

Weak MSO

Syntax - same as MSO

Semantics

$$\exists X \varphi(X, \dots)$$

means 'there is a finite set X '

$$\text{WS1S} = \text{Th}_{\text{Weak MSO}}(\mathbb{N}, \text{succ})$$

Presburger Arithmetic \hookrightarrow WS1S

$n \rightarrow n$ in binary \rightarrow set

$$9 \rightarrow \underset{3210}{1001} \rightarrow \{3, 0\} \quad \overset{11}{\text{N}} \hookrightarrow 1011 \rightarrow \{3, 1, 0\}$$

$$n_1 \dots n_k$$

$$X_1 \dots X_k \subseteq \mathcal{N}$$

0 1 2 3 . . .

$$X_1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad .$$

$$X_2 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad .$$

$$X_k \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad .$$

$\varphi(n_1 \dots n_k)$ of Fresh with
 $\psi(X_1 \dots X_k)$ of WSIS

$\varphi(\bar{n})$ is true



$\psi(\bar{X})$ is true

$$\varphi(x, y, z) \equiv x + y = z$$

WSIS

$$\psi(X, Y, Z)$$

WSIS is decidable

$\varphi(x_1, \dots, x_n)$ - weak MSO formula
over $(\mathbb{N}, succ)$

	0	1	2	3	4
x_1	1	0	1	0	1
x_2	0	0	1	0	0
x_n	1	1	0	.	.

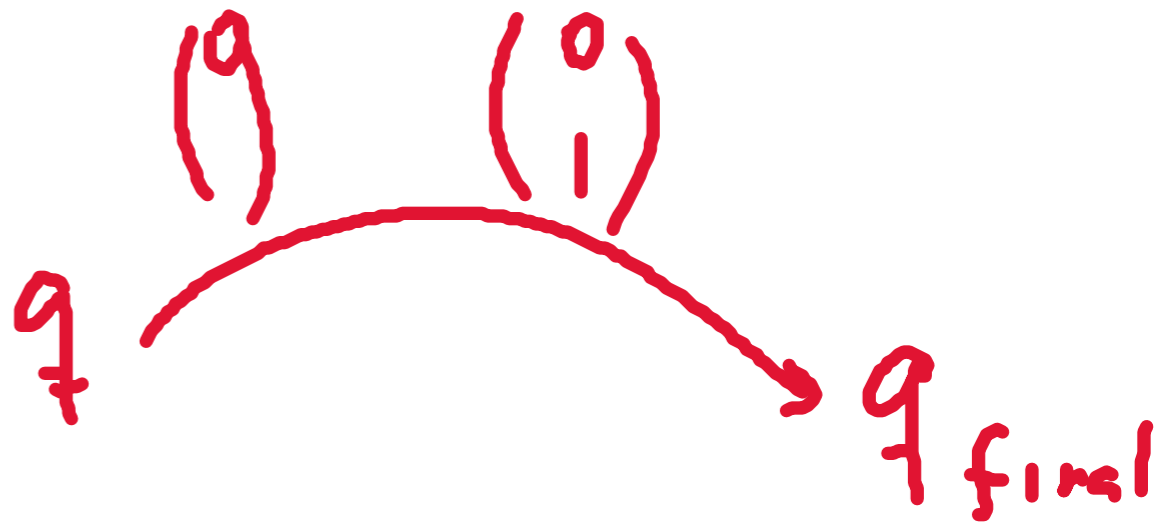
→ word over $\{0, 1\}^n$
[\bar{x}]

A_φ s.t. $A_\varphi \text{ accept } [\bar{x}] \Leftrightarrow \varphi(\bar{x}) \text{ is true}$

$$\psi(x) = \exists Y \varphi(x, Y)$$

$$A_{\varphi} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)^{\neq} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{array}{l}
 X \quad \boxed{0100101000} \\
 Y \quad 01000000001
 \end{array}$$



δ transition of A_{φ}

before

$$\delta'(q, a) = \delta\left(q, \begin{pmatrix} a \\ 0 \end{pmatrix}\right) \cup \delta\left(q, \begin{pmatrix} a \\ 1 \end{pmatrix}\right)$$