# Querying Graph Databases

# Graph DBs and applications

- Graph DBs are crucial when topology is as important as data itself.
- Renewed interest due to new applications:
  - Semantic Web and RDF.
  - Social networks.
  - Security and crime detection.
  - Knowledge representation.
  - etc etc
  - ...

# Querying graph DBs and relational technology

Why not to use relational technology?

- Translate graph DB $\mathcal{G} \rightarrow$ relational database $\mathcal{D}(\mathcal{G})$, and query $\mathcal{D}(\mathcal{G})$.

Problems:

1. Languages for graph DBs are navigational and require recursion.
2. They can be translated into Datalog, but there are problems:
   (a) Implementation:
   • SQL's recursion is hard to optimize, especially in complex queries, on large databases.
   (b) Complexity mismatch:
   • Datalog evaluation is $\textsc{Ptime}$-complete, but in $\textsc{NLogspace}$ for many graph languages.
   • Basic static analysis tasks undecidable for Datalog, but decidable for several graph languages.

# Early graph query languages

Graph query languages flourished from the mid 80s to the late 90s:

- **G**, $\mathbf{G}^{+}$, and GraphLog for hypertext and semistructured data, late 1980s
- GOOD for graph-based models of object DBs, 1990
- Hyperlog for hypergraphs, 1994
- Languages for heterogeneous and unstructured data, Lorel, StruQL, etc (late 1990s)

# Features of graph query languages

- Navigation: Recursively traverse the edges of the graph.
- Pattern matching: Check if a pattern appears in the graph DB.

And more sophisticated features:

- Path comparisons.
- Comparisons of the underlying data.

# Key problems theory studies:

**Expressiveness:** What can be said in a query language $\mathcal{L}$?

**Complexity of evaluation:**

| | |
|---|---|
| PROBLEM: | $\textsc{Eval}(\mathcal{L})$ |
| INPUT: | A graph DB $\mathcal{G}$, a tuple $\bar{t}$ of objects, an $\mathcal{L}$-query $Q$. |
| QUESTION: | Is $\bar{t} \in Q(\mathcal{G})$? |

- Combined complexity: Both $\mathcal{G}$ and $Q$ are part of the input.
- Data complexity: Only $\mathcal{G}$ is part of the input and $Q$ is fixed.

**Containment:** We study the problem $\textsc{Cont}(\mathcal{L})$:

- Given $\mathcal{L}$-queries $Q_1, Q_2$, is $Q_1(\mathcal{G}) \subseteq Q_2(\mathcal{G})$ for every graph DB $\mathcal{G}$?

# Graph data model

Different applications have given rise to a many (slightly) different graph DB models. But the essence is the same:

Finite, directed, edge labeled graphs.

Despite the simplicity of the model:

- It is flexible enough to accommodate many other more complex models and express interesting practical scenarios.
- The most fundamental theoretical issues related to querying graph DBs appear in it already.

# Graph databases

**Definition**

A graph DB $\mathcal{G}$ over finite alphabet $\Sigma$ is a pair:

$$(V, E)$$

finite set of node ids

set of edges of the form $v_1 \xrightarrow{a} v_2$
$(v_1, v_2 \in V,\ a \in \Sigma)$

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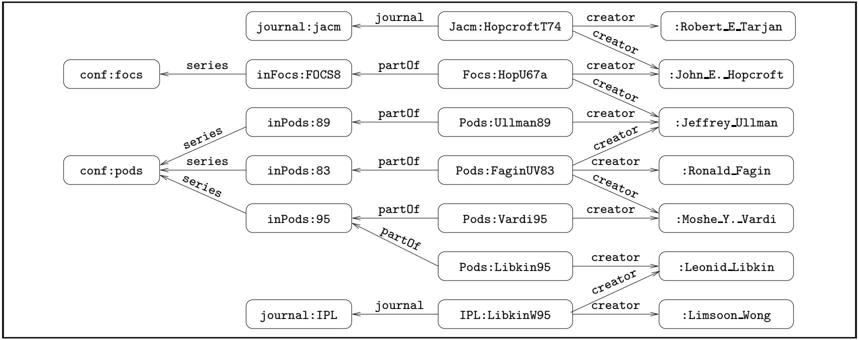
- A path in $\mathcal{G}$ is a sequence of the form:

$$\rho \ = \ v_1 \xrightarrow{a_1} v_2 \xrightarrow{a_2} v_3 \cdots v_k \xrightarrow{a_k} v_{k+1}.$$

- The label of $\rho$ is $\lambda(\rho) = a_1 a_2 \cdots a_{k-1} \in \Sigma^*$.

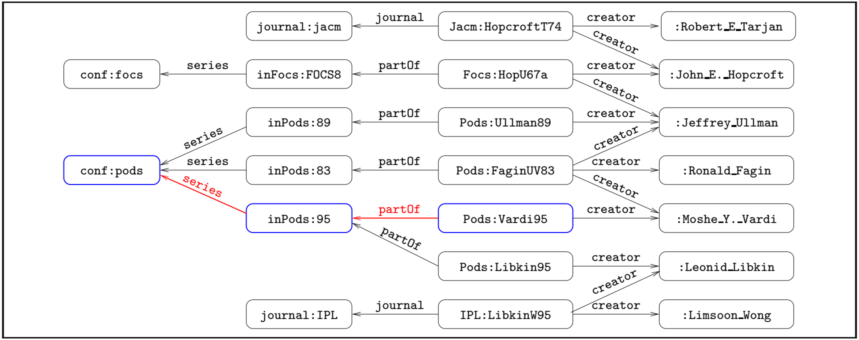
# Graph DBs: Example

A graph DB representation of a fragment of DBLP:



