Graphical Calculi for Reasoning about Quantum Information

Lucas Dixon, University of Edinburgh
(Joint work Duncan and Kissinger)

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School of informatics

Motivation and Overview

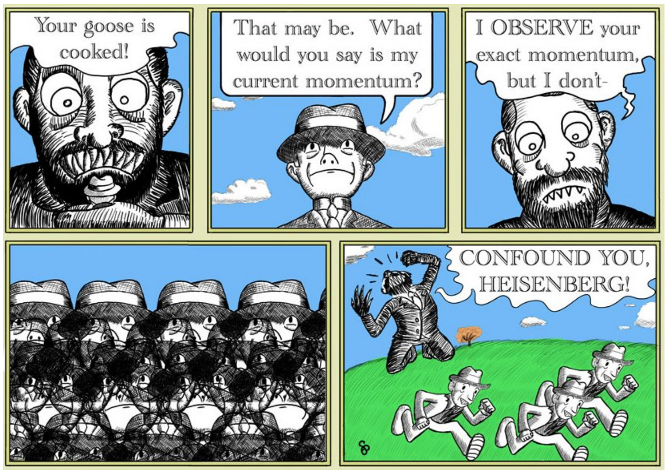
- Quantum computation as circuit-like graphs
  (Abramsky, Duncan, Coecke, ...)

$$\frac{\text{graphical calculi}}{\text{quantum mechanics}} = \frac{\lambda\text{-calculus}}{\text{Turing machines}}$$

  - Easier to understand and manipulate.
  - Certain properties have a natural graphical representation
    (e.g. disjointness in graph $\Rightarrow$ separable state)
  - Abstract algebra of graphs has other applications

- Hard to reason with manually $\Rightarrow$ develop tool support: Quantomatic
  - Generalised formalism for graph rewriting
    (includes some ellipses notation).

Quantumness: Complementary Observables

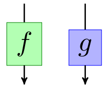


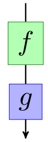
http://dresdencodak.com/

From Quantum Information to Graphs

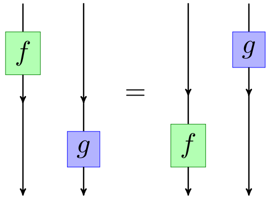
- Quantum computing usually expressed in (finite-dimensional) Hilbert spaces
- FdHilb is a symmetric monoidal category (SMC)
- Coherence results provide a graphical notation
  (Mac Lane, Kelly and Laplaza, Joyal and Street)
- To mechanise graphical reasoning, we need a corresponding notion of **graph rewriting**, as well as **sound and efficient algorithms**
- Traditional graph transformation machinery lacks: graphical derived rules, interfaces for graphs, graphical ellipsis notation.

Graphical Representation of SMC

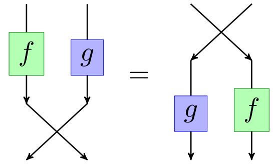
(Tensor) $f \otimes g :=$ 

(Composition) $g \circ f :=$ 

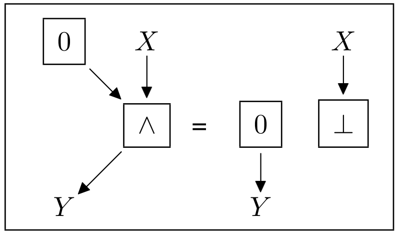
Bifunctoriality of $\otimes$

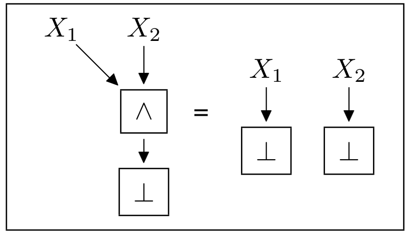


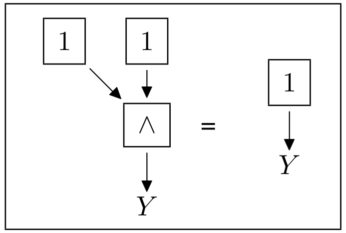
symmetry of braiding (σ)

