Graph Data Management
Why graphs?
The world is a graph – everything is connected

- people, places, events
- companies, markets
- countries, history, politics
- life sciences, bioinformatics, clinical data
- art, teaching
- technology, networks, machines, applications, users
- software, code, dependencies, architecture, deployments
- criminals, fraudsters and their behavior
The topology of the data is at least as important as the data itself.
Humans think in graphs

• We understand and learn by
  • how something new is similar to what we already know
  • how it differs
  • i.e. by relating things
    • in a graph!
What are people using graph databases for?
Use Cases

Internal Applications
- Master Data Management
- Network and IT Operations
- Fraud Detection

Customer-Facing Applications
- Real-Time Recommendations
- Graph-Based Search
- Identity and Access Management
Social Network
Impact Analysis
Logistics & Routing
Recommendations
Access Control
Fraud Analysis
Querying Graph Databases
Graph DBs and applications

- Graph DBs are crucial when topology is as important as data itself.
- Renewed interest due to new applications:
  - Semantic Web and RDF.
  - Social networks.
  - Security and crime detection.
  - Knowledge representation.
  - etc etc
  - ...


Querying graph DBs and relational technology

Why not to use relational technology?

- Translate graph DB $G \rightarrow$ relational database $D(G)$, and query $D(G)$.

Problems:

1. Languages for graph DBs are navigational and require recursion.
2. They can be translated into Datalog, but there are problems:
   (a) Implementation:
   - SQL’s recursion is hard to optimize, especially in complex queries, on large databases.
   (b) Complexity mismatch:
   - Datalog evaluation is $\text{P}_{\text{TIME}}$-complete, but in $\text{NLOGSPACE}$ for many graph languages.
   - Basic static analysis tasks undecidable for Datalog, but decidable for several graph languages.
Early graph query languages

Graph query languages flourished from the mid 80s to the late 90s:

- \( \textbf{G}, \textbf{G}^+, \) and GraphLog for hypertext and semistructured data, late 1980s
- GOOD for graph-based models of object DBs, 1990
- Hyperlog for hypergraphs, 1994
- Languages for heterogeneous and unstructured data, Lorel, StruQL, etc (late 1990s)
Features of graph query languages

- **Navigation**: Recursively traverse the edges of the graph.
- **Pattern matching**: Check if a pattern appears in the graph DB.

And more sophisticated features:
- **Path comparisons**.
- **Comparisons of the underlying data**.
Key problems theory studies:

Expressiveness: What can be said in a query language $\mathcal{L}$?

Complexity of evaluation:

<table>
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<th>PROBLEM:</th>
<th>EVAL($\mathcal{L}$)</th>
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<td>A graph DB $\mathcal{G}$, a tuple $\bar{t}$ of objects, an $\mathcal{L}$-query $Q$.</td>
</tr>
<tr>
<td>QUESTION:</td>
<td>Is $\bar{t} \in Q(\mathcal{G})$?</td>
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- **Combined complexity:** Both $\mathcal{G}$ and $Q$ are part of the input.
- **Data complexity:** Only $\mathcal{G}$ is part of the input and $Q$ is fixed.

Containment: We study the problem CONT($\mathcal{L}$):

- Given $\mathcal{L}$-queries $Q_1$, $Q_2$, is $Q_1(\mathcal{G}) \subseteq Q_2(\mathcal{G})$ for every graph DB $\mathcal{G}$?
Graph data model

Different applications have given rise to a many (slightly) different graph DB models. But the essence is the same:

Finite, directed, edge labeled graphs.

Despite the simplicity of the model:

- It is flexible enough to accommodate many other more complex models and express interesting practical scenarios.
- The most fundamental theoretical issues related to querying graph DBs appear in it already.
Graph databases

Definition

A graph DB $G$ over finite alphabet $\Sigma$ is a pair:

$$(V, E)$$

- finite set of node ids
- set of edges of the form $v_1 \xrightarrow{a} v_2$ ($v_1, v_2 \in V$, $a \in \Sigma$)
Graph databases

Definition

A graph DB $\mathcal{G}$ over finite alphabet $\Sigma$ is a pair:

$$(V, E)$$

finite set of node ids

set of edges of the form $v_1 \xrightarrow{a} v_2$

$(v_1, v_2 \in V, a \in \Sigma)$

• A path in $\mathcal{G}$ is a sequence of the form:

$$\rho = v_1 \xrightarrow{a_1} v_2 \xrightarrow{a_2} v_3 \cdots v_k \xrightarrow{a_k} v_{k+1}.$$  

• The label of $\rho$ is $\lambda(\rho) = a_1 a_2 \cdots a_{k-1} \in \Sigma^*$.  

Graph DBs: Example

A graph DB representation of a fragment of DBLP:
Graph DBs: Example

A path in this graph DB:
Graph DBs: Example

The label of such path:

```
conf:focs → journal:jacm → Jacm:HopcroftT74 → creator:Robert_E._Tarjan

conf:pods → inPods:89 → partOf:Focs:HopU67a → creator:John_E._Hopcroft


conf:pods → inPods:95 → partOf:Pods:Vardi95 → creator:Moshe_Y._Vardi

conf:pods → inPods:95 → partOf:Pods:Libkin95 → creator:Leonid_Libkin

conf:pods → inPods:95 → partOf:IPL:LibkinW95 → creator:Limsoon_Wong
```
Important: Graph DBs can be naturally seen as NFAs. Recall: NFA = Nondeterministic finite automaton.

- Nodes are states.
- Edges $u \xrightarrow{a} v$ are transitions.
- There are no initial and final states.
Regular path queries

Basic building block for graph queries: Regular path queries (RPQs).

- First studied in 1989.
- An RPQ is a Regular expressions over $\Sigma$.
- Evaluation $L(G)$ of RPQ $L$ on graph DB $G = (V, E)$:
  - Pairs of nodes $(v, v') \in V$ linked by path labeled in $L$. 
RPQs with inverse

More often studied its extension with inverses, or 2RPQs.

- First studied in 2000.
- $2\text{RPQs} = \text{RPQs over } \Sigma^\pm$, where:
  - $\Sigma^\pm = \Sigma$ extended with the inverse $a^-$ of each $a \in \Sigma$. 
RPQs with inverse

More often studied its extension with inverses, or 2RPQs.

