\alpha(P), r_\alpha(Q)) & \alpha \in L \end{cases} \\[2ex]
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\end{aligned}$$

This is used to calculate pairwise cooperation rates: the overall rate of cooperation must not exceed either of the constituent apparent rates.

Cooperation in PEPA

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- Synchronisation, or cooperation cannot make a component exceed its bounded capacity.
- Thus the apparent rate of a cooperation is the minimum of the apparent rates of the co-operands.