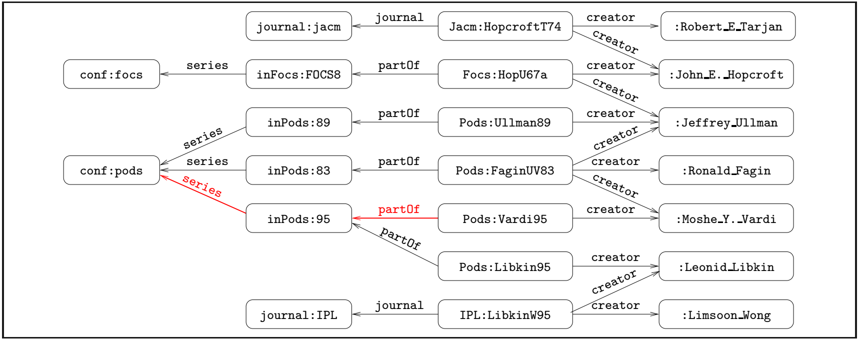
# Graph DBs: Example

A path in this graph DB:



# Graph DBs: Example

The label of such path:



# Graph DBs vs NFAs

Important: Graph DBs can be naturally seen as NFAs.
Recall: NFA = Nondeterministic finite automaton.

- Nodes are states.
- Edges $u \xrightarrow{a} v$ are transitions.
- There are no initial and final states.

# Regular path queries

Basic building block for graph queries: Regular path queries (RPQs).

- First studied in 1989.
- An RPQ is a Regular expressions over $\Sigma$.
- Evaluation $L(\mathcal{G})$ of RPQ $L$ on graph DB $\mathcal{G} = (V, E)$:
  - Pairs of nodes $(v, v') \in V$ linked by path labeled in $L$.

# RPQs with inverse

More often studied its extension with inverses, or 2RPQs.

- First studied in 2000.
- 2RPQs = RPQs over $\Sigma^{\pm}$, where:
    - $\Sigma^{\pm} = \Sigma$ extended with the inverse $a^{-}$ of each $a \in \Sigma$.

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Evaluation $L(\mathcal{G})$ of 2RPQ $L$ over graph DB $\mathcal{G} = (V, E)$:

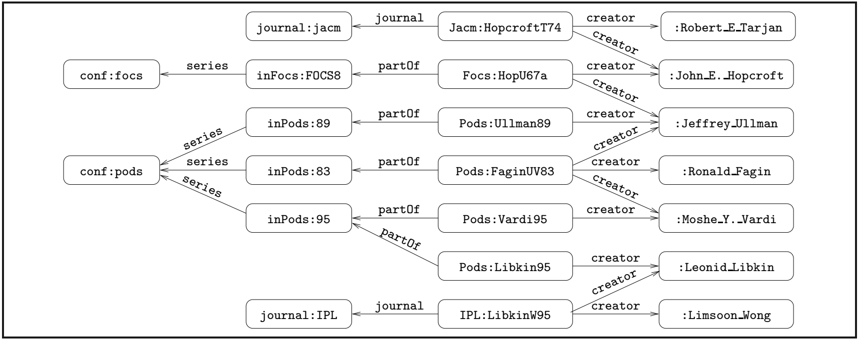
- Pairs of nodes in $\mathcal{G}$ that satisfy RPQ $L(\mathcal{G}^{\pm})$, where:
  - $\mathcal{G}^{\pm}$ obtained from $\mathcal{G}$ by adding $u \xrightarrow{a^{-}} v$ for each $v \xrightarrow{a} u \in E$.

# Example of 2RPQ

The 2RPQ

$$\left(\texttt{creator}^{-} \cdot \left(\left(\texttt{partOf} \cdot \texttt{series}\right) \cup \texttt{journal}\right)\right)$$

computes $(a, v)$ s.t. author $a$ published in conference or journal $v$.



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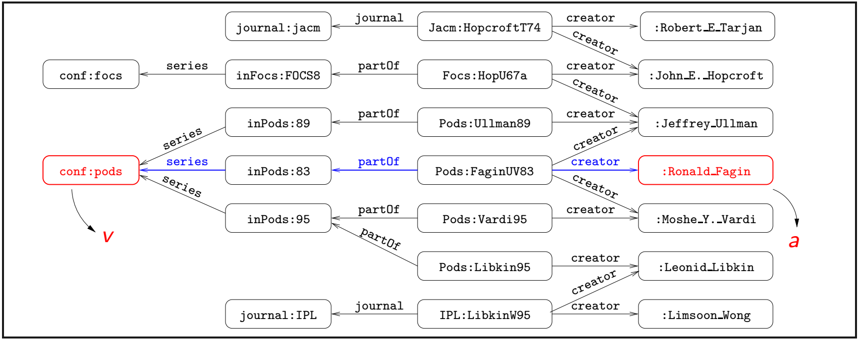

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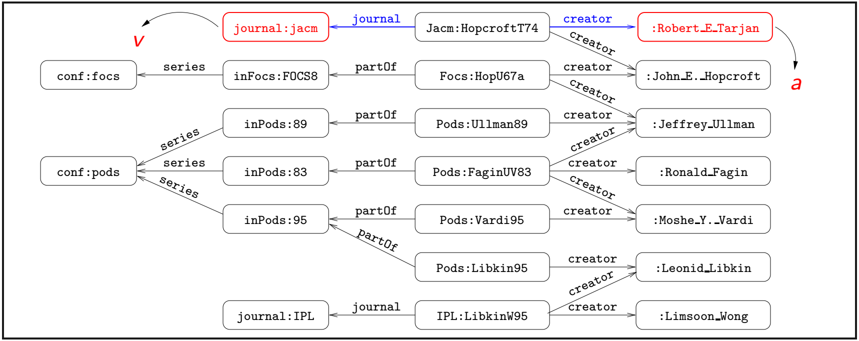

# 2RPQ evaluation

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| PROBLEM: | EVAL(2RPQ) |
| INPUT: | A graph DB $\mathcal{G}$, nodes $v, v'$ in $\mathcal{G}$, a 2RPQ $L$. |
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It boils down to:

| | |
|---|---|
| PROBLEM: | REGULARPATH |
| INPUT: | A graph DB $\mathcal{G}$, nodes $v, v'$ in $\mathcal{G}$, a regular expression $L$ over $\Sigma^{\pm}$. |
| QUESTION: | Is there a path $\rho$ from $v$ to $v'$ in $\mathcal{G}^{\pm}$ such that $\lambda(\rho) \in L$? |

# Complexity of finding regular paths

*Theorem*

$\textsc{RegularPath}$ *can be solved in time* $O(|\mathcal{G}| \cdot |L|)$.

