SPAs for performance modelling: Lecture 8 — Stochastic Probes

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Outline

1. Stochastic probes
2. Passage Time and Passage End Analysis
3. Models with a Spatial Aspect
   - Spatial Challenge: Capturing physical space

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1 Stochastic probes

2 Passage Time and Passage End Analysis

3 Models with a Spatial Aspect
- Spatial Challenge: Capturing physical space

Introduction: Stochastic probes

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In this lecture we will look at the alternative approach of using stochastic probes (and extended stochastic probes XSP).

Performance Measure Probes

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- A performance measurement probe is a separate PEPA component which may be attached to the model to be measured.
- The probe component observes the actions of the measured model and changes its state accordingly
- To measure the model the probe can then be interrogated for its state.
- Sometimes the probes are created implicitly by the tool in order to carry out a calculation.

Simple passage model: making tea

```
Idle    = (begin        , beginRate) . Water  ;
Water   = (fillKettle   , fillRate)  . TurnOn ;
TurnOn  = (turnOnKettle, onRate)     . Wait   ;
Wait    = (boil         , boilRate)  . Pour   ;
Pour    = (pour         , pourRate)  . Milk   ;
AddMilk = (milk         , milkRate)  . Stir   ;
Stir    = (stir         , stirRate)  . Drink  ;
Drink   = (drink        , drinkRate) . Idle   ;

Idle
```

A Simple Measurement

- Suppose we wish to find out what the probability that the person in the model is currently making a cup of tea.

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- Suppose we wish to find out what the probability that the person in the model is currently making a cup of tea.
- One possibility is to simply add up the probabilities of the states in which the person is making their tea.
- However this approach is not very robust because if the model is revised this measurement must also be revised.

Simple passage probe

```
ProbeIdle    = (begin, ⊤) . ProbeRunning ;

ProbeRunning = (stir , ⊤) . ProbeIdle ;
```

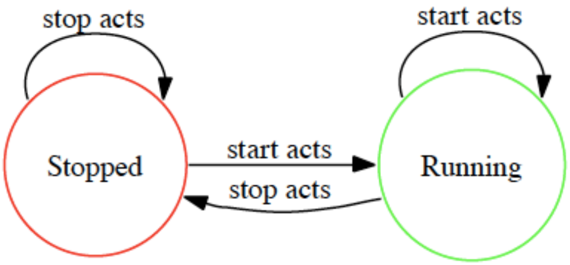
$\texttt{Idle} \underset{\{begin, stir\}}{\bowtie} \texttt{ProbeIdle}$

Simple passage probe

```
ProbeIdle    = (begin, ⊤) . ProbeRunning
             + (stir , ⊤) . ProbeIdle     ;
ProbeRunning = (stir , ⊤) . ProbeIdle
             + (begin, ⊤) . ProbeRunning ;
```

$\texttt{Idle} \underset{\{begin, stir\}}{\bowtie} \texttt{ProbeIdle}$

Generic probe graph



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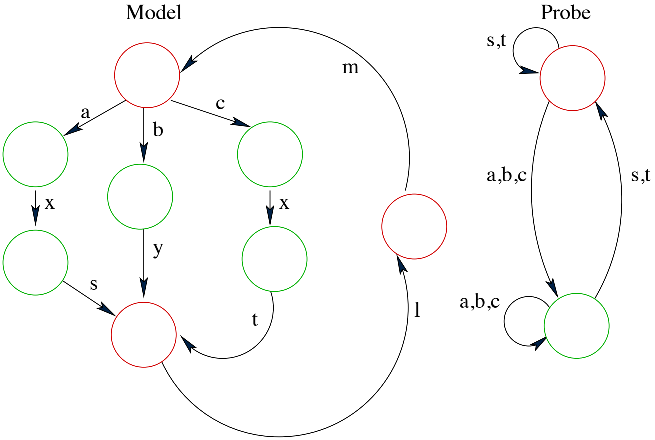
- If the model is revised the probe need not necessarily be revised (unless it directly affects the measurement in question).

A probe graph



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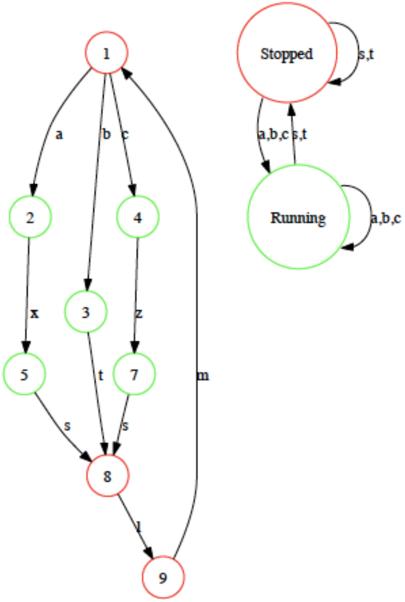


The boiler definitions

```
Boiler    = (cool  , coolRate  ) . Boiling
          + (pour  , ⊤         ) . Refilling
          ;
Boiling   = (boil  , boilRate  ) . Boiler  ;
Refilling = (refill, refillRate) . Boiling ;
```

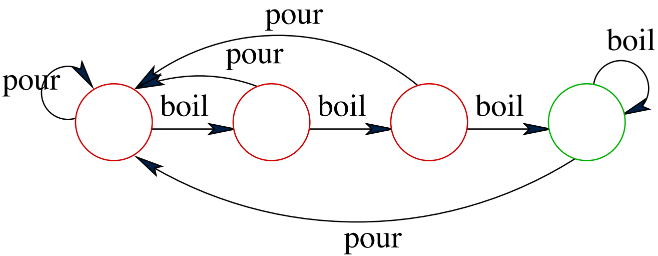
```
Idle    = (begin       , teaRate)   . Water  ;
Pour    = (pour        , pourRate)  . Milk   ;
AddMilk = (milk        , milkRate)  . Stir   ;
Stir    = (stir        , stirRate)  . Drink  ;
Drink   = (drink       , drinkRate) . Idle   ;
```

$\texttt{Idle}[30] \underset{\{pour\}}{\bowtie} \texttt{Boiler}$

A more complex probe

Suppose we wish to ask the probability of being in a state which has seen the boiler re-boil three or more times without a tea drinker taking some water.

