Interactive and Hierarchical Tracing of Techniques in IsaPlanner

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Abstract

We describe interactively tracing and exploring the application of proof planning techniques in IsaPlanner. The machinery is implemented by extending the language for expressing techniques with constructs that produce meaningful traces. We consider how the resulting tool can be used as an interface for theorem proving and as an aid to the development of techniques.

Key words: Proof Planning, Hierarchical Tracing, Theorem Proving, User Interaction, Isar, Isabelle, IsaPlanner.

1 Introduction

Many theorem proving tools rely on traces to inform the user of their behaviour, including first order resolution provers, rewriting systems, inductive theorem provers and proof planners. Unfortunately such traces are often large and difficult for the user to understand. Furthermore, they do not offer the user a way to easily interact with the proof attempt. Tracing proof attempts is an important aspect of interaction with theorem provers. Some systems, such as ACL2 [11], use traces as the main means of communication with the user. Additionally, an important tasks for many users of proof systems is encoding knowledge into the system. A common approach in those that support being extended by the addition of new proof techniques is thus the support for the development of tactics.

In this paper, we describe an approach to interactively tracing the application of techniques in the IsaPlanner proof planner. We also examine how it

[1] This research was funded by the EPSRC grants GR/S01771. We would also like to thank the anonymous referees as well as Graham Steel, Alan Bundy, and David Aspinall for their helpful comments.

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can be used to assist user-interaction. The general motivation is to help the user direct the proof process by providing them with a clear notion of their location in the proof attempt. In particular, we consider the following issues:

Debugging: How can the internal steps of a proof tool be examined at a suitable level of detail for debugging?

Development: How can the interface help developers gain insight into ways that theorem proving techniques might be extended and improved?

Explanation: How can the effect of powerful proof techniques be made comprehensible to the user?

Specialised user interaction: How can user interactions be guided to particular points in the proof process, such as those that the proof planner is most likely to get wrong?

In §2 we introduce proof planning with IsaPlanner. We then present the interactive tracing machinery in §3, and examine how it can help the process of debugging techniques and developing them in §4 and §5 respectively. We consider using the tool for explaining a technique's behaviour in §6, and how it might provide a specialised user interaction in §7. Related work is presented in §8. Finally, we conclude and describe further work in §9.

2 Proof Planning with IsaPlanner

Proof planning was introduced by Bundy [2,3] as a paradigm for proof automation that encodes common patterns of reasoning. These derive abstract descriptions of proofs called *proof plans*. IsaPlanner [7] is a proof planner, written in ML, that searches for proof plans in terms of Isar proof scripts [14], which it verifies using the Isabelle proof assistant [13].

The process of proof planning, in IsaPlanner, is broken into a series of *reasoning states* that can be viewed as 'snapshots' of the proof planning process. IsaPlanner provides an informal 'name' for each reasoning state. This is either automatically generated, or explicitly defined by the developer of a technique. These names serve as a tool to show the user what the proof planner is attempting to do.

We will refer to common patterns of reasoning encoded in IsaPlanner as *techniques*. For instance, *rippling*, is a rewriting technique based on different reduction and commonly used in inductive theorem proving [4,8]. Each reasoning state contains an optional technique, called the state's *continuation*, that represents the next step to be performed in the reasoning process. Techniques that take several steps set the continuation to be the next step to be performed.

More specifically, techniques in IsaPlanner are functions of type $reasoning\_state \rightarrow reasoning\_state\ List$, where the resulting list of states represents the orchoices in the search space. Thus, applying a technique is a sequential process

of applying a state's continuation to that state. This lazily unfolds the search space, which can be explored using standard search-strategies, such as depth-first or breadth-first search.

IsaPlanner provides an extensible language of functions for constructing techniques. This includes the usual notions of `THEN`, `OR` and `REPEAT` from tactic languages. Various techniques have been implemented in IsaPlanner, including proof critics to conjecture lemmas, mechanisms for generalisation, and higher-order version of rippling. These have been combined to provide an inductive theorem proving technique.