Proof idea:

- Compute in linear time from $L$ an equivalent NFA $\mathcal{A}$.
- Compute in linear time $(\mathcal{G}^{\pm}, v, v')$ : NFA obtained from $\mathcal{G}^{\pm}$ by setting $v$ and $v'$ as initial and final states, respectively.
- Then $(v, v') \in L(\mathcal{G})$ iff $\mathcal{L}(\mathcal{G}^{\pm}, v, v') \cap \mathcal{L}(\mathcal{A}) \neq \emptyset$.
- For this need to solve the nonemptiness problem for the NFA $(\mathcal{G}^{\pm}, v, v') \times \mathcal{A}$.
- This can be done time $O(|\mathcal{G}^{\pm}| \cdot |\mathcal{A}|) = O(|\mathcal{G}| \cdot |L|)$.

# Complexity of 2RPQ evaluation

2RPQs can be evaluated in linear time:

**Corollary**

$\textsc{Eval}$(2RPQ) *can be solved in linear time* $O(|\mathcal{G}| \cdot |L|)$.

# Data complexity of 2RPQ evaluation

Data complexity of 2RPQs belongs to a parallelizable class:

**Proposition**

*Let L be a fixed 2RPQ.*
*There is* $\textsc{NLogspace}$ *procedure that computes* $L(\mathcal{G})$ *for each* $\mathcal{G}$.

Proof idea:

- Construct $(\mathcal{G}^{\pm}, v, v')$ from $\mathcal{G}$ in $\textsc{NLogspace}$.
- Check nonemptiness of $(\mathcal{G}^{\pm}, v, v') \times \mathcal{A}$ in $\textsc{NLogspace}$.

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This is the class of conjunctive regular path queries (CRPQs).

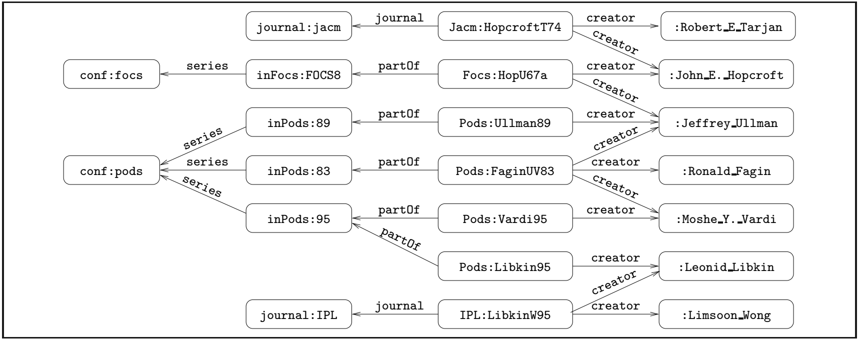
- Extended with inverses they are known as C2RPQs.

# Example of C2RPQ

The C2RPQ

$$Ans(x,u) \leftarrow (x, \texttt{creator}^{-}, y), (y, \texttt{partOf} \cdot \texttt{series}, z), (y, \texttt{creator}, u)$$

computes pairs $(a_1, a_2)$ that are coauthors of a conference paper.



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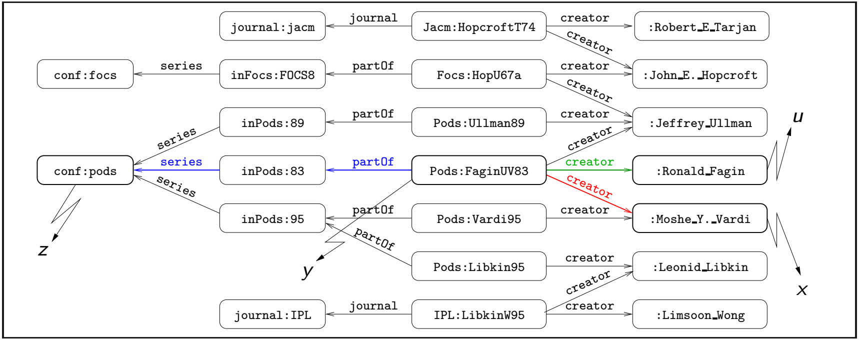

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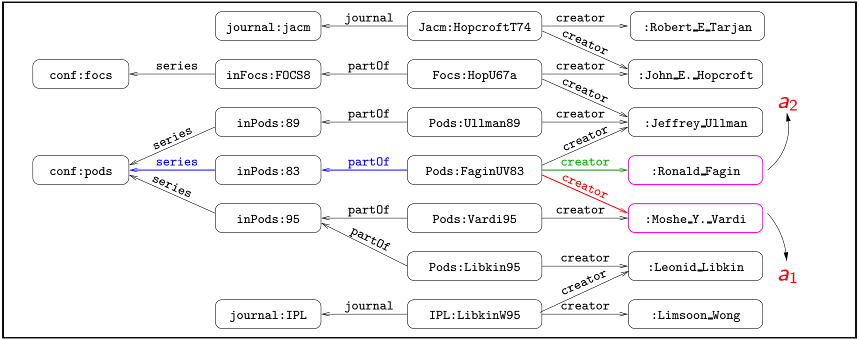

# C2RPQ: Formal definition

C2RPQ over $\Sigma$: Rule of the form:

$$Ans(\bar{z}) \leftarrow (x_1, L_1, y_1), \ldots, (x_m, L_m, y_m),$$

such that

- the $x_i, y_i$ are variables,
- each $L_i$ is a 2RPQ over $\Sigma$,
- the output $\bar{z}$ has some variables among the $x_i, y_i$.

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CRPQ: C2RPQ without inverse.