The corresponding probe graph



The boiler probe definitions

```
Probe0 = (boil, ⊤) . Probe1 ;

Probe1 = (boil, ⊤) . Probe2 ;

Probe2 = (boil, ⊤) . Probe3 ;


Probe3 = (pour, ⊤) . Probe0 ;
```

$$\texttt{Probe0} \underset{\{boil,pour\}}{\bowtie} (\texttt{Idle} \underset{\{pour\}}{\bowtie} \texttt{Boiler})$$

The boiler probe definitions

$$\mathtt{Probe0} = (\mathtt{boil}, \top) . \mathtt{Probe1} ;$$

$$\begin{aligned}\mathtt{Probe1} &= (\mathtt{boil}, \top) . \mathtt{Probe2} ; \\ &+ (\mathtt{pour}, \top) . \mathtt{Probe0} ; \\ \mathtt{Probe2} &= (\mathtt{boil}, \top) . \mathtt{Probe3} ; \\ &+ (\mathtt{pour}, \top) . \mathtt{Probe0} ;\end{aligned}$$

$$\mathtt{Probe3} = (\mathtt{pour}, \top) . \mathtt{Probe0} ;$$

$$\mathtt{Probe0} \underset{\{boil, pour\}}{\bowtie} (\mathtt{Idle} \underset{\{pour\}}{\bowtie} \mathtt{Boiler})$$

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\end{aligned}
$$

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\texttt{Probe3} &= (\texttt{pour}, \top) . \texttt{Probe0} \\
&\quad + (\texttt{boil}, \top) . \texttt{Probe3} ;
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\texttt{Probe0} \underset{\{boil, pour\}}{\bowtie} (\texttt{Idle} \underset{\{pour\}}{\bowtie} \texttt{Boiler})
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Writing probes as regular expressions

Writing a set of probe definitions as PEPA definitions is error prone. In particular it is hard to get the self-loops correct.

- Instead we allow a regular expression-like language for defining probes in the ipc tool (Imperial PEPA compiler).

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- This is then automatically translated into the PEPA component that is the probe and attached to the model.

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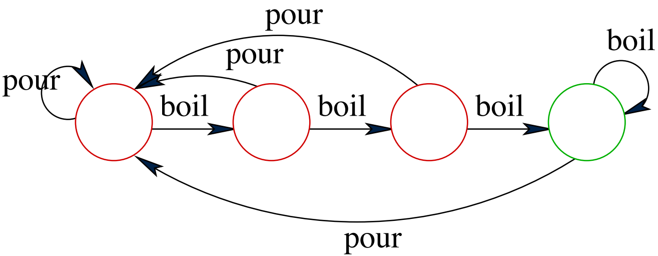
- Instead we allow a regular expression-like language for defining probes in the ipc tool (Imperial PEPA compiler).
- The previous probe would be written as:
  $((\mathit{boil}, \mathit{boil}, \mathit{boil})/\mathit{pour}) : \mathit{start}, \mathit{pour} : \mathit{stop})$
- The 'start' and 'stop' labels indicate that we are entering or exiting the state of the probe/model in which we are interested.
- This is then automatically translated into the PEPA component that is the probe and attached to the model.
- In addition this allows us to separate probes from the model and attach several probes to a single model, either separately or simultaneously.

The corresponding probe graph



A regular-expression-like syntax for probes”

R	$:=$	*activity*	Observe action
	$\mid$	R_1, R_2	sequence
	$\mid$	$R_1 \mid R_2$	choice
	$\mid$	$R : label$	labelled
	$\mid$	$R\ n$	iterate n times
	$\mid$	$R\{m, n\}$	iterate between m and n times
	$\mid$	$R+$	one or more
	$\mid$	$R*$	zero or more
	$\mid$	$R?$	zero or one
	$\mid$	$R/activity$	observe R without the activity

Probe Implementation

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2. Construct the Deterministic Finite Automaton (DFA) corresponding to the NFA
3. Minimise the DFA
4. Add necessary self-loops to the DFA
5. Pretty print as a PEPA component

Constructing probes

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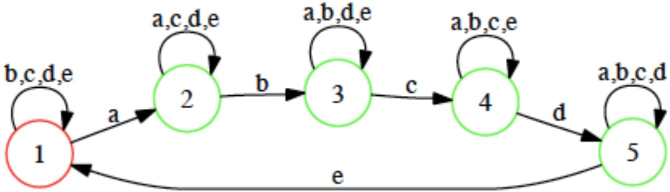
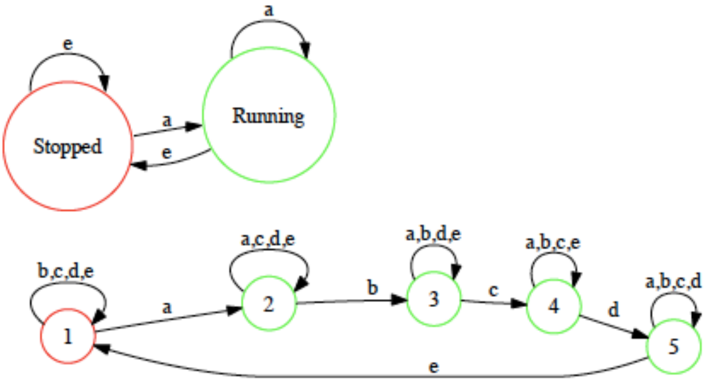
- This asks the question, what is the expected time from first observing an a activity, to observing a (possibly interrupted) sequence of activities b, c, d, e?
- The 'middle' part b, c, d may be some arbitrarily complex probe.

The probe graph



A probe on the probe



We can attach a master probe to an inner probe to retain the property of having only one running state.

Non-unique start and stop actions

- What if the probe may perform any of the start actions without wishing to move to the running state?
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- Or both:
  $a : start, (a, a), a : stop$

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- Or both:
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- This asks the very simple question: How long can we expect the model to take to perform four *a* actions?

Introducting immediate actions

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- A *start* or *stop* label can then be implemented as an immediate action.
- The probe $a : start$ becomes the prefix component: $(a, \top).(start, 1 : immediate).R$ where R is the probe's running state.
- For the remainder we'll abbreviate $(start, 1 : immediate).R$ as $start.R$

Standard master probe

```
ProbeStopped  = start . ProbeRunning
              + stop . ProbeStopped ;

ProbeRunning  = stop . ProbeStopped
              + start . ProbeRunning ;
```

Global probes vs. Local probes

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Sometimes there are problems with global probes because they "see too much".

This is illustrated by the following example based on a Client-Server system

Client-Server System

```
Client     = (work     , workRate    ) . ClientReq  ;
ClientReq  = (request  , requestRate) . ClientWait ;
ClientWait = (response, ⊤           ) . Client     ;

ServerIdle = (request  , ⊤             ) . ServerComp ;
ServerComp = (compute  , computeServer ) . ServerResp ;
ServerResp = (response, responseServer) . ServerIdle ;
```

$\texttt{Client}[4] \underset{\{\mathit{request}, \mathit{response}\}}{\bowtie} \texttt{ServerIdle}[2]$

Probing the Client-Server System

We may wish to ask: What is the expected response time?

This is a passage-time query which we might expect to be answered with the following probe specification.