- First studied in 2000.
- 2RPQs = RPQs over $\Sigma^{\pm}$, where:
  - $\Sigma^{\pm} = \Sigma$ extended with the inverse $a^{-}$ of each $a \in \Sigma$.

Evaluation $L(\mathcal{G})$ of 2RPQ $L$ over graph DB $\mathcal{G} = (V, E)$:

- Pairs of nodes in $\mathcal{G}$ that satisfy RPQ $L(\mathcal{G}^{\pm})$, where:
  - $\mathcal{G}^{\pm}$ obtained from $\mathcal{G}$ by adding $u \xrightarrow{a^{-}} v$ for each $v \xrightarrow{a} u \in E$. 
Example of 2RPQ

The 2RPQ

\[
(creator^- \cdot ((partOf \cdot series) \cup journal))
\]

computes \((a, v)\) s.t. author \(a\) published in conference or journal \(v\).
Example of 2RPQ

The 2RPQ

\[
(\text{creator}^\neg \cdot ((\text{partOf} \cdot \text{series}) \cup \text{journal}))
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Example: The 2RPQ

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\]

computes \((a, v)\) s.t. author \(a\) published in conference or journal \(v\).
**Problem:** \( \text{Eval}(2\text{RPQ}) \)

**Input:** A graph DB \( G \), nodes \( v, v' \) in \( G \), a 2RPQ \( L \).

**Question:** Is \( (v, v') \in L(G) \)?
2RPQ evaluation

**Problem:**  
**Eval**(2RPQ)

**Input:**  
A graph DB $G$, nodes $v, v'$ in $G$, a 2RPQ $L$.

**Question:** Is $(v, v') \in L(G)$?

It boils down to:

**Problem:** RegularPath

**Input:**  
A graph DB $G$, nodes $v, v'$ in $G$, a regular expression $L$ over $\Sigma^\pm$.

**Question:** Is there a path $\rho$ from $v$ to $v'$ in $G^\pm$ such that $\lambda(\rho) \in L$?
Complexity of finding regular paths

**Theorem**

**RegularPath** can be solved in time \(O(|G| \cdot |L|)\).

**Proof idea:**

- Compute in linear time from \(L\) an equivalent NFA \(A\).
- Compute in linear time \((G^\pm, \nu, \nu')\): NFA obtained from \(G^\pm\) by setting \(\nu\) and \(\nu'\) as initial and final states, respectively.
- Then \((\nu, \nu') \in L(G)\) iff \(L(G^\pm, \nu, \nu') \cap L(A) \neq \emptyset\).
- For this need to solve the nonemptiness problem for the NFA \((G^\pm, \nu, \nu') \times A\).
- This can be done time \(O(|G^\pm| \cdot |A|) = O(|G| \cdot |L|)\).
Complexity of 2RPQ evaluation

2RPQs can be evaluated in linear time:

**Corollary**

\[ \text{Eval}(2\text{RPQ}) \text{ can be solved in linear time } O(|G| \cdot |L|). \]
Data complexity of 2RPQ evaluation

Data complexity of 2RPQs belongs to a parallelizable class:

**Proposition**

Let $L$ be a fixed 2RPQ. There is \text{NLogspace} procedure that computes $L(G)$ for each $G$.

**Proof idea:**

- Construct $(G^\pm, \nu, \nu')$ from $G$ in \text{NLogspace}.
- Check nonemptiness of $(G^\pm, \nu, \nu') \times A$ in \text{NLogspace}.
Conjunctive regular path queries (CRPQs)

RPQs still do not express arbitrary patterns over graph DBs.
- To do this we need to close RPQs under joins and projection.
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RPQs still do not express arbitrary patterns over graph DBs.

- To do this we need to close RPQs under joins and projection.

This is the class of conjunctive regular path queries (CRPQs).

- Extended with inverses they are known as C2RPQs.
Example of C2RPQ

The C2RPQ

\[ \text{Ans}(x, u) \leftarrow (x, \text{creator}^-, y), (y, \text{partOf} \cdot \text{series}, z), (y, \text{creator}, u) \]

computes pairs \((a_1, a_2)\) that are coauthors of a conference paper.
Example of C2RPQ

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C2RPQ: Formal definition

C2RPQ over $\Sigma$: Rule of the form:

$$\text{Ans}(\bar{z}) \leftarrow (x_1, L_1, y_1), \ldots , (x_m, L_m, y_m),$$

such that

- $x_i, y_i$ are variables,
- each $L_i$ is a 2RPQ over $\Sigma$,
- the output $\bar{z}$ has some variables among the $x_i, y_i$. 
C2RPQ: Formal definition

**C2RPQ over Σ:** Rule of the form:

\[
\text{Ans}(\bar{z}) \leftarrow (x_1, L_1, y_1), \ldots, (x_m, L_m, y_m),
\]

such that

- the \( x_i, y_i \) are variables,
- each \( L_i \) is a 2RPQ over Σ,
- the output \( \bar{z} \) has some variables among the \( x_i, y_i \).

**CRPQ:** C2RPQ without inverse.
Evaluation of C2RPQs

To evaluate C2RPQ \( \varphi(\bar{z}) \) of the form

\[
\text{Ans}(\bar{z}) \leftarrow (x_1, L_1, y_1), \ldots, (x_m, L_m, y_m),
\]

simply evaluate the conjunctive query

\[
\text{Ans}(\bar{z}) \leftarrow L_1(x_1, y_1), \ldots, L_m(x_m, y_m),
\]

where each \( L_i(x_i, y_i) \) is the result of evaluating the 2RPQ \( L_i \).

Can also see it as

\[
\pi_{\bar{z}}(L_1 \Join \ldots \Join L_m)
\]

Will write \( \varphi(G) \).
Proposition

The C2RPQ

\[ \text{Ans}(x, u) \leftarrow (x, \text{creator}^{-}, y), (y, \text{partOf} \cdot \text{series}, z), (y, \text{creator}, u) \]

is not expressible as a 2RPQ \( L \) over the graph database:

Conclusion: Binary C2RPQs are strictly more expressive than 2RPQs.
Complexity of evaluation of C2RPQS

Increase in expressiveness has a cost in evaluation.

