For the first three problems, we use the schema below.

- **Dept(dept#, manager, budget)** (that is, department name, employee ID of its manager, and the budget)
- **Emp(emplId, dept, salary)** (that is, employee ID, department name, and employee’s salary)
- Assume that dept# is the primary key for Dept, and emplId is the primary key for Emp
- Assume furthermore the following inclusion dependencies:
  - Dept[manager] ⊆ Emp[emplId]
  - Emp[dept] ⊆ Dept[dept#]

**Problem 1.** Write the following query in relational calculus: Find names of departments with at least one employee having higher salary than his/her manager.

Answer:

\[
\{d \mid \exists m, b, s_1, s_2, e \quad (\text{Dept}(d, m, b) \\
\wedge \text{Emp}(m, d, s_1) \\
\wedge \text{Emp}(e, d, s_2) \\
\wedge s_2 > s_1) \}
\]

**Problem 2.** Write the query from Problem 1 in SQL.

```sql
SELECT D.dept#
FROM Dept D
WHERE EXISTS (SELECT *
FROM Emp E1, Emp E2
WHERE E1.emplId = D.manager AND
E2.dept = D.dept# AND
E2.salary > E1.salary)
```

**Problem 3.** Write the following in SQL: Find managers of departments that spend more than half of their budget on salaries.

```sql
SELECT D.manager
FROM Dept D
WHERE 0.5 * D.budget < (SELECT SUM(E.Salary)
FROM Emp E
WHERE E.dept# = D.dept#
GROUP BY E.dept#)
```

**Problem 4.** Given two relations $R$ and $S$, each with attributes $A$ and $B$. Express the following relational calculus query in relational algebra:

\[
\{x \mid (\exists z \ (R(z, x) \lor S(z, x))) \land (\forall y \ (S(x, y) \lor \neg R(x, y)))\}
\]

Answer: $\pi_B(S \cup R) - \rho_{B \setminus A}(\pi_A(R - S))$

Because: $R(z, x) \lor S(z, x)$ is $R \cup S$, so $\exists z \ (R(z, x) \lor S(z, x))$ is $\pi_B(S \cup R)$.

$\forall y \ (S(x, y) \lor \neg R(x, y))$ is $\neg \exists y \ (\neg (S(x, y) \lor \neg R(x, y)))$ which is the same as $\neg \exists y \ (\neg S(x, y) \land R(x, y))$.

We know that $\neg (S(x, y) \land R(x, y))$ is $R - S$, so $\exists y \ (\neg S(x, y) \land R(x, y))$ is $\pi_A(R - S)$. Combining the two subexpressions, we get the answer. We have to rename to be able to apply the difference.