Motivation and Overview

- Quantum computation as circuit-like graphs (Abramsky, Duncan, Coecke, ...)

$$\frac{\text{graphical calculi}}{\text{quantum mechanics}} = \frac{\lambda\text{-calculus}}{\text{Turing machines}}$$

- Easier to understand and manipulate.
- Certain properties have a natural graphical representation (e.g., disjointness in graph $\Rightarrow$ separable state)
- Abstract algebra of graphs has other applications

- Hard to reason with manually $\Rightarrow$ develop tool support: Quantomatic
- Generalised formalism for graph rewriting (includes some ellipses notation).

From Quantum Information to Graphs

- Quantum computing usually expressed in (finite-dimensional) Hilbert spaces
- $\text{FdHilb}$ is a symmetric monoidal category (SMC)
- Coherence results provide a graphical notation (Mac Lane, Kelly and Laplaza, Joyal and Street)
- To mechanise graphical reasoning, we need a corresponding notion of graph rewriting, as well as sound and efficient algorithms
- Traditional graph transformation machinery lacks: graphical derived rules, interfaces for graphs, graphical ellipsis notation.
Graphical Representation of SMC

(Tensor) \( f \otimes g := f \otimes g \) (Composition) \( g \circ f := g \circ f \)

Bifunctoriality of \( \otimes \) symmetry of braiding (\( \sigma \))

Example: Boolean Circuit Graphical Equations

Graphs for Quantum Computation

Two families of nodes (Z and X): (complementary observables)

\[ \epsilon_Z = \delta_Z = \epsilon_Z^\dagger = \delta_Z^\dagger = \alpha_Z = \delta_Z \]
\[ \epsilon_X = \delta_X = \epsilon_X^\dagger = \delta_X^\dagger = \alpha_X \]

Hadamard gate: \( \mathbb{H} \)

Graph Equalities e.g.

Matching

\( G \) \( \text{relax}(G) \) \( H' \) an open-subgraph of \( \text{relax}(H) \) \( \text{relax}(H) \) \( H \)

- Efficient algorithm by graph traversal:
  - built in love
  - cuts implicit by left-over graph.
Composing Graphs: a picture

- Plugging of G and H via the two-sided graph $\pi$ with embeddings $p_1$ and $p_2$.
- Nice structure: plugging is a SMC (nodes can be graphs!), matching is an order.
- Main theorem: plugging preserves matching and commutes with rewriting.

Representational Issues

Redundant Representations: (equivalent by composition with curves)

Impossible Representations: (an infinite family of rewrite rules)

!-Box Graphs: Example

Example showing how A matches D

Summary of Quantomatic

Formalised graph rewriting for SMC (applied to quantum computation)

Soundness proved, complete set of graphical equations for QM?

Extensible: rules are pairs of graphs - derived rules provide extensible approach to reasoning.

Tool: with fixed logical kernel allows:
- manipulation is otherwise error prone
- (sometimes) efficient symbolic computation by rewriting
- supports reasoning by proving equivalences between quantum computations.
- http://dream.inf.ed.ac.uk/projects/quantomatic