**Proposition**

\[ \text{Eval}(\text{C2RPQ}) \text{ is } \text{NP-complete, even if restricted to CRPQs}. \]

- Upper bound by translation to evaluation of CQs.
- Lower bound holds since CRPQs contain CQs over graphs.
Data complexity of evaluation of (U)C2RPQS

But adding conjunctions is free in data complexity.

**Proposition**

\textsc{Eval}(C2RPQ) can be solved in \textsc{NLogspace} in data complexity.
Summary of basic query languages for graph DBs

- 2RPQs can be evaluated in linear time.
- 2RPQ evaluation is in $\text{NLOGSPACE}$ in data complexity.
- For C2RPQs:
  - Retain good data complexity of 2RPQs.
  - Combined complexity is intractable.
- C2RPQs do not exhaust the $\text{NLOGSPACE}$ properties.
Complexity of C2RPQs revisited

C2RPQs can be evaluated in polynomial time in data complexity, but is this a good measure for massive datasets?

CRPQ evaluation is of the order $|G|^O(|Q|)$, which is impractical if $G$ is very big even for small $Q$.

Idea: Look for languages that are tractable in combined complexity or, at least, fixed-parameter tractable (fpt).

$L$ is fpt if there is computable function $f : \mathbb{N} \rightarrow \mathbb{N}$ and constant $c \geq 0$ such that $L$-queries can be evaluated in time $O(|G|^c \cdot f(|\varphi|))$.

The landscape so far:

- 2RPQs are tractable in combined complexity ($O(|G| \cdot |L|)$).
- CRPQs are intractable in combined complexity.
  CRPQs are not fpt (even CQs are not).
Structural restrictions of C2RPQs

Recall:

- Relational CQs are neither tractable in combined complexity nor fpt.
- Tractable cases of CQ evaluation can be obtained by restricting the syntactic shape of CQs.
- The most common such restriction is acyclicity.
  - An acyclic CQ $Q$ can be evaluated in linear time $O(|D| \cdot |Q|)$ over relational DB $D$ (Yannakakis (1981)).
- Other restrictions include bounded (hyper-)treewidth.
Acyclic C2RPQs

A UC2RPQ is **acyclic** if its underlying CQ is acyclic.

A different way of stating this:

A C2RPQ $Q$ is acyclic iff its underlying simple and undirected graph $U(Q)$ is acyclic, where $U(Q) = (V, E)$ for:

- $V = \{x_1, y_1, \ldots, x_m, y_m\}$;
- $E = \{\{x_i, y_i\} \mid 1 \leq i \leq m \text{ and } x_i \neq y_i\}$.

**Remark:** Acyclicity allows cycles of length $\leq 2$ in C2RPQs.

- The C2RPQ $Ans() \leftarrow (x, a, x), (x, b, y), (y, c, x)$ is acyclic.
Acyclic C2RPQs: Examples

- The following C2RPQ is acyclic:
  
  $\text{Ans}(x, u) \leftarrow (x, \text{creator}^-, y), (y, \text{partOf} \cdot \text{series}, z), (y, \text{creator}, u)$.

- The following C2RPQ is not acyclic:

  $\text{Ans}() \leftarrow (x, L_1, y), (y, L_2, z), (z, L_3, x)$.
Evaluation of acyclic C2RPQs

Evaluation of acyclic C2RPQs is tractable in combined complexity:

**Proposition**

*Evaluation of an acyclic C2RPQ $Q$ over a graph DB $G$ takes time $O(|G|^2 \cdot |Q|^2)$.*
The simple path semantics

**Simple paths:** No node is repeated.

Simple paths semantics:

- Motivated by applications for which simple paths are more natural.
- Studied back in the late 1980s already.
- Revival due to application in early versions of SPARQL, a language for RDF.
RPQs under simple paths semantics

- RPQ evaluation in this context = Finding regular simple paths:

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<th><strong>RegularSimplePath</strong></th>
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RPQs under simple paths semantics

- RPQ evaluation in this context = Finding regular simple paths:

**Problem:** RegularSimplePath

**Input:** A graph database $G$, nodes $v, v'$ in $G$, a regular expression $L$.

**Question:** Is there a simple path $\rho$ from $v$ to $v'$ in $G$ such that $\lambda(\rho) \in L$?

- RegularSimplePath($L$): For fixed $L$. 

Complexity of finding regular simple paths

Theorem

The problem \texttt{RegularSimplePath} is in \textit{NP}, and for some \( L \) the problem \texttt{RegularSimplePath}(L) can be \textit{NP}-complete.

- \texttt{RegularSimplePath}((00)*):
  - Is there simple directed path of even length? It is \textit{NP}-complete.
  - Query evaluation is \textit{NP}-complete in data complexity – hence impractical.
Static analysis: Containment for 2RPQs

\textbf{Cont}(\mathcal{L})$: Given \mathcal{L}\text{-queries } Q_1 \text{ and } Q_2, 

\begin{itemize}
  \item is \( Q_1(G) \subseteq Q_2(G) \) for each graph DB \( G \)?
\end{itemize}
Static analysis: Containment for 2RPQs

**Containment for 2RPQs**

\[ \text{Cont}(\mathcal{L}): \text{Given } \mathcal{L}-\text{queries } Q_1 \text{ and } Q_2, \]

\[ \text{is } Q_1(\mathcal{G}) \subseteq Q_2(\mathcal{G}) \text{ for each graph DB } \mathcal{G}? \]

Containment for 2RPQs is decidable:

**Theorem**

\[ \text{Cont}(2\text{RPQ}) \text{ is PSPACE-complete. It is PSPACE-hard even for RPQs.} \]

- For RPQs easy to prove:
  - \( L_1(\mathcal{G}) \subseteq L_2(\mathcal{G}) \) for each \( \mathcal{G} \) \( \iff \)
    regular expression \( L_1 \) contained in regular expression \( L_2 \).
  - Containment of regular expressions:
    \( \text{PSPACE-complete} \) (Stock+1)Meyer (1971).

- For 2RPQs more work is required: Reason with two-way automata.
Containment for C2RPQs

Containment of C2RPQs still decidable with exponential blow-up:

**Theorem**

\( \text{Cont}(\text{C2RPQ}) \) is \text{ExpSpace}-complete, even for CRPQs.

