Outline

1. Process algebra and Markov processes
2. A semantics for PEPA — informally
3. A formal semantics for PEPA
4. The nature of synchronisation
Outline

1. Process algebra and Markov processes
2. A semantics for PEPA — informally
3. A formal semantics for PEPA
4. The nature of synchronisation
Models consist of **agents** which engage in **actions**.

\[ \alpha . P \]
Process Algebra

- Models consist of agents which engage in actions.

\[ \alpha . P \]

- action type or name
- agent/component
Models consist of agents which engage in actions.

\[ \alpha \cdot P \]

- action type or name
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Process Algebra

- Models consist of agents which engage in actions.

\[ \alpha.P \]

- The structured operational (interleaving) semantics of the language is used to generate a labelled transition system.
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\[ \alpha.P \]

Action type or name \( \alpha \)

Agent/component \( P \)

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Process algebra model \rightarrow \text{SOS rules} \rightarrow \text{Labelled transition system}
Example

Consider a web server which offers html pages for download:

\[ \text{Server} \overset{\text{def}}{=} \text{get/download/rel/Server} \]
Example

Consider a web server which offers html pages for download:

\[
\text{Server} \equiv \text{get.download.rel.Server}
\]

Its clients might be web browsers, in a domain with a local cache of frequently requested pages. Thus any display request might result in an access to the server or in a page being loaded from the cache.

\[
\text{Browser} \equiv \text{display.}(\text{cache.Browser } + \text{ get.download.rel.Browser})
\]
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\[
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\]

A simple version of the Web can be considered to be the interaction of these components:

\[
WEB \overset{\text{def}}{=} (\text{Browser} \parallel \text{Browser}) | Server
\]
Qualitative Analysis

- The labelled transition system underlying a process algebra model can be used for functional verification e.g.: reachability analysis, specification matching and model checking.
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Will the system arrive in a particular state?
The labelled transition system underlying a process algebra model can be used for functional verification e.g.: reachability analysis, specification matching and model checking.

Does system behaviour match its specification?
The labelled transition system underlying a process algebra model can be used for functional verification e.g.: reachability analysis, specification matching and model checking.

Does a given property $\phi$ hold within the system?
Stochastic process algebras

Process algebras where models are decorated with quantitative information used to generate a stochastic process are stochastic process algebras (SPA).
SPA Languages

SPA
SPA Languages

- integrated time
- orthogonal time
SPA Languages

- integrated time
  - exponential only
  - exponential + instantaneous
  - general distributions

- orthogonal time
SPA Languages

- **integrated time**
  - exponential only
  - exponential + instantaneous

- **orthogonal time**
  - general distributions
  - exponential only
  - general distributions
**SPA Languages**

![Diagram showing different types of SPA languages with integrated and orthogonal time, exponential, and general distributions options.](image-url)
SPA Languages

- Integrated time
  - Exponential only
    - PEPA, S\(\pi\)-calculus, SCCP
  - Exponential + instantaneous
    - EMPA, Markovian TIPP
  - General distributions
    - TIPP, SPADES, GSMPA

- Orthogonal time
  - Exponential only
  - General distributions
SPA Languages

- **integrated time**
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- **orthogonal time**
  - exponential only
    - IMC
  - general distributions
SPA Languages

- Integrated time:
  - Exponential only: PEPA, $\pi$-calculus, SCCP
  - Exponential + instantaneous:
    - EMPA, Markovian TIPP
    - General distributions: TIPP, SPADES, GSMPA
  - Exponential only:
    - IMC

- Orthogonal time:
  - General distributions:
    - IGSMP, Modest
SPA Languages

- **integrated time**
  - exponential only
    - PEPA, Sπ-calculus, SCCP
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- **orthogonal time**
  - general distributions
    - TIPP, SPADES, GSMPA
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The theoretical development underpinning PEPA has focused on the interplay between the process algebra and the underlying mathematical structure, the Markov process.
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From the process algebra side the Markov chain had a profound influence on the design of the language and in particular on the interactions between components.

From the Markov chain perspective the process algebra structure has been exploited to find aspects of independence even between interacting components.
Models are constructed from components which engage in activities.

\[(\alpha, r).P\]

- Action type or name
- Activity rate (parameter of an exponential distribution)
- Component/derivative
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PEPA MODEL \(\rightarrow\) SOS rules \(\rightarrow\) LABELLED TRANSITION SYSTEM
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Performance Evaluation Process Algebra

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\[(\alpha, r).P\]

- The language is used to generate a **CTMC** for performance modelling.