3 The Interactive Tracer

The main task for the interactive tracing machinery is to allow the user to navigate through the search space within the application of a technique. This forms a custom search strategy for proof planning where the user decides which state to examine next.

In order to give the user a clear idea of their location in the search space, we define a trace structure that describes a path through the search space. We make this hierarchical so that some states in the path may be considered child-states of an earlier state. The interpretation is that child-states provide specific details about how the parent-state is performed. For example, a parent state named "rippling" would have child-states that specify the specific rules that are applied during rippling.

Each reasoning state during proof planning contains a hierarchical trace of the path traversed so far. This is a simple tree structure with ordered child nodes. The hierarchical trace is constructed as techniques are applied. In order to simplify the process of writing techniques that build traces, we extend the language for writing techniques with three basic constructors:

`NAME R N` applies technique `R` once, then names the next state `N`. This is the basic primitive for providing explicit textual descriptions of reasoning states. This provides a name which will be used when the next state is added to the trace structure.

`THEN A B` applies the technique `A` fully, then creates a new node in the trace structure for `B` which will then start to be applied. This is a simple replacement for the traditional `THEN` which incorporates the construction of the trace.

`REFINE A` applies the technique `A`, making the steps in its application child-states of the current state. Because we do not know the number of steps within `A`, this is actually composed of two sub-operations: `START_REFINE` and `END_REFINE`.

Figure 1 illustrates the definition and application of a simple technique and shows the construction of a hierarchical trace. The textual description of the trace shows child-nodes using indentation. A node starting with "+"

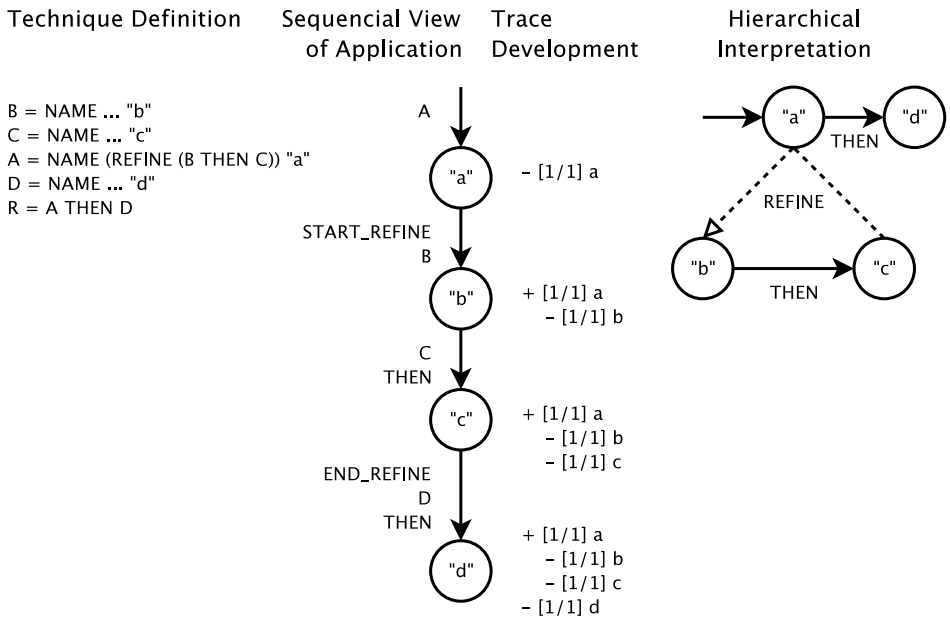

Fig. 1. The definition of a technique R, its application viewed as a sequential chain of states and its intended hierarchical interpretation. We also show the construction of IsaPlanner's text-based presentation of the trace structure.

indicates that it has child states, where those starting with "-" are leaf nodes in the hierarchy.