- Notice contrast with complexity of containment for CQs:
  - NP-complete (Chandra, Merlin (1977)).
Summary of containment

- Containment of C2RPQs is decidable in double exponential time.
- For 2RPQs containment can be checked in single exponential time.
- High lower bounds are due to the presence of regular expressions.
Path queries and comparisons

CRPQs fall short of expressive power for applications that need:

- to include paths in the output of a query, and
- to define complex relationships among labels of paths.
Path queries and comparisons

CRPQs fall short of expressive power for applications that need:

- to include paths in the output of a query, and
- to define complex relationships among labels of paths.

Examples:

- Semantic Web queries:
  - establish semantic associations among paths.
- Biological applications:
  - compare paths based on similarity.
- Route-finding applications:
  - compare paths based on length or number of occurrences of labels.
- Data provenance and semantic search over the Web:
  - require returning paths to the user.
Path comparisons

We use a set $S$ of relations on words.

- **Example:** $S$ may contain
  - Unary relations: Regular, context-free languages, etc.
  - Binary relations: prefix, equal length, subsequence, etc.

- Comparisons among labels of paths
  - **Example:** $w_1$ is a substring of $w_2$.

- We assume $S$ contains all regular languages.
Extended CRPQs

The \textit{S-extended CRPQs (ECRPQ(\(S\))} are rules obtained from a CRPQ:

\[
\text{Ans}(\bar{z}, \ ) \leftarrow (x_1, L_1, y_1), \ldots, (x_m, L_m, y_m),
\]

- by annotating each pair \((x_i, y_i)\) with a path variable \(\pi_i\),
- comparing labels of paths in \(\bar{\pi}_j\) \text{ wrt } S_j \in S
  \begin{itemize}
  \item for \(\bar{\pi}_j\) a tuple of path variables among the \(\pi_i\)'s,
  \end{itemize}
- projecting some of \(\pi_i\)'s as a tuple \(\bar{\chi}\) in the output.
Extended CRPQs

The $S$-extended CRPQs (ECRPQ($S$)) are rules obtained from a CRPQ:

$$\text{Ans}(\bar{z}, ) \leftarrow (x_1, \pi_1, y_1), \ldots, (x_m, \pi_m, y_m),$$

- by annotating each pair $(x_i, y_i)$ with a path variable $\pi_i$,
- comparing labels of paths in $\bar{\pi}_j$ wrt $S_j \in S$
  - for $\bar{\pi}_j$ a tuple of path variables among the $\pi_i$’s,
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Extended CRPQs

The $S$-extended CRPQs ($ECRPQ(S)$) are rules obtained from a CRPQ:

$$Ans(\bar{z}, \cdot) \leftarrow (x_1, \pi_1, y_1), \ldots, (x_m, \pi_m, y_m), \land_{1 \leq j \leq t} S_j(\bar{\pi}_j)$$

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Extended CRPQs

The $S$-extended CRPQs (ECRPQ($S$)) are rules obtained from a CRPQ:

$$\text{Ans} (\bar{z}, \bar{\chi}) \leftarrow (x_1, \pi_1, y_1), \ldots, (x_m, \pi_m, y_m), \bigwedge_{1 \leq j \leq t} S_j (\bar{\pi}_j)$$

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  - for $\bar{\pi}_j$ a tuple of path variables among the $\pi_i$’s,
- projecting some of $\pi_i$’s as a tuple $\bar{\chi}$ in the output.
Extended CRPQs and our requirements

ECRPQs meet our requirements:

\[
\text{Ans}(\bar{z}, \bar{\chi}) \leftarrow (x_1, \pi_1, y_1), \ldots, (x_m, \pi_m, y_m), \land_{1 \leq j \leq t} S_j(\bar{\pi}_j)
\]
Extended CRPQs and our requirements

ECRPQs meet our requirements:

\[ \text{Ans}(\tilde{z}, \tilde{\chi}) \leftarrow (x_1, \pi_1, y_1), \ldots, (x_m, \pi_m, y_m), \land_{1 \leq j \leq t} S_j(\pi_j) \]

- They allow paths in the output.
- They allow to compare labels of paths with relations \( S_j \in S \).
Extended CRPQs and our requirements

ECRPQs meet our requirements:

\[ \text{Ans}(\bar{z}, \bar{\chi}) \leftarrow (x_1, \pi_1, y_1), \ldots, (x_m, \pi_m, y_m), \land_{1 \leq j \leq t} S_j(\bar{\pi}_j) \]

- They allow paths in the output.
- They allow to compare labels of paths with relations \( S_j \in S \).
Evaluation of ECRPQs

Evaluation of the ECRPQ($\mathcal{S}$)

$$\theta(\vec{z}, \vec{\chi}) : \text{Ans}(\vec{z}, \vec{\chi}) \leftarrow (x_1, \pi_1, y_1), \ldots, (x_m, \pi_m, y_m), \bigwedge_j S_j(\vec{\pi}_j)$$

Same than for CRPQs but:

- Each $\pi_i$ is mapped to a path $\rho_i$ in the graph DB.
- For each $j$, if $\vec{\pi}_j = (\pi_{j_1}, \ldots, \pi_{j_k})$ then: $\lambda(\rho_{j_1}), \ldots, \lambda(\rho_{j_k}) \in S_j$. the labels of $(\rho_{j_1}, \ldots, \rho_{j_k})$
Evaluation of ECRPQs

Evaluation of the ECRPQ(\(S\))

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- Each \(\pi_i\) is mapped to a path \(\rho_i\) in the graph \(\text{DB}\).

- For each \(j\), if \(\vec{\pi}_j = (\pi_{j_1}, \ldots, \pi_{j_k})\) then: \(\bigwedge_j S_j(\vec{\pi}_j)\)

  \[\lambda(\rho_{j_1}), \ldots, \lambda(\rho_{j_k}) \in S_j.\]

  The labels of \((\rho_{j_1}, \ldots, \rho_{j_k})\)
Evaluation of ECRPQs

Evaluation of the ECRPQ($\mathcal{S}$)

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Same than for CRPQs but:

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  then: $(\lambda(\rho_{j1}), \ldots, \lambda(\rho_{jk})) \in S_j. (\lambda(\rho_{j1}), \ldots, \lambda(\rho_{jk})) \in S_j$.
  the labels of $(\rho_{j1}, \ldots, \rho_{jk})$
Considerations about ECRPQ(\(S\))

- ECRPQ(\(S\)) extends the class of CRPQs.
  - \(\text{Ans}(\bar{z}) \leftarrow \wedge_i(x_i, L_i, y_i)\) same as \(\text{Ans}(\bar{z}) \leftarrow \wedge_i(x_i, \pi_i, y_i), L_i(\pi_i)\).

