

Graphical Calculi for Reasoning about Quantum Information

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(Joint work Duncan and Kissinger)

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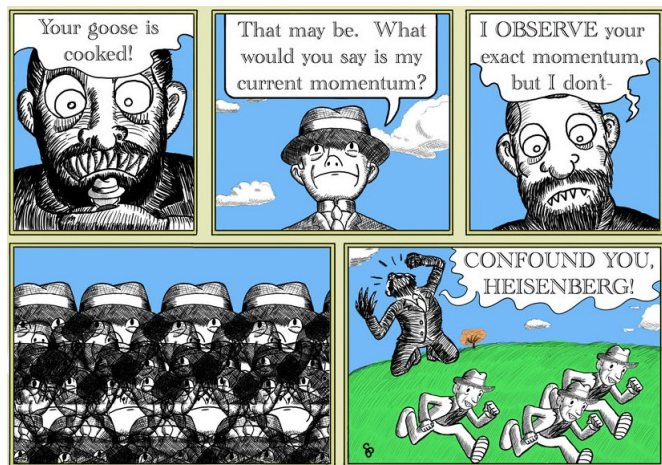
Motivation and Overview

- Quantum computation as circuit-like graphs
(Abramsky, Duncan, Coecke, ...)

$$\frac{\text{graphical calculi}}{\text{quantum mechanics}} = \frac{\lambda\text{-calculus}}{\text{Turing machines}}$$

- Easier to understand and manipulate.
 - Certain properties have a natural graphical representation (e.g. disjointness in graph \Rightarrow separable state)
 - Abstract algebra of graphs has other applications
- Hard to reason with manually \Rightarrow develop tool support: **Quantomatic**
 - Generalised formalism for graph rewriting (includes some ellipses notation).

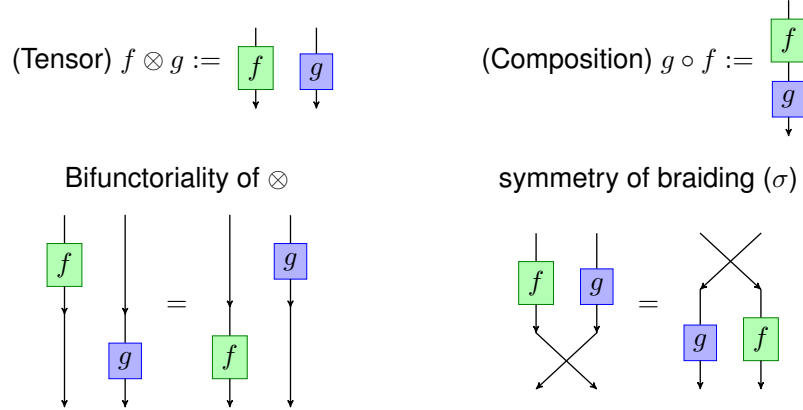
Quantumness: Complementary Observables



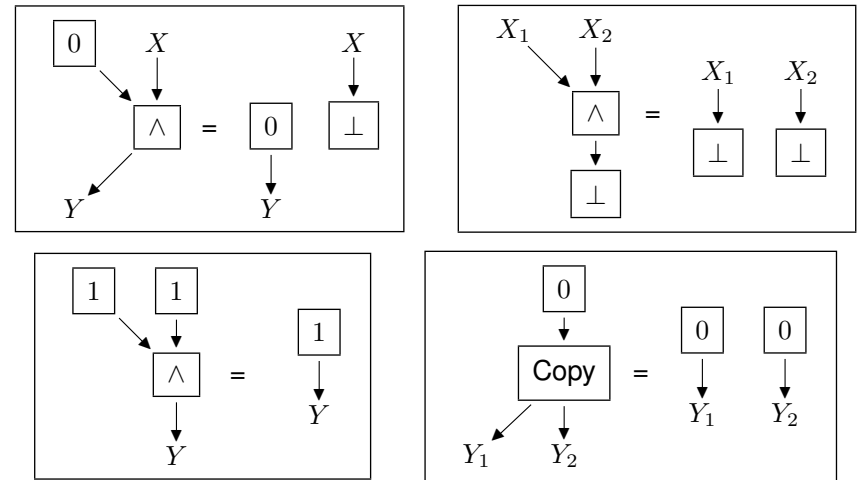
From Quantum Information to Graphs

- Quantum computing usually expressed in (finite-dimensional) Hilbert spaces
- FdHilb is a symmetric monoidal category (SMC)
- Coherence results provide a graphical notation (Mac Lane, Kelly and Laplaza, Joyal and Street)
- To mechanise graphical reasoning, we need a corresponding notion of **graph rewriting**, as well as **sound and efficient algorithms**
- Traditional graph transformation machinery lacks: **graphical derived rules, interfaces for graphs, graphical ellipsis notation.**