# Evaluation of C2RPQs

To evaluate C2RPQ $\varphi(\bar{z})$ of the form

$$Ans(\bar{z}) \leftarrow (x_1, L_1, y_1), \ldots, (x_m, L_m, y_m),$$

simply evaluate the conjunctive query

$$Ans(\bar{z}) \leftarrow L_1(x_1, y_1), \ldots, L_m(x_m, y_m),$$

where each $L_i(x_i, y_i)$ is the result of evaluating the 2RPQ $L_i$.

Can also see it as

$$\pi_{\bar{z}}(L_1 \bowtie \ldots \bowtie L_m)$$

Will write $\varphi(\mathcal{G})$.

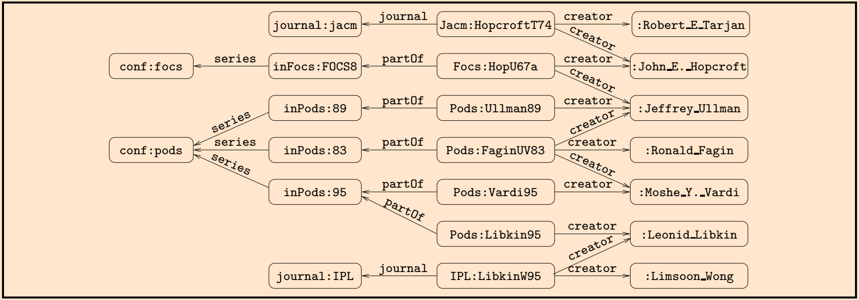
# C2RPQs vs 2RPQs

*Proposition*

*The C2RPQ*

$$Ans(x, u) \leftarrow (x, \texttt{creator}^-, y), (y, \texttt{partOf} \cdot \texttt{series}, z), (y, \texttt{creator}, u)$$

*is not* *expressible* *as a 2RPQ L over the graph database:*




Conclusion: Binary C2RPQs are strictly more expressive than 2RPQs.

# Complexity of evaluation of C2RPQS

Increase in expressiveness has a cost in evaluation.

**Proposition**

$\textsc{Eval}(\text{C2RPQ})$ *is* $\text{NP}$*-complete, even if restricted to CRPQs.*

- Upper bound by translation to evaluation of CQs.
- Lower bound holds since CRPQs contain CQs over graphs.

# Data complexity of evaluation of (U)C2RPQS

But adding conjunctions is free in data complexity.

*Proposition*

EVAL(C2RPQ) *can be solved in* NLOGSPACE *in data complexity.*

# Summary of basic query languages for graph DBs

- 2RPQs can be evaluated in linear time.
- 2RPQ evaluation is in NLogspace in data complexity.
- For C2RPQs:
  - Retain good data complexity of 2RPQs.
  - Combined complexity is intractable.
- C2RPQs do not exhaust the NLogspace properties.

# Complexity of C2RPQs revisited

C2RPQs can be evaluated in polynomial time in data complexity, but is this a good measure for massive datasets?

CRPQ evaluation is of the order $|\mathcal{G}|^{O(|Q|)}$, which is impractical if $\mathcal{G}$ is very big even for small $Q$.

Idea: Look for languages that are tractable in combined complexity or, at least, fixed-parameter tractable (fpt).

- $\mathcal{L}$ is fpt if there is computable function $f : \mathbb{N} \to \mathbb{N}$ and constant $c \geq 0$ such that $\mathcal{L}$-queries can be evaluated in time $O(|\mathcal{G}|^c \cdot f(|\varphi|))$.

The landscape so far:

- 2RPQs are tractable in combined complexity ($O(|\mathcal{G}| \cdot |L|)$).
- CRPQs are intractable in combined complexity.
  CRPQs are not fpt (even CQs are not).

# Structural restrictions of C2RPQs

Recall:

- Relational CQs are neither tractable in combined complexity nor fpt.
- Tractable cases of CQ evaluation can be obtained by restricting the syntactic shape of CQs.
- The most common such restriction is acyclicity.
  - An acyclic CQ $Q$ can be evaluated in linear time $O(|\mathcal{D}| \cdot |Q|)$ over relational DB $\mathcal{D}$ (Yannakakis (1981)).
- Other restrictions include bounded (hyper-)treewidth.

# Acyclic C2RPQs

A UC2RPQ is acyclic if its underlying CQ is acyclic.

A different way of stating this:

A C2RPQ $Q$ is acyclic iff its underlying simple and undirected graph $\mathcal{U}(Q)$ is acyclic, where $\mathcal{U}(Q) = (V, E)$ for:

- $V = \{x_1, y_1, \ldots, x_m, y_m\}$;
- $E = \{\{x_i, y_i\} \mid 1 \leq i \leq m \text{ and } x_i \neq y_i\}$.

Remark: Acyclicity allows cycles of length $\leq 2$ in C2RPQs.

- The C2RPQ $Ans() \leftarrow (x, a, x), (x, b, y), (y, c, x)$ is acyclic.

# Acyclic C2RPQs: Examples

- The following C2RPQ is acyclic:

$$Ans(x, u) \leftarrow (x, \texttt{creator}^-, y), (y, \texttt{partOf} \cdot \texttt{series}, z), (y, \texttt{creator}, u).$$

- The following C2RPQ is not acyclic:

$$Ans() \leftarrow (x, L_1, y), (y, L_2, z), (z, L_3, x).$$

# Evaluation of acyclic C2RPQs

Evaluation of acyclic C2RPQs is tractable in combined complexity:

**Proposition**

*Evaluation of an acyclic C2RPQ $Q$ over a graph DB $G$ takes time $O(|G|^2 \cdot |Q|^2)$.*

# The simple path semantics

Simple paths: No node is repeated.

Simple paths semantics:

- Motivated by applications for which simple paths are more natural.
- Studied back in the late 1980s already.
- Revival due to application in early versions of SPARQL, a language for RDF.

# RPQs under simple paths semantics

- RPQ evaluation in this context = Finding regular simple paths:

| | |
|---|---|
| PROBLEM: | REGULARSIMPLEPATH |
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- REGULARSIMPLEPATH($L$): For fixed $L$.