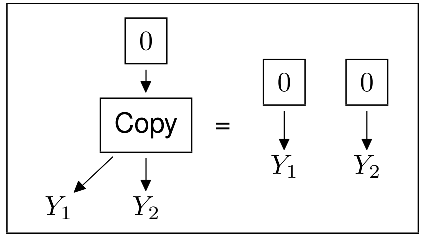


Example: Boolean Circuit Graphical Equations



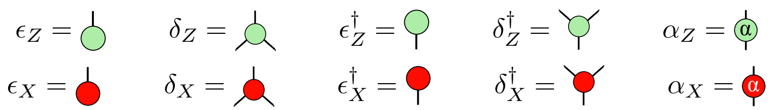






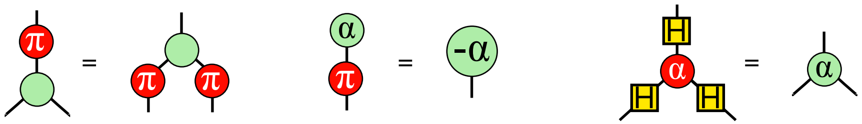
Graphs for Quantum Computation

Two families of nodes (Z and X): (complementary observables)

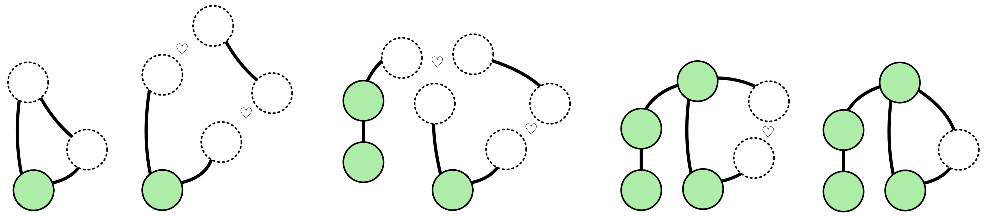


Hadamard gate: 

Graph Equalities e.g.



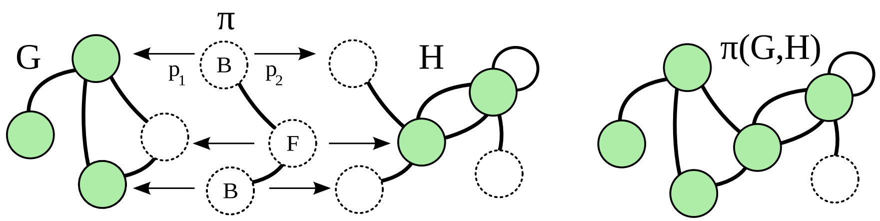
Matching



G $relax(G)$ H' an open-subgraph of $relax(H)$ $relax(H)$ H

- Efficient algorithm by graph traversal:
  - built in love
  - cuts implicit by left-over graph.

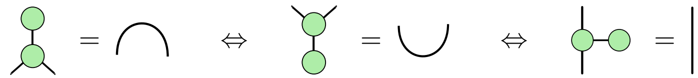
Composing Graphs: a picture



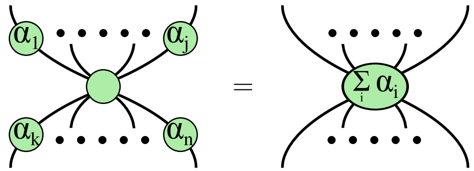
- Plugging of G and H via the two-sided graph π with embeddings p_1 and p_2.
- Nice structure: plugging is a SMC (nodes can be graphs!), matching is an order
- Main theorem: plugging preserves matching and commutes with rewriting

Representational Issues

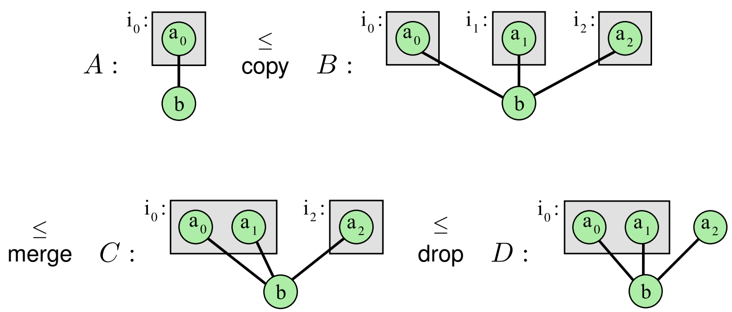
Redundant Representations: (equivalent by composition with curves)



Impossible Representations: (an infinite family of rewrite rules)



!-Box Graphs: Example



Example showing how A matches D

Summary of Quantomatic

Formalised graph rewriting for SMC (applied to quantum computation)

Soundness proved, complete set of graphical equations for QM?.

Extensible : rules are pairs of graphs - derived rules provide extensible approach to reasoning

Tool : with fixed logical kernel allows:

- manipulation is otherwise error prone
- (sometimes) efficient symbolic computation by rewriting
- supports reasoning by proving equivalences between quantum computations.
- `http://dream.inf.ed.ac.uk/projects/quantomatic`