Graphical Representation of SMC

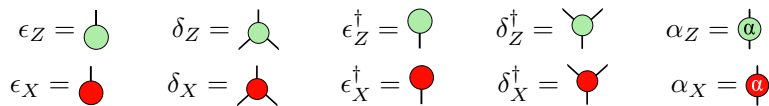



Example: Boolean Circuit Graphical Equations



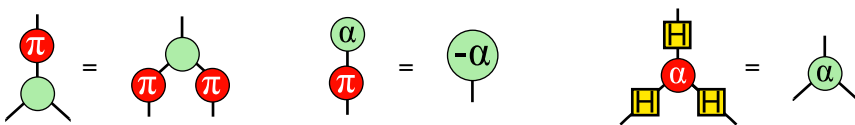
Graphs for Quantum Computation

Two families of nodes (Z and X): (complementary observables)

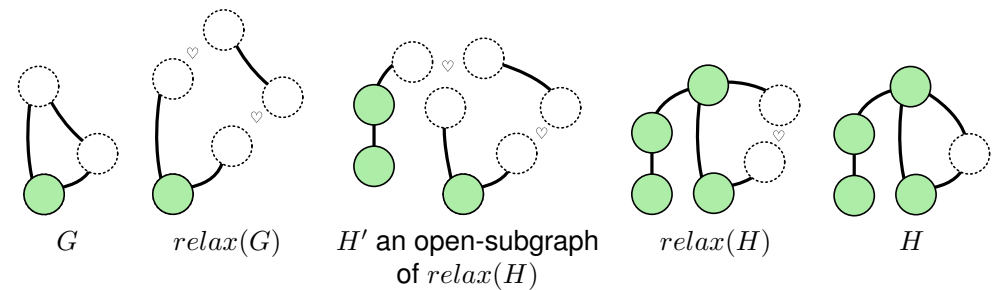


Hadamard gate: 

Graph Equalities e.g.

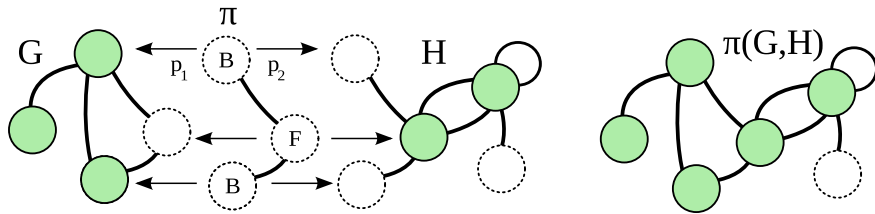


Matching



- Efficient algorithm by graph traversal:
 - built in love
 - cuts implicit by left-over graph.

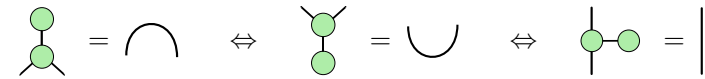
Composing Graphs: a picture



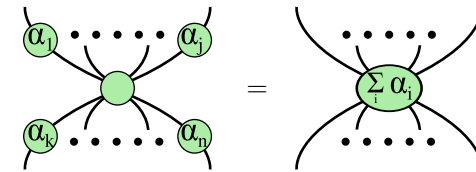
- Plugging of G and H via the two-sided graph π with embeddings p_1 and p_2 .
- Nice structure: plugging is a SMC (nodes can be graphs!), matching is an order
- Main theorem: plugging preserves matching and commutes with rewriting

Representational Issues

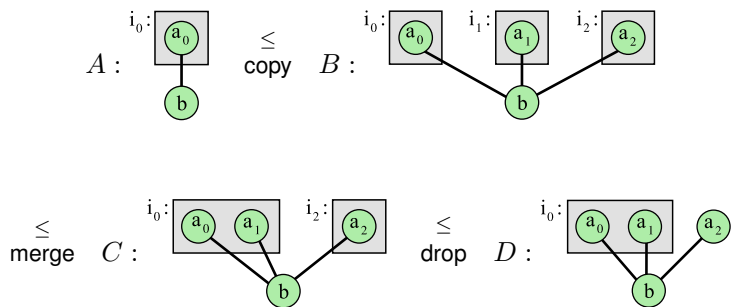
Redundant Representations: (equivalent by composition with curves)



Impossible Representations: (an infinite family of rewrite rules)



!-Box Graphs: Example



Example showing how A matches D

Summary of Quantomatic

Formalised graph rewriting for SMC (applied to quantum computation)

Soundness proved, complete set of graphical equations for QM?.

Extensible : rules are pairs of graphs - derived rules provide extensible approach to reasoning

Tool : with fixed logical kernel allows:

- manipulation is otherwise error prone
- (sometimes) efficient **symbolic computation** by rewriting
- **supports reasoning** by proving equivalences between quantum computations.
- <http://dream.inf.ed.ac.uk/projects/quantomatic>