Write first-order formulae defining the following regular languages over the alphabet $\Sigma = \{a, b\}$.

1. $(a + b)^+ \cdot a$ (1 mark)
2. $a^*b^*$ (1 mark)
3. Write an MSO formula defining the following regular languages over the alphabet $\Sigma = \{a, b\}$:

   $$((aa)^* (bb)^*)^*$$ (2 marks)

4. What is an existential MSO formula that is equivalent to the following:

   $$\forall X \forall Y \left( \forall x (X(x) \land Y(x) \rightarrow P_a(x)) \right)$$ (1 mark)