The hierarchical nature of the trace structure allows the interactive tracing tool to step through 'chunks' of the proof attempt without the user having to examine the intermediate steps. In particular, it can perform a normal search algorithm, such as depth first search, until a state is found which has returned to the original trace-depth. This enables the tracer to step over the individual rewrites involved in rippling, allowing the user to navigate the search space at a higher level. For instance, rather than examine each step in the proof of a lemma, the user can step over the proof as if it were a single step.

The hierarchical trace does not show the relationship between proof techniques and open goals. This allows us to represent steps that transform several goals simultaneously. This is needed for deductive synthesis problems where an unknown meta-variable occurring in several goals is instantiated. This also enables the expression of proof critics that may manipulate the whole proof plan rather than just add steps at open goals. However, the lack of a clear relationship between the trace and the goals can also make the trace a confusing representation of the proof. In future work, we hope to provide a representation for this relationship and mechanisms to view it in the trace structure.

3.1 The User Interface for the Tracer

In order to give the user a more logic-orientated view of the proof, IsaPlanner's tracer also shows the current partial proof plan as an Isar proof script, the current open goals, as well as the trace structure.

For example, Figure 2 shows the output of the tracer during a proof of the commutativity of addition. The partial proof plan has proved the base case by simplification using the lemma $b + 0 = b$. To make the tree structure more readable, previous subtrees in the trace are not shown, unless the user enters a command to see the full hierarchical trace.

The numbers in between the square parenthesis show which state was chosen from the or-choices and the total number of choices that were available. For example, `[1/2]` indicates that the first of two possible states was chosen. The trace structure does not show the search space related to these alternative branches. The purpose is to show the path in the search space without giving the user too much detail about other branches.

The second part of the output from the tracer shows the options available to the user, and presents them with a prompt (`PP>`). The `eXit` command leaves the interactive tracer and returns the last reasoning state examined to the underlying ML interpreter. This allows the user to interact directly with the reasoning state, for example by changing the next technique to apply.

The numbered choices in the user options show the different or-choices in the search space. This allows the user to manually select which branch to explore next. There are three other navigational commands:

- `Back` returns to the previously examined node.
- `Step` is given a number and performs a depth first search from the selected or-choice in the numbered list. It searches for the next state that returns to the same depth in the hierarchical trace.
- `Go` performs a depth first search for a reasoning state that solves the initial given conjecture.

There are also three viewing commands. These allow the user to view: the conjectures made made so far with their state (proved, failed to be proved, or refuted), the full hierarchical trace, and the full Isar proof script including the automatically proved lemmas.

One of the main differences between this kind of command driven interaction in comparison with the traditional tactic driven approach, employed by interfaces such as proof general [1], is that our interface shows the different ways in which a technique can be applied. In contrast to this, other command driven interfaces show only the first result. Such interfaces require the user to either modify the parameters to the tactic in order to get a different behaviour or explicitly enter backtracking commands. Our approach, which allows the user to see and select desired result, is particularly useful when parameterising the tactic requires intricate knowledge of its behaviour.

```
------------------------------------------------------------------
State (id: 196):

Partial proof plan:
  -- {* Proving a + b = b + a *}
  proof (induct a)
    have b = b + 0 by (rule sym[OF add_0_right])
    thus 0 + b = b + 0 by (simp (no_asm))
  next
    fix N
    assume H1: "∀ b. N + b = b + N"

Hi-Trace:
+[1/1] Induction and rippling...
  -[1/2] Induction on "a"
  +[1/1] Solve the base case using simplification...
  +[1/1] Solve the step case using rippling....
    -[1/1] Start Rippling with state:
           measure: out:[2, 0, 0], in:[0, 0, 0]
           annotated goal: [< Suc >] N + |_b_| = |_b_| + [< Suc >] N

Open Goals:
  1. Suc N + b = b + Suc N
------------------------------------------------------------------
Commands: [x] eXit  [b] Back  [s] Step  [g] Go
View:     [c] Conjectures  [i] Isar Script  [v] Full HiTrace

[1]  Ripple Step by (subst add_Suc)
     measure: out:[1, 1, 0], in:[0, 0, 0]
     annotated goal: [< Suc >] (N + |_b_|) = |_b_| + [< Suc >] N

[2]  Ripple Step by (subst add_Suc_right)
     measure: out:[1, 1, 0], in:[0, 0, 0]
     annotated goal: [< Suc >] N + |_b_| = [< Suc >] (|_b_| + N)
------------------------------------------------------------------
PP>
```

