Vocabulary (signature)
( +, * )
( +, *, < )

Structures Syntax Semantics (satisfaction)
(N, +, *) \exists x \exists y
(R, +, *) (x = y + y) y is some
(N, +, *, < ) \exists x \forall y y > x \forall x \forall y y = x false
(R, +, *, < ) \forall y (N, +, *, < )
Structures over a vocabulary $\sigma$
- a universe (set) $U$
- an interpretation of
  each function symbol $f_i$ of arity $n_i$
    as a function $U^{n_i} \rightarrow U$
  each constant as an element of $U$
  each predicate symbol $P_i$ as a subset of $U^{n_i}$
Finite strings as structures

Vocabulary
\( \left( \{ P_a, P_b \}, \prec \right) \)

alphabet \( \sum \)
\[ \sum \in \{ a, b \} \]

so \( abaab \)
\[ M_s = (\{0,1,2,3,4,3\}, P_a = \{0,2,3\}, P_b = \{1,4\}, \prec) \]
First Order Logic (FO)

Terms
- Each variable $x$ is a term
  $FV = \{x\}$
- Each constant symbol $c_i$ is a term, $FV = \emptyset$
- If $f$ is a function symbol of any $n$ and $t_1, \ldots, t_n$ are terms, then $f(t_1, \ldots, t_n)$ is a term, and $FV = \bigcup FV(t_i)$

If no function symbols, atomic flair.
- If $t_1, \ldots, t_n$ are terms and $P$ a pred symbol of any $n$, then $P(t_1, \ldots, t_n)$ is a flair.
  $FV(P(t_1, \ldots, t_n)) = \bigcup FV(t_i)$
- If $t_1$ and $t_2$ are terms, then $t_1 = t_2$ is a flair.
  and $FV = FV(t_1) \cup FV(t_2)$
. If \( \varphi, \varphi_1, \varphi_2 \) are formulas then \( \varphi_1 \lor \varphi_2, \varphi_1 \land \varphi_2, \neg \varphi \)
are formulas. \( \text{FV}(\varphi_1 \lor \varphi_2) = \text{FV}(\varphi_1) \cup \text{FV}(\varphi_2) \)
\( \text{FV}(\neg \varphi) = \text{FV}(\varphi) \)

. If \( \varphi \) is a flc, then \( \exists x \varphi, \forall x \varphi \) are flc and \( \text{FV}(\exists x \varphi) = \text{FV}(\forall x \varphi) = \text{FV}(\varphi) - \{x\} \)

Bound = not free

\( p(x, x_2) \) if \( \text{FV}(\varphi) = \{x, x_2\} \)

\( \text{F} \) w/o free variable = sentence
\( a \in \{ (a, b), (c, d) \} \)

\[ G = \{ (a, b), (c, d) \} \]

\[ G \subseteq \bigcup \{ x \in E \} \]

\[ \bigcup \{ x \in E \} \cap \bigcup \{ x \in A \} \]

\[ (x) \phi \land E \subseteq (x) \phi \times A \]

\[ E \subseteq \bigcup \{ x \in A \} \]

\[ \text{for each } n \in \mathbb{N} \text{ s.t. } n > 0 \]

\[ (\phi(0) \neq (\geq 'N)) \iff \phi(0) \iff (\geq 'N) \]

\[ (x, y) \in (x \forall y) \land (y \leq x) \iff y = x \land y \geq x \]

\[ (y = x \land y \geq x) \iff (y \leq x) \implies y = x \land \exists E \neq \emptyset \]