- Expressiveness and complexity of ECRPQ(\(S\)):
  - Depends on the class \(S\).

- We study two such classes with roots in formal language theory:
  - Regular relations (Elgot, Mezei (1965)).
  - Rational relations (Nivat (1968)).
Comparing paths with regular relations

- **Regular relations**: Regular languages for relations of any arity.
  - **REG**: Class of regular relations.

- **Bottomline**: 
  ECRPQ(REG): Reasonable expressiveness and complexity.
Regular relations

\textit{n}-ary regular relation:

Set of \(n\)-tuples \((w_1, \ldots, w_n)\) of strings accepted by \textit{synchronous} automaton over \(\Sigma^n\).
Regular relations

\textit{n-ary regular relation:}

Set of \( n \)-tuples \((w_1, \ldots, w_n)\) of strings accepted by \textit{synchronous} automaton over \( \Sigma^n \).

- The input strings are written in the \( n \)-tapes.
- Shorter strings are padded with symbol \( \bot \).
- At each step:
  The automaton simultaneously reads next symbol on each tape.
Synchronous automata

\[ w_1 = a \ a \ b \ \ldots \ a \ b \ c \]
\[ w_2 = a \ b \ a \ \ldots \ a \]
\[ w_3 = b \ b \ \ldots \]
\[ \vdots \]
\[ w_n = a \ b \ b \ \ldots \ a \ c \]
Synchronous automata

\[ w_1 = a \quad a \quad b \quad \ldots \quad a \quad b \quad c \]
\[ w_2 = a \quad b \quad a \quad \ldots \quad a \quad \bot \quad \bot \]
\[ w_3 = b \quad b \quad \bot \quad \ldots \quad \bot \quad \bot \quad \bot \]
\[ \vdots \]
\[ w_n = a \quad b \quad b \quad \ldots \quad a \quad c \quad \bot \]
Synchronous automata

\[ w_1 = a \ a \ b \ \cdots \ a \ b \ c \]
\[ w_2 = a \ b \ a \ \cdots \ a \ \bot \ \bot \]
\[ w_3 = b \ b \ \bot \ \cdots \ \bot \ \bot \ \bot \]
\[ \vdots \]
\[ w_n = a \ b \ b \ \cdots \ a \ c \ \bot \]
\[ \uparrow \]
Synchronous automata

\[ w_1 = a \ a \ b \ \cdots \ a \ b \ c \]
\[ w_2 = a \ b \ a \ \cdots \ a \ \perp \ \perp \]
\[ w_3 = b \ b \ \perp \ \cdots \ \perp \ \perp \ \perp \]
\[ \vdots \]
\[ w_n = a \ b \ b \ \cdots \ a \ c \ \perp \]
\[ \uparrow \]
Synchronous automata

\[
\begin{align*}
  w_1 &= a \ a \ b \ \cdots \ a \ b \ c \\
  w_2 &= a \ b \ a \ \cdots \ a \ \bot \ \bot \\
  w_3 &= b \ b \ \bot \ \cdots \ \bot \ \bot \ \bot \\
  \vdots & \vdots \\
  w_n &= a \ b \ b \ \cdots \ a \ c \ \bot
\end{align*}
\]
Synchronous automata

\[ w_1 = a \ a \ b \ \cdots \ a \ b \ c \]
\[ w_2 = a \ b \ a \ \cdots \ a \ \bot \ \bot \]
\[ w_3 = b \ b \ \bot \ \cdots \ \bot \ \bot \ \bot \]
\[ \vdots \]
\[ w_n = a \ b \ b \ \cdots \ a \ c \ \bot \]
Synchronous automata

\[ w_1 = a \ a \ b \ \cdots \ a \ b \ c \]
\[ w_2 = a \ b \ a \ \cdots \ a \ \bot \ \bot \]
\[ w_3 = b \ b \ \bot \ \cdots \ \bot \ \bot \ \bot \]
\[ \vdots \]
\[ w_n = a \ b \ b \ \cdots \ a \ c \ \bot \]

\[ \uparrow \]
Synchronous automata

\[ w_1 = a \ a \ b \ \ldots \ a \ b \ c \]
\[ w_2 = a \ b \ a \ \ldots \ a \ \bot \ \bot \]
\[ w_3 = b \ b \ \bot \ \ldots \ \bot \ \bot \ \bot \]
\[ \vdots \]
\[ w_n = a \ b \ b \ \ldots \ a \ c \ \bot \]

\[ \uparrow \]
Examples of regular relations

• All regular languages.

• The **prefix** relation defined by:

\[
\left( \bigcup_{a \in \Sigma} (a, a) \right)^* \cdot \left( \bigcup_{a \in \Sigma} (a, \bot) \right)^*.
\]

• The **equal length** relation defined by:

\[
\left( \bigcup_{a, b \in \Sigma} (a, b) \right)^*.
\]

• Pairs of strings at **edit distance at most** $k$, for fixed $k \geq 0$. 
Examples of regular relations

- All regular languages.
- The prefix relation defined by:
  \[( \bigcup_{a \in \Sigma} (a, a))^* \cdot ( \bigcup_{a \in \Sigma} (a, \bot))^*. \]
- The equal length relation defined by:
  \[( \bigcup_{a,b \in \Sigma} (a, b))^*. \]
- Pairs of strings at edit distance at most \(k\), for fixed \(k \geq 0\).