Fig. 2. An example of the user interface during an interactive trace of the proof of the commutativity of addition. The top half presents information about the reasoning state and the bottom half shows the user's options.

This utility of this can be seen in the following simple example. Consider the proof of $x+0 = x$ in Peano arithmetic where addition is defined recursively on the first argument. In the inductive proof of this theorem, we arrive at the step case goal $Suc\ (x + 0) = Suc\ x$. To complete the proof, we can apply the induction hypothesis, $x + 0 = x$ to remove the 0 on the left or introduce a 0 on the right. However substitution with the hypothesis can be done in many different ways, each of which results in an alternative subgoal. However, only two of these help complete the proof. The full list of choices are:

(i) $Suc\ ((x + 0) + 0) = Suc\ x$
(ii) $Suc\ (x + (0 + 0)) = Suc\ x$
(iii) $Suc\ ((x + 0) + 0) = Suc\ x$
(iv) $(Suc\ (x + 0)) + 0 = Suc\ x$
(v) $Suc\ (x + 0) = Suc\ (x + 0)$ [*]
(vi) $Suc\ (x + 0) = (Suc\ x) + 0$
(vii) $Suc\ x = Suc\ x$ [*]

but only those marked with a '*' lead to a direct proof by reflexivity. The approach often employed in tactic driven interfaces is to allow the user to select a redex by either partially instantiating the rule or by giving a number referring to a redex in some ordering of the search for substitutions. However, such an approach requires the user consider how to apply the tatic in order to then guide the prover. In the above example, the user must know the ordering in which the subtiution tactic considers redexes, or provide the instantiation by hand. In contrast, our approach provides the user with the possible results of the technique's application and allows them to select which result they would like to arrive at. This makes the interface do more of the work and thus removes the need for the user to know the details of the proof tool.

We remark that this does not always work ideally. In some cases a tool can be applied in a large number of ways which can result in the user being swamped by choices. Such situations require the developer of the technique to provide further pruning of the search space. However, because such pruning is not always possible, we do not consider our proposed approach as an alternative to tactic based theorem proving but as an extension of it. It is still useful to allow the user to specify a redex or give an instantiation by hand.

4 Debugging

One of the chief difficulties in developing proof tools for Isabelle is debugging them. The black-box evaluation of ML functions makes it difficult to observe which part of the proof tool fails and why.

A common approach to debugging is to insert print statements into the technique and use these to observe which part of the code is failing. However, for complex proof tools which perform many steps, this leads to an excessive

amount of output which the developer then has to examine. Using flags to turn on and off different kinds of printing can help, but tends to clutter the code with conditional pretty printing statements.

Another traditional approach is to use language-level debugging tools, such as an ML debugger. Such tools provide similar support to our interactive tracer. However, they give an overly detailed view of the proof process, showing many internal details. Our tracing mechanism provides an interface for debugging at a higher level and supports stepping over parts of the proof in a different way to the underlying function calls.

An example where we found the tool to be particularly useful was in debugging automatic lemma speculation. Errors we made in the development of a this critic resulted in the wrong lemma being speculated. Running the technique resulted in non-termination. However, there are many possible errors that could have this result. Our tracing mechanism allowed us to identify the lemma speculation problem easily.

5 Development

The ability to observe the failure of a technique often gives a good indication of how the technique might be modified to solve the problem. The patching of failed proof attempts is the central idea behind Ireland's development of proof critics [9]. We found tracing helpful in the development of generalisation critics for higher-order logics.