$request : start, response : stop$

Client-Server System with Probe

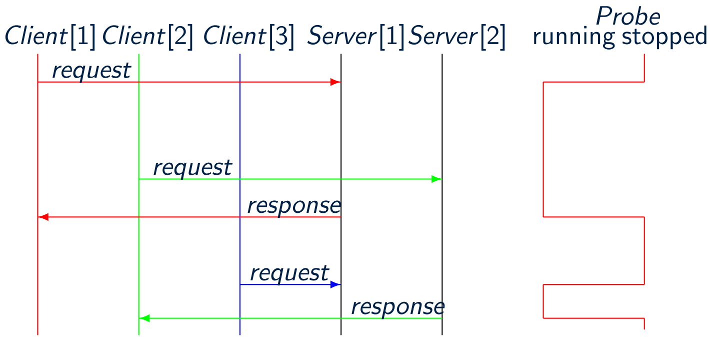
```
Client     = (work     , workRate     ) . ClientReq  ;
ClientReq  = (request  , requestRate) . ClientWait ;
ClientWait = (response, ⊤           ) . Client     ;

ServerIdle = (request  , ⊤              ) . ServerComp ;
ServerComp = (compute  , computeServer ) . ServerResp ;
ServerResp = (response, responseServer) . ServerIdle ;

Probe0     = (request, ⊤) . Probe1
           + (response, ⊤) . Probe0 ;
Probe1     = (response, ⊤). Probe0
           + (request, ⊤). Probe1 ;
```

$$(\texttt{Client}[4] \underset{\{request,\,response\}}{\bowtie} \texttt{ServerIdle}[2]) \underset{\{request,\,response\}}{\bowtie} \texttt{Probe0}$$

The problem with global probes



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$$(\, (\, \texttt{Client} \underset{\{request, response\}}{\bowtie} \texttt{Probe0} \,) \bowtie \texttt{Client[3]} \,) \underset{\{request, response\}}{\bowtie} \texttt{ServerIdle[2]}$$

Using local probes

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$$\left(\, \left(\, \texttt{Client} \underset{\{request,response\}}{\bowtie} \texttt{Probe0} \,\right) \bowtie \texttt{Client}[3] \,\right) \underset{\{request,response\}}{\bowtie} \texttt{ServerIdle}[2]$$

The probe specification language for ipc allows the user to place a probe using the :: notation, the model shown was generated with the following probe.

$$Client :: (request : start, response : stop)$$

Generalising the start and stop labels

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Since probes may now 'send' *start* and *stop* communication signals using immediate actions we can generalise these labels to enable the sending of arbitrary signals.

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Since probes may now 'send' *start* and *stop* communication signals using immediate actions we can generalise these labels to enable the sending of arbitrary signals.

In addition since we have the ability to localise a probe we may use such communication labels to communicate important events from a local probe to a master probe.

Local and master probes

We may now have several local probes which communicate with a control probe.

$$(((P \underset{\mathcal{N}}{\bowtie} ProbeP) \underset{\mathcal{K}}{\bowtie} Q) \underset{\mathcal{L}}{\bowtie} (R \underset{\mathcal{M}}{\bowtie} (S \underset{\mathcal{O}}{\bowtie} ProbeS))) \underset{\mathcal{T}}{\bowtie} Control$$

Communicating Probes Example: big and small servers

```
Small      = (request , ⊤) . SmallComp
           + (break, r) . SmallBroken ;
SmallComp    = (compute , compRate ) . SmallResp ;
SmallResp   = (response, responseRate) . Small ;
SmallBroken = (repair, repairRate) . Small ;

Big     = (request , ⊤) . BigComp
           + (break, r) . BigBroken ;
BigComp    = (compute , 3 * compRate ) . BigResp ;
BigResp   = (response, responseRate) . Big ;
BigBroken = (repair, repairRate) . Big ;

Servers = (Small || Big)
```

$\texttt{Client[4]} \underset{\{request,response\}}{\bowtie} \texttt{Servers}$

Communicating Probes

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  - $Client :: (work : cwork, response : cresp)$
  - $Small :: (break : in, repair : out)$
  - $((in, cwork)/out) : start, cresp : stop)$

Client Probe

```
ClientProbe0 = (work, ⊤) . cwork . ClientProbe1
             + (response, ⊤) . ClientProbe0 ;

ClientProbe1 = (response, ⊤) . cresp . ClientProbe0
             + (work, ⊤) . ClientProbe1 ;
```

Small Probe

```
SmallProbe0 = (break, ⊤) . in . SmallProbe1
            + (repair, ⊤) . SmallProbe0 ;
SmallProbe1 = (repair, ⊤) . out . SmallProbe0
            + (break, ⊤) . SmallProbe1 ;
```

Control Probe

```
Control0 = in . Control1
         + cwork . Control0
         + cresp . Control0
         + out . Control0 ;
Control1 = cwork . start . Control2
         + out . Control0
         + in . Control1
         + cresp . Control1 ;
Control2 = cresp . stop . Control0
         + cwork . Control2
         + in . Control2
         + in . Control2 ;
```

Placing the Local Probes

$$\texttt{PClient} = (\texttt{Client} \underset{\{work,response\}}{\bowtie} \texttt{ClientProbe0})$$

$$\texttt{PSmall} = (\texttt{Small} \underset{\{break,repair\}}{\bowtie} \texttt{SmallProbe0})$$

$$\texttt{Clients} = \texttt{PClient} \bowtie \texttt{Client}[3]$$

$$\texttt{Servers} = \texttt{PSmall} \parallel \texttt{Big}$$

$$\texttt{System} = (\texttt{Clients} \underset{\{request,response\}}{\bowtie} \texttt{Servers}) \underset{\{cwork,cresp,in,out\}}{\bowtie} \texttt{Control0}$$

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- $Big :: (break : in, repair : out)$
- How about, if either of the two servers are broken:
- $Servers :: (break : in, (break, repair)*, repair : out)$

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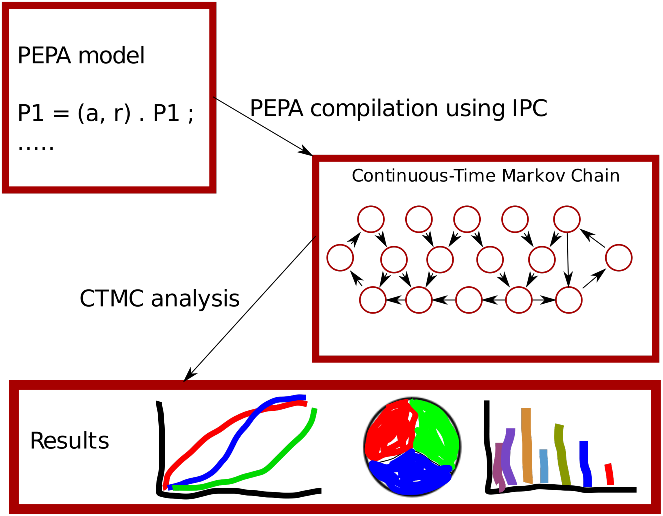
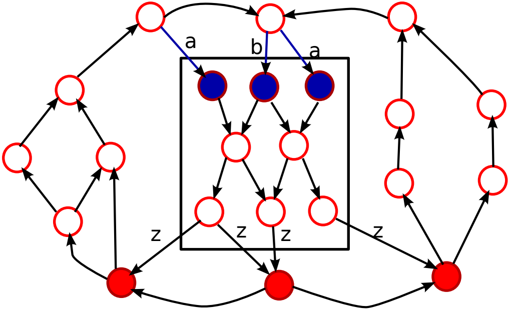
3 Models with a Spatial Aspect
- Spatial Challenge: Capturing physical space