**PEPA** MODEL \(\rightarrow\) **SOS rules** \(\rightarrow\) **LABELLED TRANSITION SYSTEM** \(\rightarrow\) state transition diagram \(\rightarrow\) **CTMC Q**
PEPA

\[ S ::= (\alpha, r).S \mid S + S \mid \alpha \]
\[ P ::= S \mid P \parallel P \mid P/L \]
PEPA

\[ S ::= (\alpha, r).S | S + S | A \]
\[ P ::= S | P \lhd P | P/L \]

**PREFIX:** \((\alpha, r).S\) designated first action
PEPA

\[ S ::= (\alpha, r).S \mid S + S \mid A \]
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**CHOICE:** \(S + S\) competing components
PEPA

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\[ P ::= S \mid P \boxdot P \mid P/L \]

**PREFIX:** \((\alpha, r).S\) designated first action

**CHOICE:** \(S + S\) competing components

**CONSTANT:** \(A \overset{\text{def}}{=} S\) assigning names
PEPA

\[ S ::= (\alpha, r).S \mid S + S \mid A \]
\[ P ::= S \mid P \otimes P \mid P/L \]

**PREFIX:** \((\alpha, r).S\) designated first action

**CHOICE:** \(S + S\) competing components

**CONSTANT:** \(A \overset{\text{def}}{=} S\) assigning names

**COOPERATION:** \(P \otimes_{L} P\) \(\alpha \notin L\) individual actions  
\(\alpha \in L\) shared actions
PEPA

\[ S ::= (\alpha, r).S \mid S + S \mid A \]
\[ P ::= S \mid P \otimes_L P \mid P/L \]

**PREFIX:** \((\alpha, r).S\) designated first action

**CHOICE:** \(S + S\) competing components

**CONSTANT:** \(A \stackrel{\text{def}}{=} S\) assigning names

**COOPERATION:** \(P \otimes_L P\) \(\alpha \notin L\) individual actions

\(\alpha \in L\) shared actions

**HIDING:** \(P/L\) abstraction \(\alpha \in L \Rightarrow \alpha \rightarrow \tau\)
Example: Browsers, server and download

\[
\text{Server} \; \overset{\text{def}}{=} \; (\text{get, } \top). (\text{download, } \mu). (\text{rel, } \top). \text{Server}
\]

\[
\text{Browser} \; \overset{\text{def}}{=} \; (\text{display, } p\lambda). (\text{get, } g). (\text{download, } \top). (\text{rel, } r). \text{Browser}
\]

\[
+ \; (\text{display, } (1 - p)\lambda). (\text{cache, } m). \text{Browser}
\]

\[
\text{WEB} \; \overset{\text{def}}{=} \; (\text{Browser} \parallel \text{Browser}) \boxtimes \text{Server}
\]

where \( L = \{ \text{get, download, rel} \} \)
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PEPA activities and rates

When enabled an activity, $a = (\alpha, \lambda)$, will delay for a period determined by its associated distribution function, i.e. the probability that the activity $a$ happens within a period of time of length $t$ is $F_a(t) = 1 - e^{-\lambda t}$. 
PEPA activities and rates

- We can think of this as the activity setting a timer whenever it becomes enabled.
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- If several activities are enabled at the same time each will have its own associated timer.
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- When the first timer finishes that activity takes place—the activity is said to complete or succeed.
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- If several activities are enabled at the same time each will have its own associated timer.
- When the first timer finishes that activity takes place—the activity is said to complete or succeed.
- This means that the activity is considered to “happen”: an external observer will witness the event of activity of type $\alpha$.
- An activity may be preempted, or aborted, if another one completes first.
PEPA and Markov processes

In a PEPA model if we define the stochastic process $X(t)$, such that $X(t) = C_i$ indicates that the system behaves as component $C_i$ at time $t$, then $X(t)$ is a Markov process which can be characterised by a matrix, $Q$. 
In a PEPA model if we define the stochastic process $X(t)$, such that $X(t) = C_i$ indicates that the system behaves as component $C_i$ at time $t$, then $X(t)$ is a Markov process which can be characterised by a matrix, $Q$.

