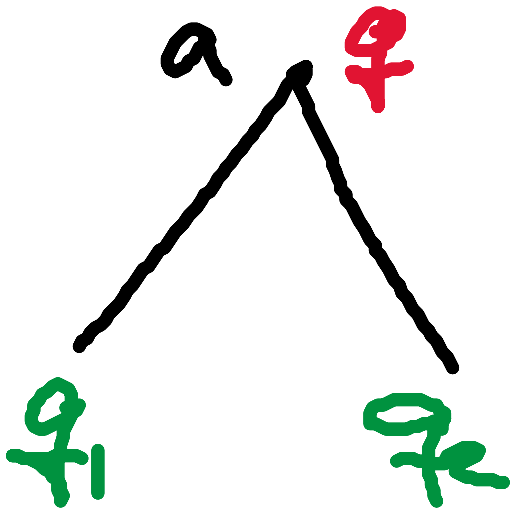
Top-down



$\delta\ Q \times \Sigma \to 2^{Q \times Q}$

$(\rho(s0), \rho(s1)) \in \delta(\rho(s), \lambda(s))$

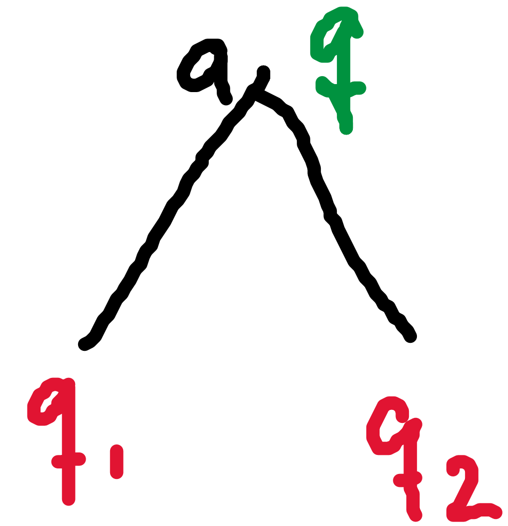
Leaves (acceptance)

$\delta(\rho(s), \lambda(s))$ contains a pair (q, q') s.t. $q, q' \in F$

Root (initial)

$\rho(\varepsilon) \in Q_0$

Bottom-up



$\delta\ Q \times Q \times \Sigma \to 2^{Q}$

$-\ \rho(s) \in \delta(\rho(s0), \rho(s1), \lambda(s))$

Leaves (initial condition)

$\rho(s) \in \delta(q, q', \lambda(s))$ where $q, q' \in Q_0$

Root (acceptance)

$\rho(\varepsilon) \in F$

Nondeterministic TD tree automata

$$\shortparallel$$

Nondet BU tree automata

$$\shortparallel$$

Regular tree languages

<u>BUT</u> A deterministic BU TA could be translated into a nondet TD TA

Det BUTA $\quad \delta \; Q \times \Sigma \times Q \to Q$

Every Nondet BUTA is equivalent to a det BUTA

$$\boxed{\delta'(X, a, Y) = \bigcup_{q \in X} \bigcup_{q' \in Y} \delta(q, q', a)}$$

<u>Proof</u> Powerset construction

Nondet $A = (Q, Q_0, F, \delta \; Q \times \Sigma \times Q \to 2^Q)$

Det $A_d = (2^Q, \{Q_0\}, \{X \mid X \cap F \neq \emptyset\}, \delta' \; 2^Q \times \Sigma \times 2^Q \to 2^Q)$

final states

Closure properties of regular tree languages

– under intersection, union, complement

Intersection – product construction

$A_1 \quad A_2 \qquad$ BU nondet TA

$$A_1 = (Q^1, Q^1_0, F^1, \delta^1 : Q^1 \times \Sigma \times Q^1 \to 2^{Q^1})$$

$$A_2 = (Q^2, Q^2_0, F^2, \delta^2 : Q^2 \times \Sigma \times Q^2 \to 2^{Q^2})$$

$$A_1 \times A_2 = \left(Q^1 \times Q^2, Q^1_0 \times Q^2_0, F^1 \times F^2, \delta : (Q^1 \times Q^2) \times \Sigma \times (Q^1 \times Q^2) \to 2^{Q_1 \times Q_2}\right)$$

$$(q_1, q_2) \in \delta((q_1', q_2'), a, (q_1'', q_2''))$$

$$\iff$$

$q_1 \in \delta(q_1', a, q_1'')$ and

$q_2 \in \delta(q_2', a, q_2'')$

$$L(A_1 \times A_2) = L(A_1) \cap L(A_2)$$

<u>Complement</u> $A \xrightarrow[\text{det.}]{} A_d = (Q, q_0, F, \delta)$

$$\bar{A}_d = (Q, q_0, Q - F, \delta)$$