For example, during the proof attempt of "$a^{(b+c)} = a^b * a^c$" within Peano arithmetic, our initial technique, having performed induction on b, arrived at the step case subgoal "$a*(a^n*a^c) = (a*a^n)*a^c$". Our initial lemma speculation critic conjectured, "$a * (g * a^c) = a * g * a^c$", generalising only over a single common subterm, rather than suggesting the more general associativity of multiplication $x * (y * z) = (x * y) * z$. Stepping through the proof attempt to the location where the generalisation was made quickly made the problem and its solution apparent. We were then able to quickly modify the speculation mechanism to further generalise the conjectured lemma.

Another example of the benefit to the development of proof techniques is shown in the proof attempt of $x + 0 = x$ which we introduced earlier. A technique can be applied in many alternative ways indicates points in the search space that have a high branching factor. In the above example, considering the application of the induction hypothesis braught to light the following heuristic for pruning the search space: during an inductive proof, only substitutions with the induction hypothesis used from left to right should be considered in the left hand side of the subgoal. Symetrically, only right to left subsitutions should be considered in the right hand side. We then observed that this heuristic also has a theoretical motivation. In particular, only these subtiutions make the left and right hand sides of an equational subgoal more similar. Thus only these applications are likely to allow the proof to then be completed

by reflexivity.

Although the tracing tool allows the user to explore the search space and observe specific points when the proof fails, it gives only a limited indication of which parts of the search space are irrelevant to the proof. In particular, it allows us to observe points with a high branching factor, but does not show which branches do not lead to success. We suggest that the tracing tool should be combined with visualisation mechanisms in order to help developers devise further ways to prune the search space of their techniques.

6 Explanation

The converse of understanding why a technique fails is comprehending why it succeeded to prove a goal. In a sense, we are asking for an explanation of the proof. The motivation for such proof explanation comes from the increasing use of powerful proof techniques that can perform steps that are too large to be comprehensible to the user. For example, IsaPlanner can prove, from only the primitive definitions, the theorem $(a^b)^c = a^{b*c}$ in Peano arithmetic, whereas most presentations of this involve several lemmas. We note that new users to the theorem prover may also find proof explanations useful to understand how the underlying proof system behaves.

The tracing structure provides an approach to explaining the application of a technique. It tells the story of the proof attempt. For example, the trace produced for by proof planning $(a^b)^c = a^{b*c}$ is shown in Figure 3. At present, these descirptions are text based, but we believe that a visual presentation, such as that used in the XBarnicle system [10], which uses nested boxes to show the structure of the proof, could also help understanding of the proof attempt. A tool for constructing and interacting with such visualisations of traces is further work. However, we note that the relationship between the trace and the underlying proof would need to be formalised in order to allow the user to independently guide the proof attempts of different subgoals.

Unlike approaches that explain proofs at a logical level, examining the hierarchical-trace explains how the proof was found in terms of its path through the search space. It particular, it does not necessarily reflect the proof's structure. Although this can be confusing, it provides an ability to present the behaviour of proof critics. This makes it a distinct object from the proof plan, which presents only the final proof. For example, contrast the trace and the proof plan produced when proof planning the theorem $(a^b)^c = a^{b*c}$, shown in Figures 3 and 4 respectively. We note that using the Isar language as the representation of poof plans makes the logical structure of the proof readable. Furthermore, it allows the proof scripts constructed by proof planning to be copy and pasted into the theory developments. Providing improved mechanisms for managing this kind of interaction is another interesting area for further work.

```
+[1/1] Prove the goal using Induction and Rippling...
  -[3/3] induction on "c"
  -[1/1] Solve the base case by simplification.
  +[1/1] Solve the step case using rippling...
    -[1/1] Start Rippling with state:
         measure: out:[1, 1, 0, 0], in:[0, 0, 0, 0]
         aterm: "(a^b)^[<Suc>] c = a^(b*[<Suc>]c)"
    -[2/2] Ripple Step by subst mult_Suc
         measure: out:[0, 2, 0, 0], in:[0, 0, 0, 0]
         aterm: "(a^b)^[<Suc>] c = a^([<op +>] (b*c) [<b>])"
    -[1/1] Ripple Step by subst exp_Suc
         measure: out:[0, 1, 1, 0], in:[0, 0, 0, 0]
         aterm: "[<op *>] ((a^b)^c) [<a^b>] = a^([<op +>] (b*c) [<b>])"
  -[1/1] Weak fertilisation
  +[1/1] By proving lemma1: "a^g*a^b = a^(g+b)"...
```