# Complexity of finding regular simple paths

**Theorem**

*The problem* $\textsc{RegularSimplePath}$ *is in* $\mathrm{NP}$, *and for some* $L$ *the problem* $\textsc{RegularSimplePath}(L)$ *can be* $\mathrm{NP}$*-complete.*

- $\textsc{RegularSimplePath}((00)^*)$:
- Is there simple directed path of even length? It is NP-complete.
- Query evaluation is $\mathrm{NP}$-complete in data complexity – hence impractical.

# Static analysis: Containment for 2RPQs

$\textsc{Cont}(\mathcal{L})$: Given $\mathcal{L}$-queries $Q_1$ and $Q_2$,

- is $Q_1(\mathcal{G}) \subseteq Q_2(\mathcal{G})$ for each graph DB $\mathcal{G}$?

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Containment for 2RPQs is decidable:

**Theorem**

$\textsc{Cont}$(2RPQ) *is* $\textsc{Pspace}$*-complete. It is* $\textsc{Pspace}$*-hard even for RPQs.*

- For RPQs easy to prove:
  - $L_1(\mathcal{G}) \subseteq L_2(\mathcal{G})$ for each $\mathcal{G}$ $\Longleftrightarrow$ regular expression $L_1$ contained in regular expression $L_2$.
  - Containment of regular expressions: $\textsc{Pspace}$-complete (Stock+1)Meyer (1971)).
- For 2RPQs more work is required: Reason with two-way automata.

# Containment for C2RPQs

Containment of C2RPQs still decidable with exponential blow-up:

**Theorem**

CONT(C2RPQ) *is* EXPSPACE*-complete, even for CRPQs.*

- Notice contrast with complexity of containment for CQs:
  - NP-complete (Chandra,Merlin (1977)).

# Summary of containment

- Containment of C2RPQs is decidable in double exponential time.
- For 2RPQs containment can be checked in single exponential time.
- High lower bounds are due to the presence of regular expressions.

# Path queries and comparisons

CRPQs fall short of expressive power for applications that need:

- to include paths in the output of a query, and
- to define complex relationships among labels of paths.

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Examples:

- Semantic Web queries:
  - establish semantic associations among paths.
- Biological applications:
  - compare paths based on similarity.
- Route-finding applications:
  - compare paths based on length or number of occurrences of labels.
- Data provenance and semantic search over the Web:
  - require returning paths to the user.

# Path comparisons

We use a set $\mathcal{S}$ of relations on words.

- Example: $\mathcal{S}$ may contain
  - Unary relations: Regular, context-free languages, etc.
  - Binary relations: prefix, equal length, subsequence, etc.
- Comparisons among labels of paths
  - Example: $w_1$ is a substring of $w_2$.
- We assume $\mathcal{S}$ contains all regular languages.

# Extended CRPQs

The $\mathcal{S}$-extended CRPQs (ECRPQ($\mathcal{S}$)) are rules obtained from a CRPQ:

$$Ans(\bar{z},\quad) \leftarrow (x_1, L_1, y_1), \ldots, (x_m, L_m, y_m),$$

- by annotating each pair $(x_i, y_i)$ with a path variable $\pi_i$,
- comparing labels of paths in $\bar{\pi}_j$ wrt $S_j \in \mathcal{S}$
  - for $\bar{\pi}_j$ a tuple of path variables among the $\pi_i$'s,
- projecting some of $\pi_i$'s as a tuple $\bar{\chi}$ in the output.

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# Extended CRPQs and our requirements

ECRPQs meet our requirements:

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# Evaluation of ECRPQs

Evaluation of the ECRPQ($\mathcal{S}$)

$$\theta(\bar{z},\bar{\chi}) : Ans(\bar{z},\bar{\chi}) \leftarrow (x_1,\pi_1,y_1),\ldots,(x_m,\pi_m,y_m),\bigwedge_j S_j(\bar{\pi}_j)$$

Same than for CRPQs but:

- Each $\pi_i$ is mapped to a path $\rho_i$ in the graph DB.
- For each $j$, if $\bar{\pi}_j = (\pi_{j_1},\ldots,\pi_{j_k})$ then: $\underbrace{(\lambda(\rho_{j_1}),\ldots,\lambda(\rho_{j_k}))}_{\text{the labels of } (\rho_{j_1},\ldots,\rho_{j_k})} \in S_j$.

# Evaluation of ECRPQs

Evaluation of the ECRPQ($\mathcal{S}$)

$$\theta(\bar{z}, \bar{\chi}) : Ans(\bar{z}, \bar{\chi}) \leftarrow (x_1, \pi_1, y_1), \ldots, (x_m, \pi_m, y_m), \bigwedge_j S_j(\bar{\pi}_j)$$

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# Considerations about ECRPQ($\mathcal{S}$)

- ECRPQ($\mathcal{S}$) extends the class of CRPQs.
  - $Ans(\bar{z}) \leftarrow \bigwedge_i (x_i, L_i, y_i)$ same as $Ans(\bar{z}) \leftarrow \bigwedge_i (x_i, \pi_i, y_i), L_i(\pi_i)$.
- Expressiveness and complexity of ECRPQ($\mathcal{S}$):
  - Depends on the class $\mathcal{S}$.
- We study two such classes with roots in formal language theory:
  - Regular relations (Elgot, Mezei (1965)).
  - Rational relations (Nivat (1968)).

# Comparing paths with regular relations

- Regular relations: Regular languages for relations of any arity.
  - REG: Class of regular relations.
- Bottomline:

  ECRPQ(REG): Reasonable expressiveness and complexity.

# Regular relations

$n$-ary regular relation:

Set of $n$-tuples $(w_1, \ldots, w_n)$ of strings
accepted by synchronous automaton over $\Sigma^n$.

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Set of $n$-tuples $(w_1, \ldots, w_n)$ of strings
accepted by synchronous automaton over $\Sigma^n$.

- The input strings are written in the $n$-tapes.
- Shorter strings are padded with symbol $\perp$.
- At each step:
  The automaton simultaneously reads next symbol on each tape.

