Σ - alphabet (finite)

Σ^* — all finite strings (words) over Σ

$L \subseteq \Sigma^*$ - languages over Σ

A nondeterministic finite automaton

$$\mathcal{A} = (Q, Q_0, F, \delta)$$

Q — set of states

Q_0 — set of initial states, $Q_0 \subseteq Q$,

F — final states, $F \subseteq Q$,

δ — transition function

A run is accepting if $\rho_A(n) \in F$

A word $s \in \Sigma^*$ is accepted by A if there is an accepting run

Notation. $L(A) = \{ s \in \Sigma^* \mid s \text{ is accepted by } A \}$

Such languages $L(A)$ are called <u>regular</u>

Regular expressions

ε – empty string

a, $a \in \Sigma$

e, e' new expr $e \cdot e'$, $e|e'$, e^*

L_1 is accepted by $A_1 = (Q^1, Q_0^1, F^1, \delta^1)$

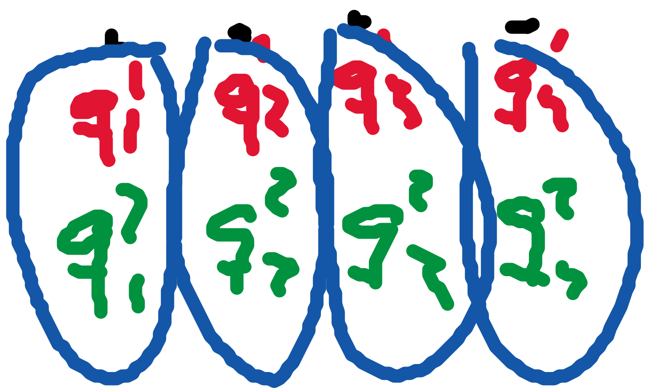
L_2 $\quad$ · $\quad$ $A_2 = (Q^2, Q_0^2, F^2, \delta^2)$

assume $Q^1 \cap Q^2 = \emptyset$

1) A for $L_1 \cup L_2$

$$(Q^1 \cup Q^2,\ Q_0^1 \cup Q_0^2,\ F^1 \cup F^2,\ \delta^1 \cup \delta^2)$$

2) $L_1 \cap L_2$ - product construction



$$\Big(Q^1 \times Q^2,\ Q_0^1 \times Q_0^2,\ F^1 \times F^2$$

$$\delta\left(\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, a\right) \ni \begin{pmatrix} q_1' \\ q_2' \end{pmatrix} \iff \begin{array}{l} q_1' \in \delta(q_1, a) \\ q_2' \in \delta(q_2, a) \end{array}$$

$A = (Q, Q_0, \delta, F) \longrightarrow A_d$

powerset construction

$A_d = (2^Q,$

initial states $\{q_0\}$, for $q_0 \in Q_0$

final states $Q' \subseteq Q$ s.t. $Q' \cap F \neq \emptyset$

transition
$\forall Q' \forall a$

$$\delta'(Q', a) = \bigcup_{q \in Q'} \delta(q, a)$$

Decision Problems

1 Non-emptiness Given A_1 is $L(A) \neq \emptyset$?

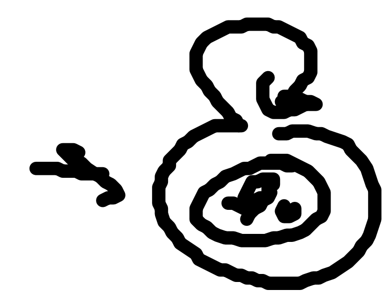
2 Containment Given A_1, A_2, is $L(A_1) \subseteq L(A_2)$?

3 Equivalence $=$

4 Universality Given A_1 is $L(A) = \Sigma^*$?

$A \longrightarrow \bar{A}$

A_1



$L(A) = \Sigma^* \iff L(\bar{A}) = \emptyset$

PSPACE-complete