Fig. 3. An example trace produced by proof planning the theorem $(a^b)^c = a^{b*c}$ where exponentiation is written using the infix operator ^. We omit the trace for lemma which involves conjecturing and proving further lemmas for the sake of brevity.

```
theorem (a^b)^c = a^(b * c)
proof (induct c)
  show (a^b)^0 = a^(b * 0) by simp
next
  fix c
  assume IH: ∀ a b. (a^b)^c = a^(b * c)
  have a^(b * c) * a^b = a^(b * c + b) by (rule lemma1)
  hence (a^b)^c * a^b = a^(b * c + b) by (subst IH)
  hence (a^b)^(Suc c) = a^(b * c + b) by (subst exp_Suc)
  thus (a^b)^c = a^(b * c) by (subst mult_Suc)
qed
```

Fig. 4. The Isar proof script produced by proof planning the theorem $(a^b)^c = a^{b*c}$. For brevity, the proof script for the lemma named *lemma1* is omitted.

7 Specialised User Interaction

Interactive navigation of proof attempts, rather than using fixed search strategies, allows the user to avoid unpromising paths in the search space. This provides a novel approach to working with the theorem prover. IsaPlanner further specialises this user interaction by allowing techniques to tag a reasoning state with a string. The tracer can then be told to search normally, until a state containing the requested tag is found.

In terms of writing techniques in IsaPlanner, we provide a tagging constructor in the technique language, "`TAG N R`", which unfolds the technique `R` once, and tags the resulting state with the identifier `N`. This allows the tracer to search for a state in which some action is performed, indicated by the tag. This can be used to focus the user interaction on points in the proof where a technique is more likely to make an error, such as a generalisation step.

8 Related Work

Many proof tools provide a trace of their behaviour. For example, the $\lambda Clam$ system has a step-by-step mode which allows the user to step through the systems proof planning attempt [6]. Like many other systems, $\lambda Clam$ also provides different levels of printed output, from verbose messages to none at all. Similarly, Isabelle's simplifier provides an option to trace its application of rewrite rules. Such traces of a proof attempt are of even greater importance in the ACL2 system [11], as they are the main mechanism for the prover to communicate with the user. However, the traces provided by these systems are one dimensional: the user can view the trace but cannot interact with the proof tool. In contrast, the approach to tracing that we have presented in this paper provides an interactive mechanism for exploring the search space and allows the user to modify the proof attempt.

A few other systems also allow techniques to be applied in a hierarchical fashion. In particular, the tactics of Nuprl [5] and the methods of the Multi proof planner in the Omega system [12]. The main difference is that our notion of unfolding a hierarchical techniques can involve choice, where these systems have a deterministic result. We allow the user to interact with the choices in the unfolding of the hierarchy and use this as a mechanism for interaction. Another significant difference is that our hierarchy is not directly related to the structure of the underlying proof. Some steps can completely change the proof plan where steps in the unfolding of hierarchical techniques in Nuprl and Omega only refine part of the proof related to a particular subgoal. In this sense, our hierarchy is a generalisation of that used in these system.

9 Conclusions and Further Work

We have described the part of IsaPlanner's language for encoding techniques that constructs hierarchical traces. This is used by the system's interactive tracing machinery to support flexible navigation through the search space involved in the application of a technique. We provide the user with a clear notion of their location in the search space of a proof attempt. We believe this can help guide the interaction with the prover. We also found this tracing mechanism particularly beneficial to the development and debugging of proof techniques. The only additional burden on the technique developer is to use the provided constructs. We found this to have significantly less damage to the clarity of the code than the use of conditional printing statements.