PEPA

We write our models in the PEPA stochastic process algebra



Introduction — Average Response Time



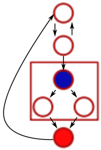
$$\mathit{Response\ Time} = \frac{\text{Probability we are within a passage}}{\text{Throughput of actions which start the passage}}$$

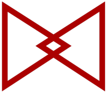
Introduction — Average Response Time

- The above passage may be difficult to specify
- We can describe a passage with a probe
- $(a \mid b) : start, z : stop$
- This is then translated into a PEPA component
- $$\begin{array}{lcl} Probe & \stackrel{def}{=} & (a, \top).Probe_1 \\ & + & (b, \top).Probe_1 \\ Probe_1 & \stackrel{def}{=} & (z, \top).Probe \end{array}$$
- And attached to the main component of the system
- $System \bowtie_{\mathcal{L}} Probe$
  where $\mathcal{L} = \{a, b, z\}$

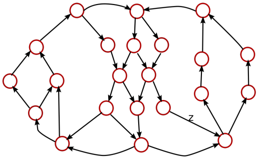
Probes

Probe

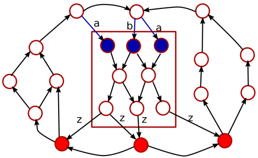




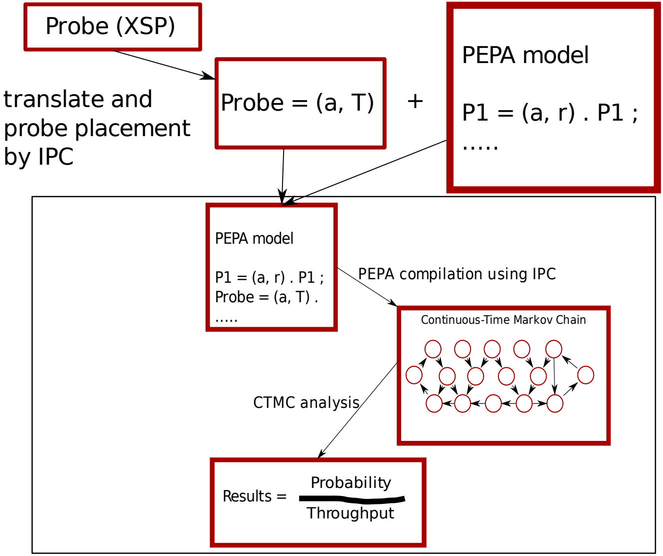
Model



Probed Model



Probe Workflow



Limitations of the average response time

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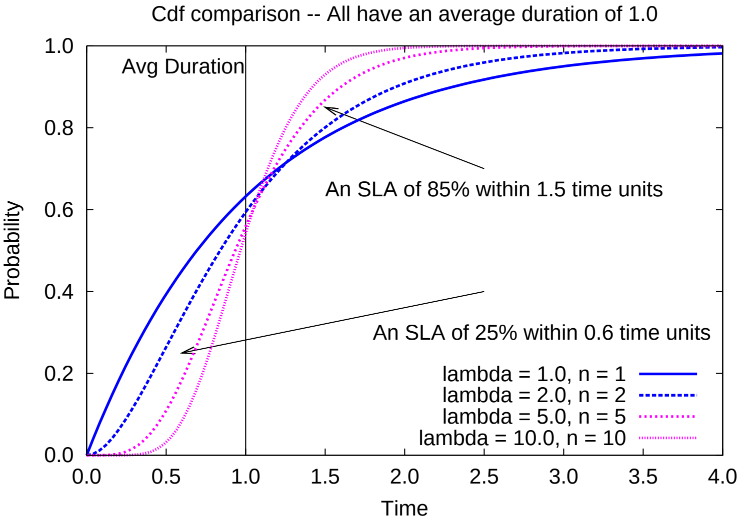
Service Level Agreements are the way that the industry often likes to specify performance requirements and these require a more detailed analysis of responses.

In particular they require the full cumulative (probability) distribution function (cdf) to be calculated for a response time rather than just its average.

Passage times and service level agreements



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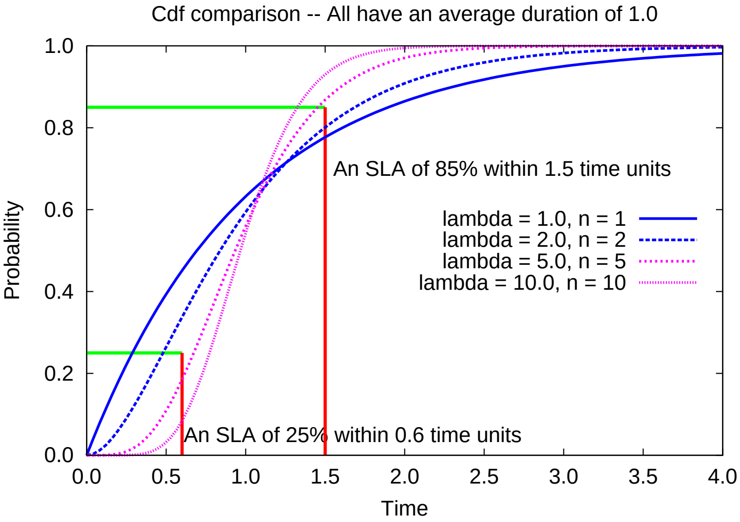
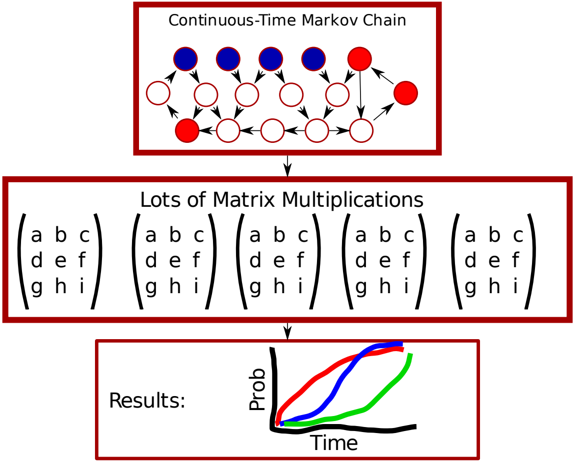


Uniformisation

We use a technique called "uniformisation" to calculate the passage-time quantiles or response-time profiles from the CTMC derived from the PEPA model with a suitable probe.