A stationary or equilibrium probability distribution, $\pi(\cdot)$, exists for every time-homogeneous irreducible Markov process whose states are all positive-recurrent.
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A stationary or equilibrium probability distribution, $\pi(\cdot)$, exists for every time-homogeneous irreducible Markov process whose states are all positive-recurrent.

This distribution is found by solving the global balance equation

$$\pi Q = 0$$

subject to the normalisation condition

$$\sum \pi(C_i) = 1.$$
All PEPA models are time-homogeneous since all activities are time-homogeneous: the rate and type of activities enabled by a component are independent of time.
PEPA and irreducibility and positive-recurrence

The other conditions, irreducibility and positive-recurrent states, are easily expressed in terms of the derivation graph of the PEPA model.

We only consider PEPA models with a finite number of states so if the model is irreducible then all states must be positive-recurrent i.e. the derivation graph is strongly connected.
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We only consider PEPA models with a finite number of states so if the model is irreducible then all states must be positive-recurrent i.e. the derivation graph is strongly connected.

In terms of the PEPA model this means that all behaviours of the system must be recurrent; in particular, for every choice, whichever path is chosen it must eventually return to the point where the choice can be made again, possibly with a different outcome.
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Structured Operational Semantics

PEPA is defined using a Plotkin-style structured operational semantics (a “small step” semantics).
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A labelled transition system is a set of process terms $\mathcal{P}$, a set of action labels $\mathcal{A}$ and a relation $\mathcal{P} \times \mathcal{A} \times \mathcal{P}$ given by the operational rules.
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In the rules, the derivation below the line can be inferred from the premise above the line.

Note that in this semantics the rate information is only treated as an additional label.
Structured Operational Semantics: Prefix and Choice

Prefix

\[(\alpha, r).E \xrightarrow{(\alpha,r)} E\]
Structured Operational Semantics: Prefix and Choice

Prefix

\[(\alpha, r).E \xrightarrow{(\alpha, r)} E\]

Choice

\[E \xrightarrow{(\alpha, r)} E' \]

\[E + F \xrightarrow{(\alpha, r)} E' \]

\[F \xrightarrow{(\alpha, r)} F' \]

\[E + F \xrightarrow{(\alpha, r)} F' \]
Structured Operational Semantics: Cooperation ($\alpha \notin L$)

Cooperation

\[
\begin{array}{c}
E \xrightarrow{(\alpha, r)} E' \\
E \boxtimes F \xrightarrow{(\alpha, r)} E' \boxtimes F
\end{array}
\quad (\alpha \notin L)
\]
Structured Operational Semantics: Cooperation ($\alpha \notin L$)

**Cooperation**

\[
\begin{align*}
E & \xrightarrow{\alpha, r} E' \\
E \mathbin{\bigotimes}_L F & \xrightarrow{\alpha, r} E' \mathbin{\bigotimes}_L F
\end{align*}
\]

\[
\begin{align*}
F & \xrightarrow{\alpha, r} F' \\
E \mathbin{\bigotimes}_L F & \xrightarrow{\alpha, r} E \mathbin{\bigotimes}_L F'
\end{align*}
\]
Structured Operational Semantics: Cooperation ($\alpha \in L$)

$E \xrightarrow{(\alpha, r_1)} E' \quad F \xrightarrow{(\alpha, r_2)} F' \quad (\alpha \in L)$

$E \bowtie_L F \xrightarrow{(\alpha, R)} E' \bowtie_L F'$
Structured Operational Semantics: Cooperation ($\alpha \in L$)

Cooperation

\[
\begin{align*}
E \xrightarrow{(\alpha,r_1)} E' & \quad F \xrightarrow{(\alpha,r_2)} F' \\
E \triangleright \bigotimes L F & \xrightarrow{(\alpha,R)} E' \bigotimes \triangleright \bigotimes L F'
\end{align*}
\]

where

\[
R = \frac{r_1}{r_\alpha(E)} \frac{r_2}{r_\alpha(F)} \min(r_\alpha(E), r_\alpha(F))
\]
Apparent Rate

\[
r_{\alpha}((\beta, r).P) = \begin{cases} 
  r & \beta = \alpha \\
  0 & \beta \neq \alpha
\end{cases}
\]

\[
r_{\alpha}(P + Q) = r_{\alpha}(P) + r_{\alpha}(Q)
\]

\[
r_{\alpha}(A) = r_{\alpha}(P)
\]

where \( A \overset{\text{def}}{=} P \)

\[
r_{\alpha}(P \boxdot_{L} Q) = \begin{cases} 
  r_{\alpha}(P) + r_{\alpha}(Q) & \alpha \not\in L \\
  \min(r_{\alpha}(P), r_{\alpha}(Q)) & \alpha \in L
\end{cases}
\]

\[
r_{\alpha}(P/L) = \begin{cases} 
  r_{\alpha}(P) & \alpha \not\in L \\
  0 & \alpha \in L
\end{cases}
\]
Structured Operational Semantics: Hiding