**Proposition**

The subsequence, subword and suffix relations are not regular.
ECRPQ(REG)

**ECRPQ(REG):** Class of queries of the form

\[
\text{Ans}(\vec{z}, \vec{\chi}) \leftarrow \bigwedge_i (x_i, \pi_i, y_i), \bigwedge_j S_j(\pi_j),
\]

where each \( S_j \) is a regular relation
ECRPQ(REG)

ECRPQ(REG): Class of queries of the form

\[ \text{Ans}(\bar{z}, \bar{\chi}) \leftarrow \bigwedge_i(x_i, \pi_i, y_i), \bigwedge_j S_j(\bar{\pi}_j), \]

where each \( S_j \) is a regular relation

Example: The ECRPQ(REG) query

\[ \text{Ans}(x, y) \leftarrow (x, \pi_1, z), (z, \pi_2, y), a^*({\pi_1}), b^*({\pi_2}), \text{equal\_length}(\pi_1, \pi_2) \]

computes pairs of nodes linked by a path labeled in \( \{a^n b^n \mid n \geq 0\} \).
ECRPQ(REG)

**ECRPQ(REG):** Class of queries of the form

\[
\text{Ans}(\vec{z}, \vec{\chi}) \leftarrow \bigwedge_i (x_i, \pi_i, y_i), \bigwedge_j S_j(\vec{\pi}_j),
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where each \( S_j \) is a regular relation

**Example:** The ECRPQ(REG) query

\[
\text{Ans}(x, y) \leftarrow (x, \pi_1, z), (z, \pi_2, y), a^*(\pi_1), b^*(\pi_2), \text{equal\_length}(\pi_1, \pi_2)
\]

computes pairs of nodes linked by a path labeled in \( \{a^n b^n \mid n \geq 0\} \).

**Corollary**

ECRPQ(REG) *properly extends the class of CRPQs.*
Complexity of evaluation of ECRPQ(REG)

- Extending CRPQs with regular relations is free for data complexity.
- Combined complexity is that of relational calculus over relational databases.

**Theorem**

\[ \text{Eval}(\text{ECPRQ}(\text{REG})) \text{ is } \text{PSPACE-complete}. \]

\[ \text{Eval}(\text{ECPRQ}(\text{REG})) \text{ is in } \text{NLOGSPACE in data complexity}. \]
Containment for ECRPQ(REG)

**Theorem**

\( \text{Cont}(\text{ECRPQ}(\text{REG})) \) is undecidable.

- Notice contrast with CRPQs for which containment is decidable.
- But this is like for full relational algebra/calculus.
Comparing with rational relations

ECRPQ(REG) queries are still short of expressive power:

▶ RDF or biological networks:
  • Compare strings based on subsequence and subword relations.
▶ These relations are rational: Accepted by asynchronous automata.
  • RAT: Class of rational relations.

Bottomline:

▶ ECRPQ(RAT) evaluation:
  • Undecidable or very high complexity.
▶ Restricting the syntactic shape of queries yields tractability.
Rational relations

\textit{n-ary rational relation:}

Set of \( n \)-tuples \((w_1, \ldots, w_n)\) of strings accepted by \textit{asynchronous} automaton with \( n \) heads.
Rational relations

\textit{n-ary rational relation:}
Set of $n$-tuples $(w_1, \ldots, w_n)$ of strings accepted by \textit{asynchronous} automaton with $n$ heads.

- The input strings are written in the $n$-tapes.
- At each step:
  - The automaton enters a new state and move some tape heads.
Rational relations

\textit{n-ary rational relation:}

Set of \( n \)-tuples \((w_1, \ldots, w_n)\) of strings
accepted by \textbf{asynchronous} automaton with \( n \) heads.

\begin{itemize}
  \item The input strings are written in the \( n \)-tapes.
  \item At each step:
    \begin{itemize}
      \item The automaton enters a new state and move some tape heads.
    \end{itemize}
\end{itemize}

\textit{n-ary rational relation:}

Described by regular expression over alphabet \((\Sigma \cup \{\epsilon\})^n\).
Examples of rational relations

- All regular relations.

- The subsequence relation $\preceq_{ss}$ defined by:

\[
\left( \bigcup_{a \in \Sigma} (a, \epsilon) \right)^* \bigcup_{b \in \Sigma} (b, b) \bigcup_{a \in \Sigma} (a, \epsilon) .
\]

- The subword relation $\preceq_{sw}$ defined by:

\[
\left( \bigcup_{a \in \Sigma} (a, \epsilon) \right)^* \bigcup_{b \in \Sigma} (b, b) \bigcup_{a \in \Sigma} (a, \epsilon) .
\]
Examples of rational relations

- All regular relations.
- The subsequence relation $\preceq_{ss}$ defined by:
  $$\left( \bigcup_{a \in \Sigma} (a, \varepsilon)^* \bigcup_{b \in \Sigma} (b, b) \right)^* \cdot \left( \bigcup_{a \in \Sigma} (a, \varepsilon)^* \right).$$
- The subword relation $\preceq_{sw}$ defined by:
  $$\left( \bigcup_{a \in \Sigma} (a, \varepsilon)^* \right) \cdot \left( \bigcup_{b \in \Sigma} (b, b)^* \right) \cdot \left( \bigcup_{a \in \Sigma} (a, \varepsilon)^* \right).$$

**Proposition**

The set of pairs $(w_1, w_2)$ such that $w_1$ is the reversal of $w_2$ is not rational.
ECRPQ(RAT)

**ECRPQ(RAT):** Class of queries of the form

\[
\text{Ans}(\tilde{z}, \bar{\chi}) \leftarrow \bigwedge_i (x_i, \pi_i, y_i), \bigwedge_j S_j(\bar{\pi}_j),
\]

where each \( S_j \) is a rational relation

**Example:** The ECRPQ(RAT) query

\[
\text{Ans}(x, y) \leftarrow (x, \pi_1, z), (y, \pi_2, w), \pi_1 \preceq_{\text{ss}} \pi_2
\]

computes \( x, y \) that are origins of paths \( \rho_1 \) and \( \rho_2 \) such that:

- \( \lambda(\rho_1) \) is a subsequence of \( \lambda(\rho_2) \).
Evaluation of ECRPQ(RAT) queries

Evaluation of queries in ECRPQ(RAT) is undecidable, but:

- True if we allow only practically motivated rational relations?
  - For example, $\leq_{ss}$ and $\leq_{sw}$. 
Evaluation of ECRPQ(RAT) queries

Evaluation of queries in ECRPQ(RAT) is undecidable, but:

- True if we allow only practically motivated rational relations?
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Adding subword relation to ECRPQ(REG) leads to undecidability:

**Theorem**

*Evaluation of* $(\text{ECRPQ}(\text{REG} \cup \{\leq_{sw}\}))$ *queries is undecidable. The same is true for suffix in place of subword.*
Evaluation of ECRPQ(RAT) queries

Evaluation of queries in ECRPQ(RAT) is undecidable, but:

- True if we allow only practically motivated rational relations?
  - For example, \( \leq_{ss} \) and \( \leq_{sw} \).