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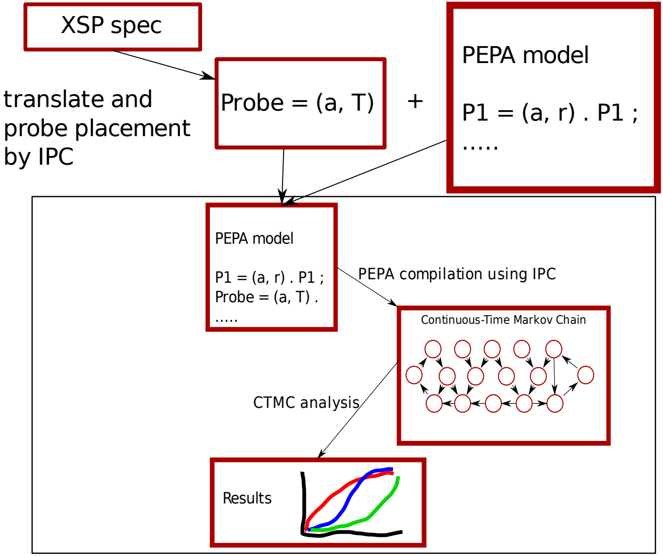
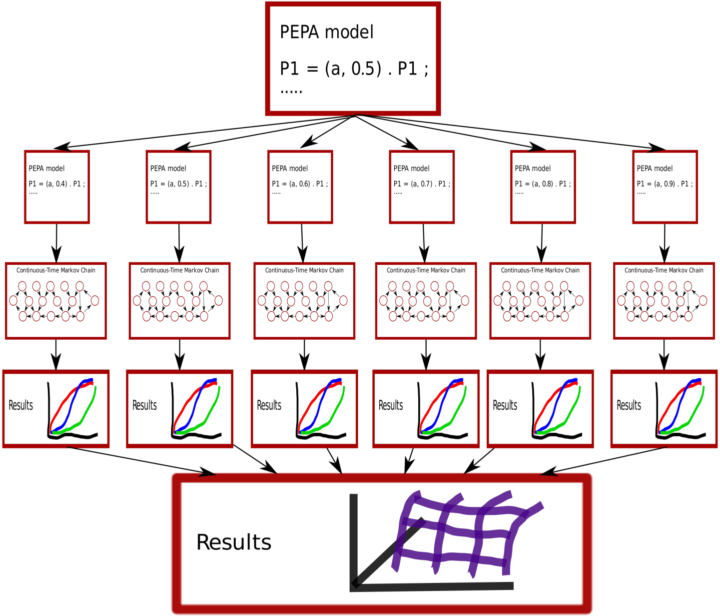


Sensitivity Analysis



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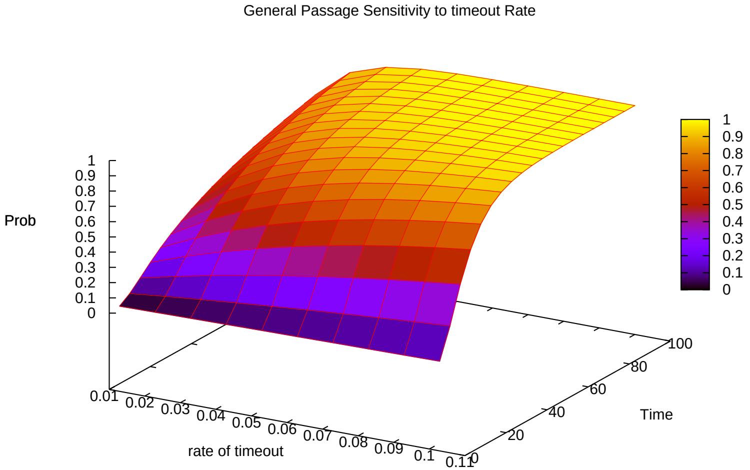

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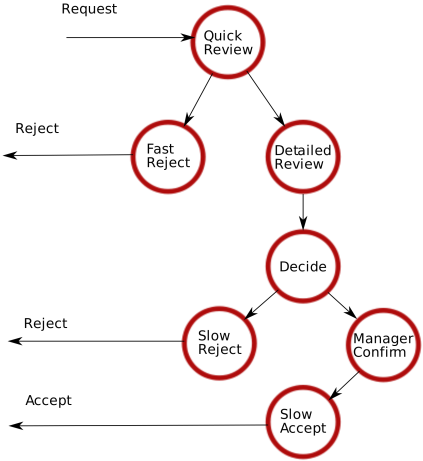

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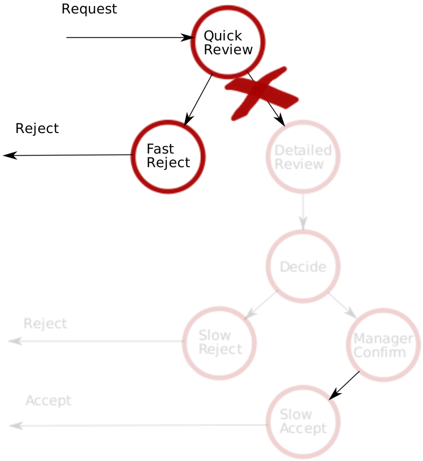

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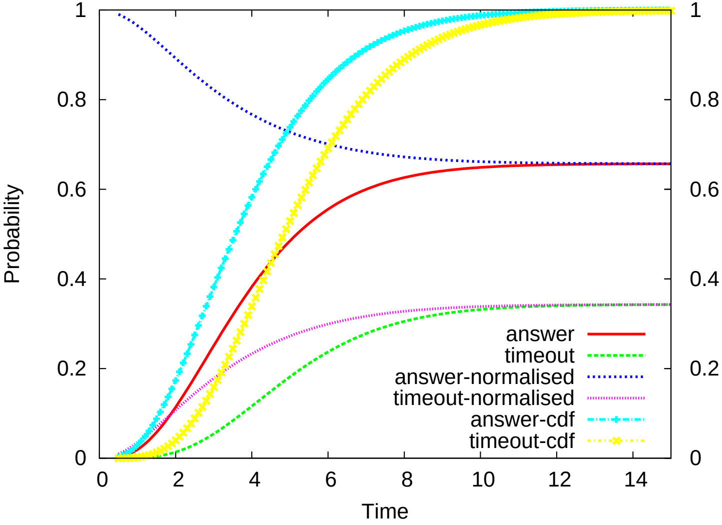
- To address these two problems we developed passage-end analysis.