# Synchronous automata

$$
\begin{array}{ccccccccc}
w_1 & = & \mathsf{a} & \mathsf{a} & \mathsf{b} & \cdots & \mathsf{a} & \mathsf{b} & \mathsf{c} \\
w_2 & = & \mathsf{a} & \mathsf{b} & \mathsf{a} & \cdots & \mathsf{a} & & \\
w_3 & = & \mathsf{b} & \mathsf{b} & & \cdots & & & \\
\vdots & & & & & \vdots & & & \\
w_n & = & \mathsf{a} & \mathsf{b} & \mathsf{b} & \cdots & \mathsf{a} & \mathsf{c} &
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w_3 & = & \mathsf{b} & \mathsf{b} & \perp & \cdots & \perp & \perp & \perp \\
\vdots & & & & & \vdots & & & \\
w_n & = & \mathsf{a} & \mathsf{b} & \mathsf{b} & \cdots & \mathsf{a} & \mathsf{c} & \perp
\end{array}$$

# Synchronous automata

$$
\begin{array}{lcccccccc}
w_1 & = & \color{red}{\mathsf{a}} & \mathsf{a} & \mathsf{b} & \cdots & \mathsf{a} & \mathsf{b} & \mathsf{c} \\
w_2 & = & \color{red}{\mathsf{a}} & \mathsf{b} & \mathsf{a} & \cdots & \mathsf{a} & \perp & \perp \\
w_3 & = & \color{red}{\mathsf{b}} & \mathsf{b} & \perp & \cdots & \perp & \perp & \perp \\
\vdots & & & & & \vdots & & & \\
w_n & = & \color{red}{\mathsf{a}} & \mathsf{b} & \mathsf{b} & \cdots & \mathsf{a} & \mathsf{c} & \perp \\
 & & \color{red}{\Uparrow} & & & & & &
\end{array}
$$

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\begin{array}{lcccccccc}
w_1 & = & \mathrm{a} & \mathrm{a} & \mathrm{b} & \cdots & \mathrm{a} & \mathrm{b} & \mathrm{c} \\
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# Examples of regular relations

- All regular languages.
- The prefix relation defined by:

$$\Big(\bigcup_{a\in\Sigma}(a,a)\Big)^* \cdot \Big(\bigcup_{a\in\Sigma}(a,\perp)\Big)^*.$$

- The equal length relation defined by:

$$\Big(\bigcup_{a,b\in\Sigma}(a,b)\Big)^*.$$

- Pairs of strings at edit distance at most $k$, for fixed $k \geq 0$.

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*Proposition*

*The subsequence, subword and suffix relations are not regular.*

# ECRPQ(REG)

ECRPQ(REG): Class of queries of the form

$$Ans(\bar{z}, \bar{\chi}) \leftarrow \bigwedge_i (x_i, \pi_i, y_i), \bigwedge_j S_j(\bar{\pi}_j),$$

where each $S_j$ is a regular relation

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Example: The ECRPQ(REG) query

$$Ans(x, y) \leftarrow (x, \pi_1, z), (z, \pi_2, y), a^*(\pi_1), b^*(\pi_2), \mathrm{equal\_length}(\pi_1, \pi_2)$$

computes pairs of nodes linked by a path labeled in $\{a^n b^n \mid n \geq 0\}$.

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*Corollary*

ECRPQ(REG) *properly extends the class of* CRPQs.

# Complexity of evaluation of ECRPQ(REG)

• Extending CRPQs with regular relations is free for data complexity.
• Combined complexity is that of relational calculus over relational databases.

*Theorem*

- EVAL(ECPRQ(REG)) *is* PSPACE-*complete.*
- EVAL(ECPRQ(REG)) *is in* NLOGSPACE *in data complexity.*

# Containment for ECRPQ(REG)

*Theorem*

$\textsc{Cont}$(ECRPQ(REG)) *is undecidable.*

- Notice contrast with CRPQs for which containment is decidable.
- But this is like for full relational algebra/calculus.

# Comparing with rational relations

ECRPQ(REG) queries are still short of expressive power:

- RDF or biological networks:
  - Compare strings based on subsequence and subword relations.
- These relations are rational: Accepted by asynchronous automata.
  - RAT: Class of rational relations.

Bottomline:

- ECRPQ(RAT) evaluation:
  - Undecidable or very high complexity.
- Restricting the syntactic shape of queries yields tractability.

# Rational relations

*$n$-ary rational relation:*
Set of $n$-tuples $(w_1, \ldots, w_n)$ of strings
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$n$-ary rational relation:
Described by regular expression over alphabet $(\Sigma \cup \{\epsilon\})^n$.

# Examples of rational relations

- All regular relations.
- The subsequence relation $\preceq_{\mathrm{ss}}$ defined by:

$$\left(\Big(\bigcup_{a\in\Sigma}(a,\epsilon)\Big)^* \bigcup_{b\in\Sigma}(b,b)\right)^* \cdot \Big(\bigcup_{a\in\Sigma}(a,\epsilon)\Big)^*.$$

- The subword relation $\preceq_{\mathrm{sw}}$ defined by:

$$\Big(\bigcup_{a\in\Sigma}(a,\epsilon)\Big)^* \cdot \Big(\bigcup_{b\in\Sigma}(b,b)\Big)^* \cdot \Big(\bigcup_{a\in\Sigma}(a,\epsilon)\Big)^*.$$

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*Proposition*

*The set of pairs $(w_1, w_2)$ such that $w_1$ is the reversal of $w_2$ is not rational.*

# ECRPQ(RAT)

ECRPQ(RAT): Class of queries of the form

$$Ans(\bar{z}, \bar{\chi}) \leftarrow \bigwedge_i (x_i, \pi_i, y_i), \bigwedge_j S_j(\bar{\pi}_j),$$

where each $S_j$ is a rational relation

Example: The ECRPQ(RAT) query

$$Ans(x, y) \leftarrow (x, \pi_1, z), (y, \pi_2, w), \pi_1 \preceq_{\mathrm{ss}} \pi_2$$

computes $x, y$ that are origins of paths $\rho_1$ and $\rho_2$ such that:

- $\lambda(\rho_1)$ is a subsequence of $\lambda(\rho_2)$.