Hiding

\[
\begin{align*}
E & \xrightarrow{(\alpha,r)} E' \\
E/L & \xrightarrow{(\alpha,r)} E'/L
\end{align*}
\]

(\alpha \notin L)
Hiding

\[
\begin{align*}
E & \xrightarrow{(\alpha, r)} E' \\
E/L & \xrightarrow{(\alpha, r)} E'/L \quad (\alpha \notin L) \\
E/\perp & \xrightarrow{(\tau, r)} E'/\perp \\
E/L & \xrightarrow{(\tau, r)} E'/L \quad (\alpha \in L)
\end{align*}
\]
Structured Operational Semantics: Constants

**Constant**

\[
E \xrightarrow{\alpha, r} E' \quad (A \overset{\text{def}}{=} E)
\]

\[
A \xrightarrow{\alpha, r} E'
\]
Properties of the definition (1)

PEPA has no “nil” (a deadlocked process).

This is because the PEPA language is intended for modelling non-stop processes (such as Web servers, operating systems, or manufacturing processes) rather than for modelling terminating processes (a compilation, a sorting routine, and so forth).
Creating a deadlocked process

When we are interested in transient behaviour we use the deadlocked process $\text{Stop}$ to signal a component which performs no further actions.

\[
\text{Stop} \overset{\text{def}}{=} \left(\left( (a, r).\text{Stop} \right) \parallel_{\{a,b\}} \left( (b, r).\text{Stop} \right) \right) / \{a, b\}
\]
Cooperation in PEPA is multi-way. Two, three, four or more partners may cooperate, and they all need to synchronise for the activity to happen.
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This comes from the fact that synchronisation has the form $a, a \rightarrow a$ (as in CSP) instead of $a, \bar{a} \rightarrow \tau$ (as in CCS and the $\pi$-calculus).
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This comes from the fact that synchronisation has the form $a, a \rightarrow a$ (as in CSP) instead of $a, \overline{a} \rightarrow \tau$ (as in CCS and the $\pi$-calculus).

This is used to have “witnesses” to events (known as stochastic probes).
Because of its mapping onto a CTMC, PEPA has an interleaving semantics.
Properties of the definition (3)

- Because of its mapping onto a CTMC, PEPA has an interleaving semantics.
- Other modelling formalisms based on CTMCs are also based on an interleaving semantics (e.g. Generalised Stochastic Petri nets).
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As we have seen a continuous time Markov chain (CTMC) is generated from a PEPA model via its structured operational semantics.
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Linear algebra is used to solve the model in terms of equilibrium behaviour.
Properties of the definition (3)

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- Other modelling formalisms based on CTMCs are also based on an interleaving semantics (e.g. Generalised Stochastic Petri nets).
- As we have seen a continuous time Markov chain (CTMC) is generated from a PEPA model via its structured operational semantics.
- Linear algebra is used to solve the model in terms of equilibrium behaviour.
- The resulting probability distribution is seldom the ultimate goal of performance analysis; a modeller derives performance measures from this distribution via a reward structure.
Integrated analysis

Qualitative verification can now be complemented by quantitative verification.
How long will it take for the system to arrive in a particular state?
Integrated analysis: Specification matching

With what **probability** does system behaviour match its specification?
Integrated analysis: Specification matching