Airbag Example: Passage-end Analysis



Passage-end Analysis: example queries

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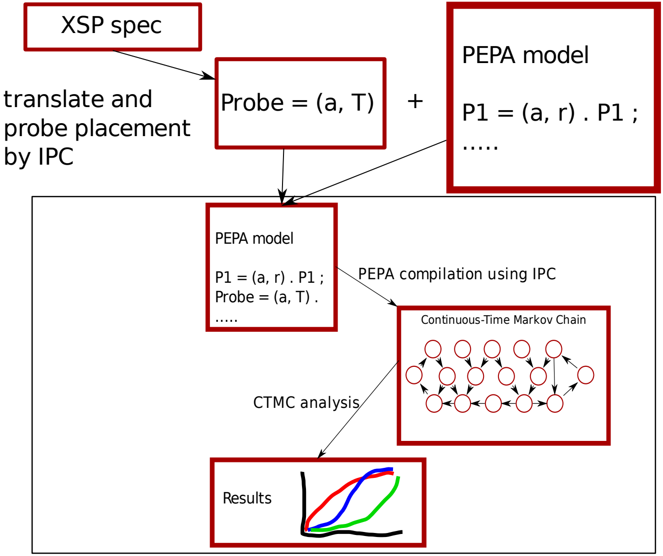
ODE probes

- However the (relevant) analyses possible were limited to average response-time
- During the course of the SENSORIA project we have used the expressiveness of the language of XSP to develop a technique to extract response-time profiles from models which are translated into ODEs
- Additionally because this uses XSP the workflow observed by the user is the same as that when performing analysis via CTMC
- In particular this is due to XSP specifications being translated into PEPA components which when added to the original model result in what is still a PEPA model.

Probe Workflow



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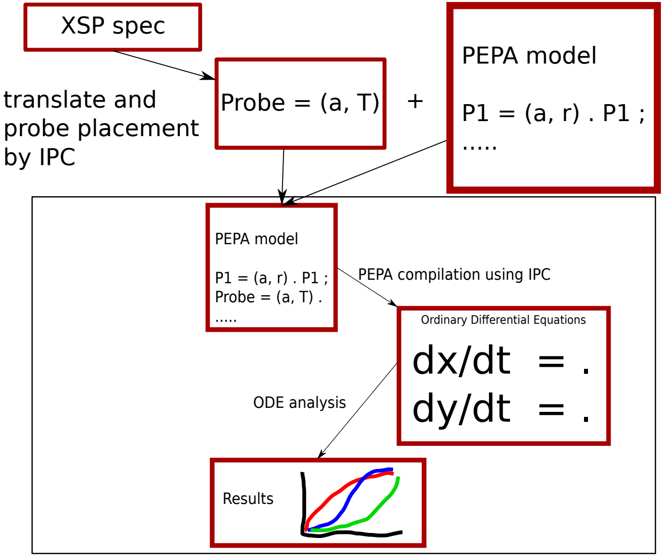

Another Example — Processor/Resource Model

$$\begin{aligned} Processor_1 &\stackrel{def}{=} (think, r_{think}).Processor_2 \\ Processor_2 &\stackrel{def}{=} (use, r_{use}).Processor_1 \end{aligned}$$

$$\begin{aligned} Resource_1 &\stackrel{def}{=} (use, r_{use}).Resource_2 \\ Resource_2 &\stackrel{def}{=} (reset, r_{reset}).Resource_1 \end{aligned}$$

$$(Processor_1[200]) \underset{\{use\}}{\bowtie} (Resource_1[120])$$

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From the steady-state analysis:

$$
\begin{aligned}
RunningProcessor_2 &= 111.75 \\
StoppedProcessor_1 &= 88.25
\end{aligned}
$$

Processor - Resource CTMC Response Times

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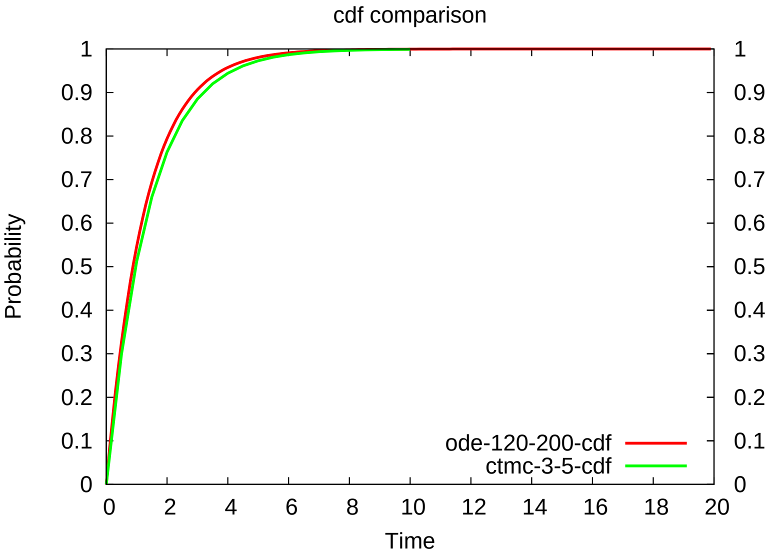
$$Resource_2 \stackrel{def}{=} (reset, r_{reset}).Resource_1$$

$$(Processor_1[5]) \underset{\{use\}}{\bowtie} (Resource_1[3])$$

$$Probe = Processor_1 :: think : start, use : stop$$

Processor - Resource Comparison



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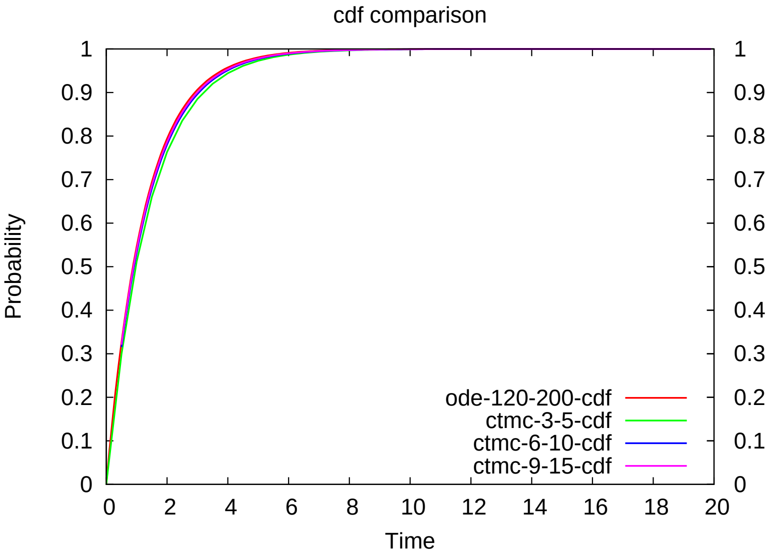