Adding subword relation to ECRPQ(REG) leads to undecidability:

**Theorem**

Evaluation of \((ECRPQ(REG \cup \{\leq_{sw}\}))\) queries is undecidable. The same is true for suffix in place of subword.

Adding subsequence preserves decidability, but at a very high cost:

**Theorem**

Evaluation of \((ECRPQ(REG \cup \{\leq_{ss}\}))\) queries is decidable, but non-primitive-recursive.

Primitive-recursive, informally: any function you can think of!
Acyclic ECRPQ(RAT) queries

Acyclic ECRPQ(RAT) queries yield tractable data complexity.

Queries of the form:

\[
\text{Ans}(\vec{z}) \leftarrow \bigwedge_{i \leq k} (x_i, \pi_i, y_i), L_i(\pi_i), \bigwedge_j S_j(\pi_{j_1}, \pi_{j_2}),
\]

where the graph on \{1, \ldots, k\} defined by edges \((\pi_{j_1}, \pi_{j_2})\) is acyclic.
Acyclic ECRPQ(RAT) queries yield tractable data complexity.

- Queries of the form:

\[
\text{Ans}(\vec{z}) \leftarrow \bigwedge_{i \leq k} (x_i, \pi_i, y_i), L_i(\pi_i), \bigwedge_j S_j(\pi_{j1}, \pi_{j2}),
\]

where the graph on \(\{1, \ldots, k\}\) defined by edges \((\pi_{j1}, \pi_{j2})\) is acyclic.

Acyclic ECRPQ(RAT) is not more expensive than ECRPQ(REG):

**Theorem**

- Evaluation of acyclic ECRPQ(RAT) queries is \(\text{PSPACE}\)-complete.
- It is in \(\text{NLOGSPACE}\) in data complexity.
Summary of path queries

- Usual query languages do not allow:
  - to export paths and compare labels of paths.

- This has led to the introduction of ECRPQ(S) queries:
  - They output paths and compare labels of paths with relations in $S$.

- Comparing paths with regular relations:
  - Preserves tractable data complexity of evaluation.
  - Leads to undecidability of containment.

- Comparing paths with practically motivated rational relations:
  - Leads to undecidability or high complexity of evaluation.
  - Tractable cases found restricting the syntactic shape of queries.
Querying graphs with data

So far queries only talk about the topology of the data.

Queries that combine topology and data are important in practice:

- **Example:**
  People of the same age connected by professional links.

We present a language that expresses topological properties of the data:

- It requires an extension of the data model (**data graphs**).
- It talks about **data paths:**
  Summarize the topology and the underlying data of a path.
Data graphs and data paths

We work with data graphs and paths over set of data values $D$.

**Definition**

A **data graph** $\mathcal{G}$ over $\Sigma$ is a tuple $(V, E, \delta)$, where:
- $(V, E)$ is a graph database over $\Sigma$, and
- $\delta$ is a mapping that assigns a value in $D$ to each node $v \in V$. 

We work with data graphs and paths over set of data values $D$.

**Definition**

A data graph $\mathcal{G}$ over $\Sigma$ is a tuple $(V, E, \delta)$, where:

- $(V, E)$ is a graph database over $\Sigma$, and
- $\delta$ is a mapping that assigns a value in $D$ to each node $v \in V$.

With each path $\rho = v_1 \xrightarrow{a_1} v_2 \cdots v_k \xrightarrow{a_k} v_{k+1}$ in $(V, E)$:

We associate a data path in $\mathcal{G}$ of the form

$$\rho_D = \delta(v_1) \xrightarrow{a_1} \delta(v_2) \cdots \delta(v_k) \xrightarrow{a_k} \delta(v_{k+1}),$$

that is obtained from $\rho$ by replacing each node by its data value.
Data paths and data words

Data paths are very close to data words:

- Object studied in XML and verification (Bojanczyk et al. (2006)).
- Data words are strings over $\Sigma \times D$.

Mechanisms that query data words can be used for data paths:

- FO, MSO, and some versions of XPath (Bojanczyk et al. (2006)).
- Pebble automata (Neven, Schwentick, Vianu (2004)).
- Register automata (Kaminski, Francez (1994)).
The choice of a formalism

Formalism for querying data paths has to be chosen with care:

**Theorem**

The problem **DistinctValues** is NP-complete:

- **DistinctValues**: Is there a path $\rho$ from $v$ to $v'$ s.t. no data value in $\rho_D$ is repeated?
The choice of a formalism

Formalism for querying data paths has to be chosen with care:

**Theorem**

The problem **DistinctValues** is NP-complete:

- **DistinctValues**: Is there a path $\rho$ from $v$ to $v'$ s.t. no data value in $\rho_D$ is repeated?

**Conclusion:**

- If a language expresses **DistinctValues**:
  - It is NP-hard in data complexity $\Rightarrow$ Impractical.

- Rules out all formalisms except for one:
  - Register automata.
Regular expressions for register automata

Regular expressions with memory (REMs):
Same as register automata
Regular expressions for register automata

Regular expressions with memory (REMs):
Same as register automata

- REMs permit to specify when data values are remembered and used.
- Data values are remembered in $k$ registers $\{x_1, \ldots, x_k\}$.
- At any point we can compare a data value with one in the registers.
Consider the REM $\downarrow x.a^+[x=]$.

**Intuition:**
- Store the current data value $d$ in register $x$.
- After reading a word in $a^+$ check that $d$ is seen again.