The tracer can be used to explain a proof in varying levels of detail, without hiding the application of proof critics. Our tracing mechanism is text based and gives an operational, rather than visual, view of the search space. Providing a visual representation traces and machinery for interaction is further work. However, the lack of a clear relationship between the tracer and the open goals would be required. In future work we intend to examine mays to include some goal information in the trace. Further work also includes combining the tracing tool with a visual presentations of the search space. We believe this could help developers focus their attention onto portions of the search space that might usefully be pruned.

In order to further evaluate the interactive tracing of techniques an we suggest performing an experiment with new users to Isabelle. The hypothesis would be that examining the trace of a technique will help users to learn how to prove theorems in an interactive prover. Users would be split into two groups. One group would be given the choice to use the interactive tracing machinery and the other would use techniques without being able to further examine their unfolding. The groups would both be given a large set of proofs to complete and they would be meaured based on how many proofs they completed. If the tracing tool helps users then we would expect that group to complete more proofs. In such an experiment the same level of automation should be provided to both groups. Such an experiment would help to identify if traces are useful for more than just the development of proof techniques.

References

[1] David Aspinall and Thomas Kleymann. *Proof General Manual*. University of Edinburgh, proofgeneral-3.5pre-2002 edition, 2002.

[2] A. Bundy. The use of explicit plans to guide inductive proofs. In *Conference on Automated Deduction*, pages 111–120, 1988.

[3] A. Bundy. A science of reasoning. In *Computational Logic - Essays in Honor of Alan Robinson*, pages 178–198, 1991.

[4] A. Bundy, D. Basin, D. Hutter, and A. Ireland. *Rippling: Meta-level Guidance for Mathematical Reasoning*. Springer-Verlag, 2005.

[5] Robert L. Constable, Stuart F. Allen, H. M. Bromley, W. R. Cleaveland, J. F. Cremer, R. W. Harper, Douglas J. Howe, T. B. Knoblock, N. P. Mendler, P. Panangaden, James T. Sasaki, and Scott F. Smith. *Implementing Mathematics with the Nuprl Development System*. Prentice-Hall, NJ, 1986.

[6] L. Dennis and J. Brotherston. *User/Programmer Manual for the λClam proof planner*. School of Informatics, University of Edinburgh, Edinburgh, v4.0.1 edition, 2002.

[7] L. Dixon and J. D. Fleuriot. IsaPlanner: A prototype proof planner in Isabelle. In *Proceedings of CADE'03*, LNCS, pages 279–283, 2003.

[8] L. Dixon and J. D. Fleuriot. Higher order rippling in isaplanner. In *Proceedings of TPHOLs'04*, LNCS, pages 83–98, 2004.

[9] A. Ireland and A. Bundy. Productive use of failure in inductive proof. *Journal of Automated Reasoning*, 16(1–2):79–111, 1996.

[10] Michael Jackson and Helen Lowe. System description: Interactive proof critics in xbarnacle. In David A. McAllester, editor, *CADE*, volume 1831 of *Lecture Notes in Computer Science*, pages 502–506. Springer, 2000.

[11] M. Kaufmann and J. Moore. An industrial strength theorem prover for a logic based on Common Lisp. *IEEE Transactions on Software Engineering*, 23(4):203–213, April 1997.

[12] Erica Melis and Andreas Meier. Proof planning with multiple strategies. In *Computational Logic*, pages 644–659, 2000.

[13] L. C. Paulson. *Isabelle: A generic theorem prover*. Springer-Verlag, 1994.

[14] M. Wenzel. Isar - a generic interpretative approach to readable formal proof documents. In *TPHOLs'99*, volume 1690 of *LNCS*, pages 167–184, 1999.