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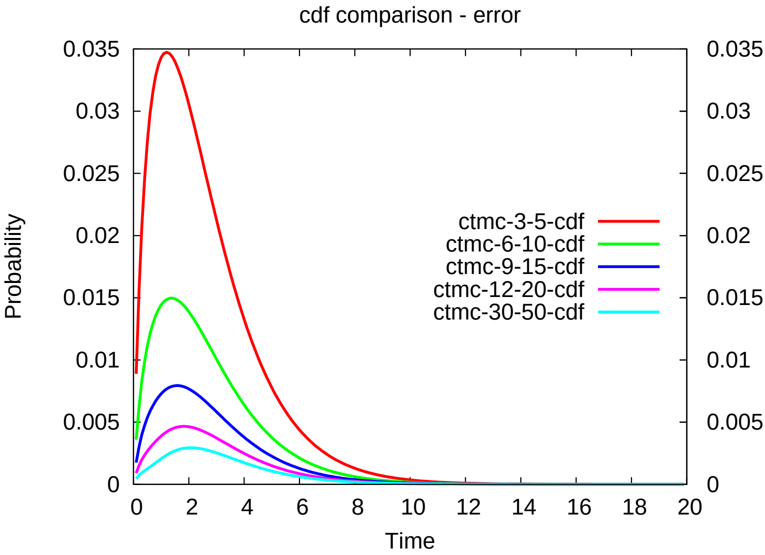

Outline

1 Stochastic probes

2 Passage Time and Passage End Analysis

3 Models with a Spatial Aspect

- Spatial Challenge: Capturing physical space

Spatial Challenge: capturing logical space

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Mobile devices and mobile computation

The location of components of a software system can have dramatic effect on the performance, particularly as communication is often slow compared with computation. Thus capturing whether components are co-located or communicating over a distance became important.

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Biochemical signalling pathways
Far from being a well-mixed soup, the inside of a cell is highly structured and divided into distinct compartments. This physical organisation can have a strong impact on the dynamic behaviour.

Mobile computation: PEPA nets

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- The combination naturally represents applications with two classes of change of state (global and local).
- For example, in a mobile code system PEPA terms are used to model the program code which moves between network hosts (the places in the net).

Petri nets

- Petri nets provide a graphical presentation of a model which has an easily accessible interpretation and like process algebras they are supported by an unambiguous formal interpretation.

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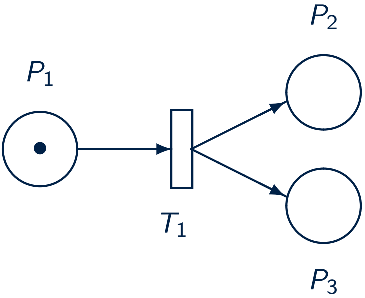

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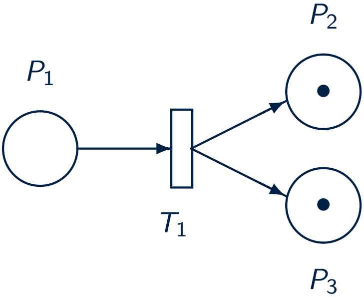

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- PEPA nets are coloured stochastic Petri nets where the colours used as the tokens of the net are PEPA components.

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- Since only co-located components can cooperate all transitions have local effect because they involve only components at one place in the net.

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- When it returns to the central co-ordinating site it dumps the data which it has harvested to the master probe. The master probe performs a computationally expensive statistical analysis of the data.
- The structure of the system allows this computation to be overlapped with the agent's communication and data gathering.

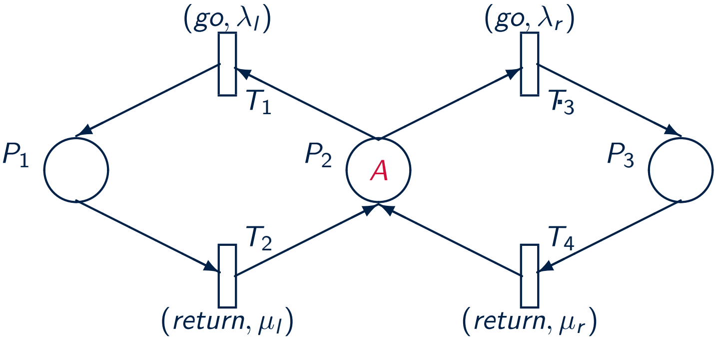
PEPA components

$$
\begin{aligned}
Agent &\stackrel{def}{=} (go, \lambda).Agent' \\
Agent' &\stackrel{def}{=} (interrogate, r_i).Agent'' \\
Agent'' &\stackrel{def}{=} (return, \mu).Agent''' \\
Agent''' &\stackrel{def}{=} (dump, r_d).Agent
\end{aligned}
$$

$$
\begin{aligned}
Master &\stackrel{def}{=} (dump, \top).Master' \\
Master' &\stackrel{def}{=} (analyse, r_a).Master
\end{aligned}
$$

$$
\begin{aligned}
Probe &\stackrel{def}{=} (monitor, r_m).Probe + \\
&\quad\; (interrogate, \top).Probe
\end{aligned}
$$

PEPA net example



In this model there is a *Master* component located in place P_2, and a *Probe* component located in each of the places P_1 and P_3.

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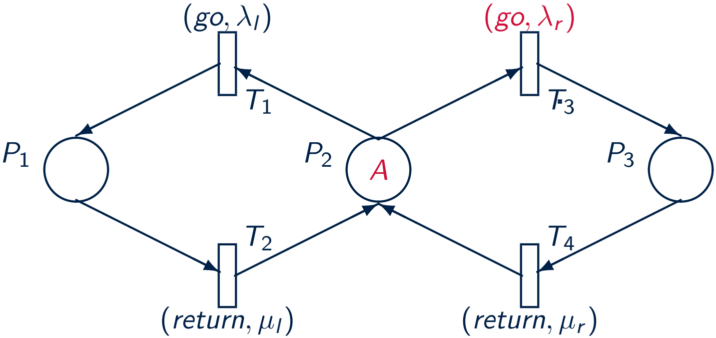

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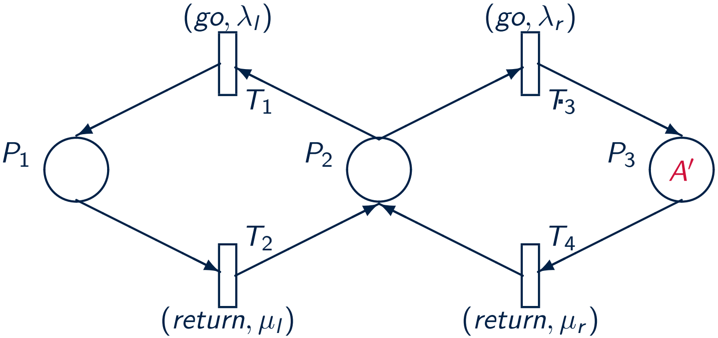