# Evaluation of ECRPQ(RAT) queries

Evaluation of queries in ECRPQ(RAT) is undecidable, but:

- True if we allow only practically motivated rational relations?
  - For example, $\preceq_{\mathrm{ss}}$ and $\preceq_{\mathrm{sw}}$.

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Adding subword relation to ECRPQ(REG) leads to undecidability:

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*Evaluation of* $(\mathrm{ECRPQ}(\mathrm{REG} \cup \{\preceq_{\mathrm{sw}}\}))$ *queries is undecidable. The same is true for suffix in place of subword.*

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**Theorem**

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Adding subsequence preserves decidability, but at a very high cost:

**Theorem**

*Evaluation of* $(\mathrm{ECRPQ}(\mathrm{REG} \cup \{\preceq_{\mathrm{ss}}\}))$ *queries is decidable, but non-primitive-recursive.*

Primitive-recursive, informally: any function you can think of!

# Acyclic ECRPQ(RAT) queries

Acyclic ECRPQ(RAT) queries yield tractable data complexity.

- Queries of the form:

$$Ans(\bar{z}) \leftarrow \bigwedge_{i \leq k} (x_i, \pi_i, y_i), L_i(\pi_i), \bigwedge_j S_j(\pi_{j_1}, \pi_{j_2}),$$

  where the graph on $\{1, \ldots, k\}$ defined by edges $(\pi_{j_1}, \pi_{j_2})$ is acyclic.

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Acyclic ECRPQ(RAT) is not more expensive than ECRPQ(REG):

*Theorem*

- *Evaluation of acyclic* ECRPQ(RAT) *queries is* $\textsc{Pspace}$*-complete.*
- *It is in* $\textsc{NLogspace}$ *in data complexity.*

# Summary of path queries

- Usual query languages do not allow:
  - to export paths and compare labels of paths.
- This has led to the introduction of ECRPQ($\mathcal{S}$) queries:
  - They output paths and compare labels of paths with relations in $\mathcal{S}$.
- Comparing paths with regular relations:
  - Preserves tractable data complexity of evaluation.
  - Leads to undecidability of containment.
- Comparing paths with practically motivated rational relations:
  - Leads to undecidability or high complexity of evaluation.
  - Tractable cases found restricting the syntactic shape of queries.

# Querying graphs with data

So far queries only talk about the topology of the data.

Queries that combine topology and data are important in practice:

- Example:
  People of the same age connected by professional links.

We present a language that expresses topological properties of the data:

- It requires an extension of the data model (data graphs).
- It talks about data paths:
  Summarize the topology and the underlying data of a path.

# Data graphs and data paths

We work with data graphs and paths over set of data values $\mathcal{D}$.

**Definition**

A data graph $\mathfrak{G}$ over $\Sigma$ is a tuple $(V, E, \delta)$, where:

- $(V, E)$ is a graph database over $\Sigma$, and
- $\delta$ is a mapping that assigns a value in $\mathcal{D}$ to each node $v \in V$.

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With each path $\rho = v_1 \xrightarrow{a_1} v_2 \cdots v_k \xrightarrow{a_k} v_{k+1}$ in $(V, E)$:
We associate a data path in $\mathfrak{G}$ of the form

$$\rho_{\mathcal{D}} \;=\; \delta(v_1) \xrightarrow{a_1} \delta(v_2) \cdot\cdot\cdot \delta(v_k) \xrightarrow{a_k} \delta(v_{k+1}),$$

that is obtained from $\rho$ by replacing each node by its data value.

# Data paths and data words

Data paths are very close to data words:

- Object studied in XML and verification (Bojanczyk et al. (2006)).
- Data words are strings over $\Sigma \times \mathcal{D}$.

Mechanisms that query data words can be used for data paths:

- FO, MSO, and some versions of XPath (Bojanczyk et al. (2006)).
- Pebble automata (Neven, Schwentick, Vianu (2004)).
- Register automata (Kaminski, Francez (1994)).

# The choice of a formalism

Formalism for querying data paths has to be chosen with care:

*Theorem*

*The problem* DistinctValues *is* NP*-complete:*

- DistinctValues*:*
  *Is there a path* $\rho$ *from* $v$ *to* $v'$ *s.t. no data value in* $\rho_{\mathcal{D}}$ *is repeated?*

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- $\textsc{DistinctValues}$*:*
  *Is there a path* $\rho$ *from* $v$ *to* $v'$ *s.t. no data value in* $\rho_{\mathcal{D}}$ *is repeated?*

Conclusion:

- If a language expresses $\textsc{DistinctValues}$:
  - It is $\mathrm{NP}$-hard in data complexity $\Rightarrow$ Impractical.
- Rules out all formalisms except for one:
  - Register automata.

# Regular expressions for register automata

Regular expressions with memory (REMs):
Same as register automata

# Regular expressions for register automata

Regular expressions with memory (REMs):

Same as register automata

- REMs permit to specify when data values are remembered and used.
- Data values are remembered in $k$ registers $\{x_1, \ldots, x_k\}$.
- At any point we can compare a data value with one in the registers.

# REM: Example

Consider the REM $\downarrow x.a^+[x^=]$.

Intuition:

- Store the current data value $d$ in register $x$.
- After reading a word in $a^+$ check that $d$ is seen again.

Semantics: Pairs $(v, v')$ of nodes:

- Linked by a path labeled in $a^+$.
- $v$ and $v'$ contain the same data value.

# REM: Conditions

- Conditions: Compare a data value with the ones in the registers.
- Conditions over $\{x_1, \ldots, x_k\}$ are given by the grammar:

$$c \ := \ x_i^{=} \ \mid \neg c \mid c \wedge c \qquad (1 \leq i \leq k)$$

- We define $(d, \tau) \models c$ for $d \in \mathcal{D}$ and $\tau = (d_1, \ldots, d_k) \in \mathcal{D}^k$:
  - $(d, \tau) \models x_i^{=}$ iff $d = d_i$.
  - Boolean combinations are standard.