Does the “frequency profile” of the system match that of the specification?
Does a given property $\phi$ hold within the system with a given probability?
For a given starting state, how long is it until a given property $\phi$ holds?
The Importance of Being Exponential

\[(\alpha, r).Stop \parallel (\beta, s).Stop\]

\[(\alpha, r) \quad (\beta, s)\]

\[Stop \parallel (\beta, s).Stop \quad (\alpha, r).Stop \parallel Stop\]

\[(\beta, s) \quad (\alpha, r)\]

\[Stop \parallel Stop\]
The Importance of Being Exponential

\[(\alpha, r).\text{Stop} \parallel (\beta, s).\text{Stop}\]

\[(\alpha, r) \quad \text{Stop} \parallel (\beta, s).\text{Stop} \quad (\beta, s) \quad (\alpha, r).\text{Stop} \parallel \text{Stop}\]

\[(\beta, s) \quad \text{Stop} \parallel (\beta, s).\text{Stop} \quad (\alpha, r) \quad (\alpha, r).\text{Stop} \parallel \text{Stop}\]
The Importance of Being Exponential

\[(\alpha, r).\text{Stop} \parallel (\beta, s).\text{Stop}\]

\[
\begin{align*}
(\alpha, r) & \quad (\beta, s) \\
\text{Stop} \parallel (\beta, s).\text{Stop} & \quad (\alpha, r).\text{Stop} \parallel \text{Stop} \\
(\beta, s) & \quad (\alpha, r) \\
\text{Stop} \parallel \text{Stop} &
\end{align*}
\]
The Importance of Being Exponential

\[ (\alpha, r).\text{Stop} \parallel (\beta, s).\text{Stop} \]

\[ (\alpha, r) \rightarrow (\alpha, r) \rightarrow (\beta, s) \rightarrow (\beta, s) \rightarrow (\alpha, r).\text{Stop} \parallel \text{Stop} \]

\[ \text{Stop} \parallel (\beta, s).\text{Stop} \rightarrow (\beta, s) \rightarrow (\alpha, r).\text{Stop} \parallel \text{Stop} \]

\[ (\beta, s) \rightarrow (\beta, s) \rightarrow \text{Stop} \parallel \text{Stop} \rightarrow (\alpha, r) \rightarrow (\alpha, r) \rightarrow (\beta, s) \rightarrow (\beta, s) \rightarrow (\alpha, r).\text{Stop} \parallel \text{Stop} \]
The Importance of Being Exponential

\[
\begin{align*}
& (\alpha, r).\text{Stop} \parallel (\beta, s).\text{Stop} \\
& (\alpha, r) \quad (\beta, s) \\
& \text{Stop} \parallel (\beta, s).\text{Stop} \quad (\alpha, r).\text{Stop} \parallel \text{Stop} \\
& (\beta, s) \quad (\alpha, r) \\
& \text{Stop} \parallel \text{Stop}
\end{align*}
\]
The Importance of Being Exponential

\[(\alpha, r) \cdot \text{Stop} \parallel (\beta, s) \cdot \text{Stop}\]

\[\text{Stop} \parallel (\beta, s) \cdot \text{Stop} \quad (\alpha, r) \cdot \text{Stop} \parallel \text{Stop}\]

\[(\beta, s) \quad \text{Stop} \parallel \text{Stop} \quad (\alpha, r)\]
The Importance of Being Exponential

\[(\alpha, r).Stop \parallel (\beta, s).Stop\]

\[(\alpha, r) \rightarrow (\beta, s)\]
\[(\beta, s) \rightarrow (\alpha, r)\]

\[Stop \parallel (\beta, s).Stop\]
\[Stop \parallel Stop\]
The memoryless property of the negative exponential distribution means that residual times do not need to be recorded.
We retain the expansion law of classical process algebra:

\[(\alpha, r).Stop \parallel (\beta, s).Stop = (\alpha, r).(\beta, s).(Stop \parallel Stop) + (\beta, s).(\alpha, r).(Stop \parallel Stop)\]

only if the negative exponential distribution is used.
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Parallel composition is the basis of the compositionality in a process algebra.
Parallel composition is the basis of the compositionality in a process algebra — it defines \textit{which} components interact and how.
Parallel Composition

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- In classical process algebra, it is often associated with communication.
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- Parallel composition is the basis of the compositionality in a process algebra — it defines *which* components interact and how.

- In classical process algebra is it often associated with *communication*.

- When the activities of the process algebra have a *duration* the definition of parallel composition becomes more complex.
Parallel composition is the basis of the compositionality in a process algebra — it defines **which** components interact and how.

In classical process algebra it is often associated with **communication**.

When the activities of the process algebra have a **duration** the definition of parallel composition becomes more complex.

The issue of what it means for two timed activities to synchronise is a vexed one....
Who Synchronises...?

Even within classical process algebras there is variation in the interpretation of parallel composition:
Who Synchronises...?