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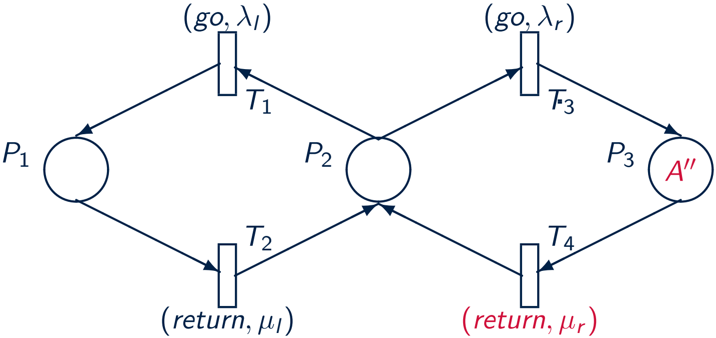

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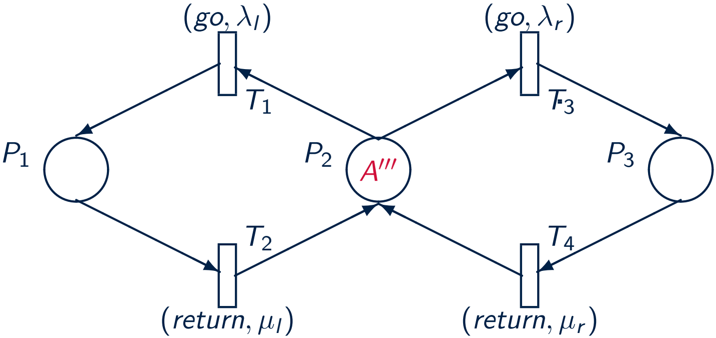

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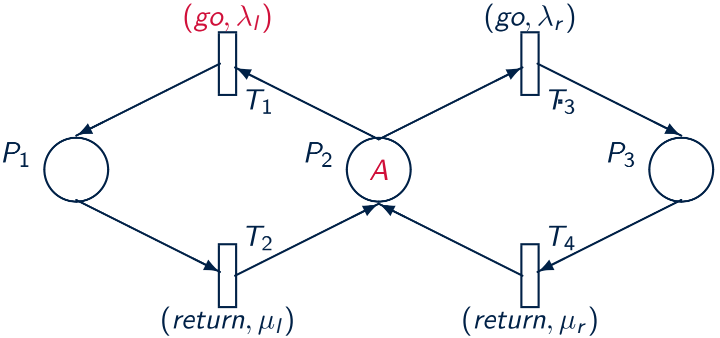

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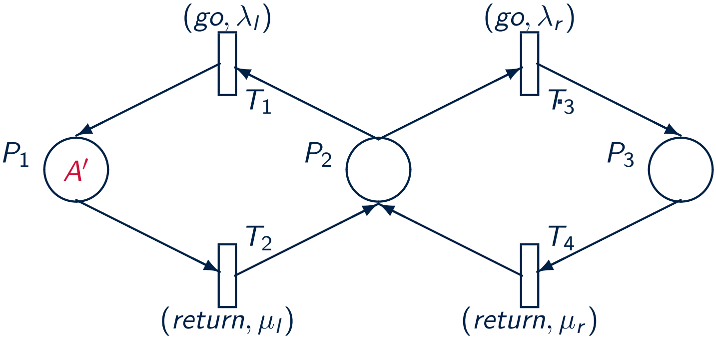

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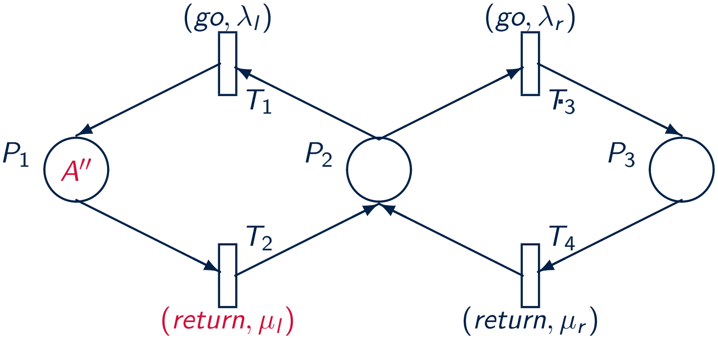

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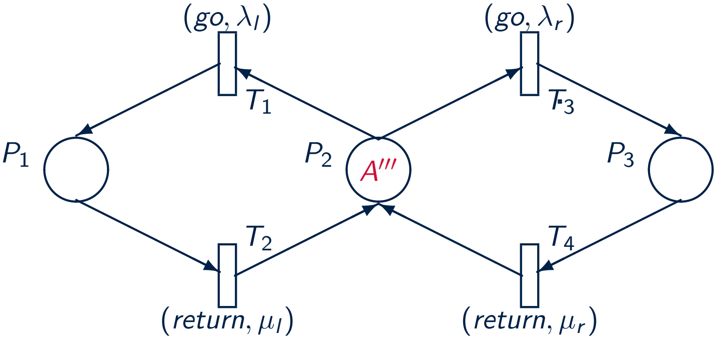

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Bio-PEPA

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The language contains some constructs to model locations, and particularly pathways involving multiple compartments.

Modelling biological locations

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Reactions can then be considered to be

- internal to one compartment or membrane
- involving elements in one compartment and one membrane
- transport between compartments.

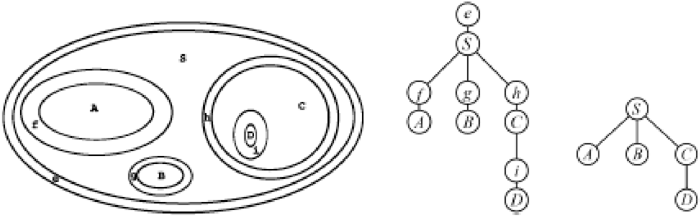
Modelling biological locations

Bio-PEPA considers locations which can be either compartments or membranes.

Reactions can then be considered to be

- internal to one compartment or membrane
- involving elements in one compartment and one membrane
- transport between compartments.

A location tree is used to represent the hierarchy of locations.



Locations in Bio-PEPA

Components in Bio-PEPA are known as species, and in essence, a species in a different location is treated as a distinct species.

However to ease the representation of models, high-level syntax allows some compact representations e.g.

$S \stackrel{def}{=} (\gamma[L_1 \to L_2], \kappa) \odot S$ for transport from location L_1 to location

$S \stackrel{def}{=} (\alpha, \kappa) op S@L_1$ for reaction α at location L_1

Analysing models with logical locations

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However, for analysis they currently rely on an expansion that treats each component, at each location, as distinct.

This exacerbates the problem of state space explosion and can limit the size of models that can be analysed.

In particular, fluid approximation techniques can only be used when the population at each location is sufficiently large to justify the continuous approximation.

Moving on to physical space

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This poses significant challenges both of model expression and model solution.

QUANTICOL Examples

The most expensive aspect of the Paris bike sharing system is relocating bikes to where they are needed.

In smart grids and sustainable energy production with limited storage capacity the location of production and demand become important.

Hybrid process algebra HYPE

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Unfortunately based on hybrid simulation only six zebras could be simulated on a standard machine and fluid techniques are not applicable.