# REMs: Syntax and semantics (Intuition)

REMs over $\Sigma$ and $\{x_1, \ldots, x_k\}$ are defined by grammar:

$$e \ := \ \varepsilon \ | \ a \ | \ e \cup e \ | \ e \cdot e \ | \ e^+ \ | \ e[c] \ | \downarrow \bar{x}.e$$

where $a \in \Sigma$, $c$ condition, and $\bar{x}$ tuple in $\{x_1, \ldots, x_k\}$.

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Intuition: Evaluation of REM $e$ on data graph $\mathfrak{G}$ is:

- pairs $(v, v')$ of nodes linked by path $\rho$ such that $\rho_{\mathcal{D}} \models e$, where:

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  - $\rho_{\mathcal{D}} \models e[c]$ if and only if

$\rho_{\mathcal{D}}$

$$\underbrace{\kappa(v_1) \xrightarrow{a_1} \kappa(v_2) \cdots \kappa(v_k) \xrightarrow{a_k} \kappa(v_{k+1})}_{\text{can be parsed wrt } e}$$

starting from empty registers

finishing in register value $\tau \in \mathcal{D}^k$ st $(\kappa(v_{k+1}), \tau) \models c$

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starting from the register value that assigns $\kappa(v_1)$ to each $x \in \bar{x}$

# REM: Example

Consider the REM $\Sigma^* \cdot (\downarrow x.\Sigma^+[x^=]) \cdot \Sigma^*$:

- Defines pairs of nodes linked by path $\rho$ such that:
  - $\rho_{\mathcal{D}}$ contains the same data value twice.
- The complement of this language is $\textsc{DistinctValues}$.

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- Defines pairs of nodes linked by path $\rho$ such that:
  - $\rho_{\mathcal{D}}$ contains the same data value twice.
- The complement of this language is DistinctValues.

**Corollary**

*REMs are not closed under complement.*

# Complexity of REM evaluation

- Data complexity of REM evaluation coincides with that of CRPQs.
- Combined complexity same than for FO over relational databases.

*Theorem*

- $\textsc{Eval}$(REM) *is* $\textsc{Pspace}$*-complete.*
- *It is in* $\textsc{NLogspace}$ *in data complexity.*

- Both bounds extend to the class of conjunctive REMs.

# Summary of queries on graphs with data

- Most query languages for graph DBs:
  - talk about topology, but not about underlying data.
- Query languages that combine topology and data:
  - talk about data paths in data graphs.
- Choosing a formalism to query data paths must be done with care:
  - intractability can be reached easily.
- To query data paths:
  - Can use REMs, which are based on register automata.
  - REMs can be evaluated efficiently in data complexity.

# Comments on papers


- Isabel F. Cruz, Alberto O. Mendelzon, Peter T. Wood: A Graphical Query Language Supporting Recursion. SIGMOD Conference 1987: 323-330
- Mariano P. Consens, Alberto O. Mendelzon: Low Complexity Aggregation in GraphLog and Datalog. Theor. Comput. Sci. 116(1): 95-116 (1993)

  Original papers introducing (C)RPQs
- Pablo Barcelo: Querying graph databases. PODS 2013: 175-188
- Renzo Angles, Claudio Gutiérrez: Survey of graph database models. ACM Comput. Surv. 40(1) (2008)
- Peter T. Wood: Query languages for graph databases. SIGMOD Record 41(1): 50-60 (2012)

  Three suveys of graph languages, two are more theoretical, one more practical.
- Diego Calvanese, Giuseppe De Giacomo, Maurizio Lenzerini, Moshe Y. Vardi: Rewriting of Regular Expressions and Regular Path Queries. J. Comput. Syst. Sci. 64(3): 443-465 (2002)

  Introducing two-way queries.

# Comments on papers

- Diego Calvanese, Giuseppe De Giacomo, Maurizio Lenzerini, Moshe Y. Vardi: Reasoning on regular path queries. SIGMOD Record 32(4): 83-92 (2003)
- Diego Calvanese, Giuseppe De Giacomo, Maurizio Lenzerini, Moshe Y. Vardi: Containment of Conjunctive Regular Path Queries with Inverse. KR 2000: 176-185

  Static analysis of regular path queries.
- Leonid Libkin, Wim Martens, Domagoj Vrgoc: Querying graph databases with XPath. ICDT 2013: 129-140

  Adding data values to (C)RPQs
- Pablo Barcelo, Leonid Libkin, Anthony Widjaja Lin, Peter T. Wood: Expressive Languages for Path Queries over Graph-Structured Data. ACM Trans. Database Syst. 37(4): 31 (2012)

  Extending RPQs with regular relations; topics to concentrate on are those not covered in class.
- Pablo Barcelo, Diego Figueira, Leonid Libkin: Graph Logics with Rational Relations .Logical Methods in Computer Science 9(3) (2013)

  Likewise for rational relations.

# Comments on papers

- Dominik D. Freydenberger, Nicole Schweikardt: Expressiveness and Static Analysis of Extended Conjunctive Regular Path Queries. AMW 2011

  Resolving some of the questions on the containment of path queries.
- Jelle Hellings, Bart Kuijpers, Jan Van den Bussche, Xiaowang Zhang: Walk logic as a framework for path query languages on graph databases. ICDT 2013: 117-128

  A different approach to expanding the power of path languages.
- Pablo Barcelo, Leonid Libkin, Juan L. Reutter: Querying Regular Graph Patterns. Journal of the ACM 61(1): 8:1-8:54 (2014)

  Incomplete information in graph databases and querying it.
- Wenfei Fan, Xin Wang, Yinghui Wu: Querying big graphs within bounded resources. SIGMOD Conference 2014: 301-312
- Wenfei Fan: Graph pattern matching revised for social network analysis. ICDT 2012: 8-21

  Two papers on making graph queries scalable.