Even within classical process algebras there is variation in the interpretation of parallel composition:

**CCS-style**

- Actions are partitioned into **input** and **output** pairs.
- Communication or synchronisation takes places between **conjugate** pairs.
- The resulting action has silent type $\tau$. 

Even within classical process algebras there is variation in the interpretation of parallel composition:

**CCS-style**
- Actions are partitioned into *input* and *output* pairs.
- Communication or synchronisation takes places between *conjugate* pairs.
- The resulting action has silent type $\tau$.

**CSP-style**
- No distinction between input and output actions.
- Communication or synchronisation takes place on the basis of *shared names*.
- The resulting action has the same name.
Even within classical process algebras there is variation in the interpretation of parallel composition:

**CCS-style**
- Actions are partitioned into **input** and **output** pairs.
- Communication or synchronisation takes places between **conjugate** pairs.
- The resulting action has silent type $\tau$.

**CSP-style**
- **No distinction** between input and output actions.
- Communication or synchronisation takes place on the basis of **shared names**.
- The resulting action has the same name.

Most stochastic process algebras adopt **CSP-style synchronisation**.
Timed Synchronisation

P_1 \quad r_1 \quad s_1

P_2 \quad r_2 \quad s_2

r?

s?
Timed Synchronisation

Barrier Synchronisation

$s = \max(s_1, s_2)$
Timed Synchronisation

\[ s = \max(s_1, s_2) \]

\( P_1 \)

\( r_1 \) \hspace{1cm} \( s_1 \)

\( P_2 \)

\( r_2 \) \hspace{1cm} \( s_2 \)

\( s \) is no longer exponentially distributed
Timed Synchronisation

algebraic considerations limit choices
Timed Synchronisation

\[ r = r_1 \times r_2 \]

TIPP: new rate is product of individual rates
Timed Synchronisation

EMPA: one participant is passive
Timed Synchronisation

bounded capacity: new rate is the minimum of the rates

$r = \min(r_1, r_2)$
Within the cooperation framework, PEPA assumes bounded capacity: that is, a component cannot be made to perform an activity faster by cooperation, so the rate of a shared activity is the minimum of the apparent rates of the activity in the cooperating components.
Apparent Rate

\[ r_\alpha((\beta, r).P) = \begin{cases} r & \beta = \alpha \\ 0 & \beta \neq \alpha \end{cases} \]

\[ r_\alpha(P + Q) = r_\alpha(P) + r_\alpha(Q) \]

\[ r_\alpha(A) = r_\alpha(P) \quad \text{where } A \overset{\text{def}}{=} P \]

\[ r_\alpha(P \not\equiv_L Q) = \begin{cases} r_\alpha(P) + r_\alpha(Q) & \alpha \notin L \\ \min(r_\alpha(P), r_\alpha(Q)) & \alpha \in L \end{cases} \]

\[ r_\alpha(P/L) = \begin{cases} r_\alpha(P) & \alpha \notin L \\ 0 & \alpha \in L \end{cases} \]
Apparent Rate

\[ r_\alpha((\beta, r).P) = \begin{cases} r & \beta = \alpha \\ 0 & \beta \neq \alpha \end{cases} \]

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\[ r_\alpha(A) = r_\alpha(P) \quad \text{where} \quad A \overset{\text{def}}{=} P \]

\[ r_\alpha(P \otimes^L Q) = \begin{cases} r_\alpha(P) + r_\alpha(Q) & \alpha \notin L \\ \min(r_\alpha(P), r_\alpha(Q)) & \alpha \in L \end{cases} \]

\[ r_\alpha(P/L) = \begin{cases} r_\alpha(P) & \alpha \notin L \\ 0 & \alpha \in L \end{cases} \]

This is used to calculate pairwise cooperation rates: the overall rate of cooperation must not exceed either of the constituent apparent rates.
In PEPA each component has a bounded capacity to carry out activities of any particular type, determined by the apparent rate for that type.
Cooperation in PEPA

- In PEPA each component has a **bounded capacity** to carry out activities of any particular type, determined by the **apparent rate** for that type.

- Synchronisation, or **cooperation** cannot make a component exceed its bounded capacity.
In PEPA each component has a **bounded capacity** to carry out activities of any particular type, determined by the **apparent rate** for that type.

- Synchronisation, or cooperation cannot make a component exceed its bounded capacity.

- Thus the apparent rate of a cooperation is the **minimum** of the apparent rates of the co-operands.