weak MSO   Syntax – same as MSO

Semantics $\exists X \varphi(X, \quad)$

means "there is a finite set X."

$$\mathrm{WS1S} = \mathrm{Th}_{\text{weak MSO}}(\mathbb{N}, \mathrm{Succ})$$

Presburger Arithmetic $\hookrightarrow$ WS1S

$n \longrightarrow n$ in binary $\longrightarrow$ set

$9 \to \underset{3\,2\,1\,0}{1001} \longrightarrow \{3,0\}$ $\underset{\mathbb{N}}{\overset{\Uparrow}{11}} \to 1011 \longrightarrow \{3,1,0\}$

$n_1 \; . \; n_k$

$X_1 \quad X_k \subseteq \mathbb{N}$

0 1 2 3 . .

X_1 1 0 1 1 0 .

X_2 1 0 0 0 1 .

X_k 1 1 0 0 0 .

$\varphi(n_1 \quad n_k)$ of Presb arith

$\psi(X_1 \; . \; X_k)$ of WS1S

$\varphi(\bar{n})$ is true

$\Updownarrow$

$\psi(\bar{X})$ is true

$\varphi(x, y, z) \equiv x + y = z$

WS1S $\quad \psi(X, Y, Z)$

WS1S is decidable

$\varphi(X_1 \quad X_n)$ — weak MSO fla over $(\mathbb{N}, succ)$

$$
\begin{array}{c|ccccc}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
X_1 & 1 & 0 & 1 & 0 & 1 \\
X_2 & 0 & 0 & 1 & 0 & 0 \\
X_n & 1 & 1 & 0 & & .
\end{array}
$$

$\to$ word over $\{0,1\}^n$

$[\bar{X}]$

A_φ s.t. A_φ accepts $[\bar{X}] \Leftrightarrow \varphi(\bar{X})$ is true

$\psi(X) = \exists Y \, \varphi(X, Y)$

A_φ $\quad \left(\binom{0}{0} + \binom{0}{1} \right)^* \binom{0}{1}$

$X \quad \boxed{0\,1\,0\,0\,1\,0\,1}\,0\,0\,0\,0\,.$

$Y \quad 0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,1$

$q \xrightarrow{(a) \quad \binom{0}{1}} q_{final}$

δ transform of A_φ

before

$$\delta'(q, a) = \delta\left(q, \binom{a}{0}\right) \cup \delta\left(q, \binom{a}{1}\right)$$