**Semantics:** Pairs $(v, v')$ of nodes:
- Linked by a path labeled in $a^+$.
- $v$ and $v'$ contain the same data value.
REM: Conditions

- **Conditions**: Compare a data value with the ones in the registers.
- Conditions over \( \{x_1, \ldots, x_k\} \) are given by the grammar:

\[
c := x_i \bar{=} | \neg c | c \land c \quad (1 \leq i \leq k)
\]

- We define \((d, \tau) \models c\) for \(d \in \mathcal{D}\) and \(\tau = (d_1, \ldots, d_k) \in \mathcal{D}^k\):
  - \((d, \tau) \models x_i \bar{=} \) iff \(d = d_i\).
  - Boolean combinations are standard.
REMs: Syntax and semantics (Intuition)

REMs over $\Sigma$ and $\{x_1, \ldots, x_k\}$ are defined by grammar:

$$ e := \varepsilon \mid a \mid e \cup e \mid e \cdot e \mid e^+ \mid e[c] \mid \downarrow \bar{x}.e $$

where $a \in \Sigma$, $c$ condition, and $\bar{x}$ tuple in $\{x_1, \ldots, x_k\}$. 
REMs: Syntax and semantics (Intuition)

REMs over \( \Sigma \) and \( \{x_1, \ldots, x_k\} \) are defined by grammar:

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\]

where \( a \in \Sigma \), \( c \) condition, and \( \bar{x} \) tuple in \( \{x_1, \ldots, x_k\} \).

**Intuition:** Evaluation of REM \( e \) on data graph \( \mathcal{G} \) is:

- pairs \((v, v')\) of nodes linked by path \( \rho \) such that \( \rho_{\mathcal{D}} \models e \), where:
REMs: Syntax and semantics (Intuition)

REMs over $\Sigma$ and $\{x_1, \ldots, x_k\}$ are defined by grammar:

\[
e := \varepsilon \mid a \mid e \cup e \mid e \cdot e \mid e^+ \mid e[c] \mid \downarrow \bar{x}.e
\]

where $a \in \Sigma$, $c$ condition, and $\bar{x}$ tuple in $\{x_1, \ldots, x_k\}$.

**Intuition:** Evaluation of REM $e$ on data graph $\mathcal{G}$ is:
- pairs $(v, v')$ of nodes linked by path $\rho$ such that $\rho \models e$, where:

  $\rho \models e[c]$ if and only if

- $\rho$ starting from empty registers
- $\rho$ can be parsed wrt $e$
- finishing in register value $\tau \in \mathcal{D}^k$ st $(\kappa(v_k+1), \tau) \models c$
REMs: Syntax and semantics (Intuition)

REMs over $\Sigma$ and $\{x_1, \ldots, x_k\}$ are defined by grammar:

$$
e \ ::= \ varepsilon \mid a \mid e \cup e \mid e \cdot e \mid e^+ \mid e[c] \mid \downarrow \vec{x}.e
$$

where $a \in \Sigma$, $c$ condition, and $\vec{x}$ tuple in $\{x_1, \ldots, x_k\}$.

Intuition: Evaluation of REM $e$ on data graph $G$ is:
- pairs $(v, v')$ of nodes linked by path $\rho$ such that $\rho_D \models e$, where:
  - $\rho_D \models \downarrow \vec{x}.e$ if and only if:
    - $\rho_D \models e$ starting from the register value that assigns $\kappa(v_1)$ to each $x \in \vec{x}$
    - can be parsed wrt $e$
REM: Example

Consider the REM $\Sigma^* \cdot (\downarrow x. \Sigma^+ [x =]) \cdot \Sigma^*$:

- Defines pairs of nodes linked by path $\rho$ such that:
  - $\rho_D$ contains the same data value twice.
- The complement of this language is $\text{DISTINCTVALUES}$.
Consider the REM $\Sigma^* \cdot (\downarrow x. \Sigma^+[x=]) \cdot \Sigma^*$:

- Defines pairs of nodes linked by path $\rho$ such that:
  - $\rho \mathcal{D}$ contains the same data value twice.
- The complement of this language is $\text{DistinctValues}$.

**Corollary**

REM$s$ are not closed under complement.
Complexity of REM evaluation

- Data complexity of REM evaluation coincides with that of CRPQs.
- Combined complexity same than for FO over relational databases.

**Theorem**

- $\text{Eval}(\text{REM})$ is $\text{PSPACE}$-complete.
- It is in $\text{NLLOGSPACE}$ in data complexity.

- Both bounds extend to the class of conjunctive REMs.
Summary of queries on graphs with data

- Most query languages for graph DBs:
  - talk about topology, but not about underlying data.

- Query languages that combine topology and data:
  - talk about data paths in data graphs.

- Choosing a formalism to query data paths must be done with care:
  - intractability can be reached easily.

- To query data paths:
  - Can use REMs, which are based on register automata.
  - REMs can be evaluated efficiently in data complexity.
Comments on papers

  Original papers introducing (C)RPQs
- Pablo Barcelo: Querying graph databases. PODS 2013: 175-188
- Peter T. Wood: Query languages for graph databases. SIGMOD Record 41(1): 50-60 (2012)
  Three suveys of graph languages, two are more theoretical, one more practical.
  Introducing two-way queries.
Comments on papers


  Static analysis of regular path queries.

- Leonid Libkin, Wim Martens, Domagoj Vrgoc: Querying graph databases with XPath. ICDT 2013: 129-140

  Adding data values to (C)RPQs


  Extending RPQs with regular relations; topics to concentrate on are those not covered in class.

- Pablo Barcelo, Diego Figueira, Leonid Libkin: Graph Logics with Rational Relations. Logical Methods in Computer Science 9(3) (2013)

  Likewise for rational relations.
Comments on papers

- Dominik D. Freydenberger, Nicole Schweikardt: Expressiveness and Static Analysis of Extended Conjunctive Regular Path Queries. AMW 2011
  Resolving some of the questions on the containment of path queries.

- Jelle Hellings, Bart Kuijpers, Jan Van den Bussche, Xiaowang Zhang: Walk logic as a framework for path query languages on graph databases. ICDT 2013: 117-128
  A different approach to expanding the power of path languages.

  Incomplete information in graph databases and querying it.

- Wenfei Fan, Xin Wang, Yinghui Wu: Querying big graphs within bounded resources. SIGMOD Conference 2014: 301-312

- Wenfei Fan: Graph pattern matching revised for social network analysis. ICDT 2012: 8-21
  Two papers on making graph queries scalable.