Unranked Tree domain is a subset of $\mathbb{N}^*$ such that:
1. $D$ is prefix-closed: $s \in D, s' \leq s \Rightarrow s' \in D$
2. If $s \cdot (c+1) \in D \Rightarrow s \cdot c \in D$

Unranked Tree $(D, \lambda)$ s.t. $D \subseteq \text{tree domain}$, $\lambda : D \rightarrow \Sigma$
Unranked tree automata

\[ A = (Q,F,\delta) \]

\[ \delta : Q \times \Sigma \rightarrow 2^Q^*, \quad \delta(q,a) \subseteq Q^* \]

Each \( \delta(q,a) \) is a regular language.
A run is a mapping $\delta : D \rightarrow Q$ such that for each node $s$ with children $s_0, s_{(n-1)}$, $\delta(p(s), \lambda(s)) \ni \delta(p(s_0)) \cdot \delta(p(s_{(n-1)}))$

$\Delta \in \delta(p(s), \lambda(s))$ if $s$ is a leaf

A run is accepting if $\delta(E) \in F$
Extended DTDs $d$ over labelings alphabet $\Sigma$ is a DTD $d'$ over an alphabet $\Sigma' \geq \Sigma$ together with a mapping $\mu: \Sigma' \to \Sigma$

$\Sigma' = \{ \text{root, new, used, car}, \text{car}_{n}, \text{car}_{o}, \text{was}, \text{mil} \}$

$\mu: \{ \text{root} \to \text{new, old}, \text{car} \to \text{car}_{n}, \text{car} \to \text{car}_{o} \}$

$\text{car} \to \text{price, model, was? mil?}$
A tree $T = (\mathcal{D}, \lambda')$ conforms to an extended DTD $\Sigma'$ if there is a tree $T' = (D, \lambda')$ that conforms to $\Sigma'$ s.t.

for each node $s$, $\lambda(s) = \mu(\lambda'(s))$.

\[ \mu(a) \]

\[ \lambda'(s) \]
Extended DTD = unranked tree automata


Theorem A set of unranked trees is definable in MSO if and only if it is regular, i.e., given by an unranked tree automaton.
Unranked trees as structures

\( T = (D, \lambda) \)

\( \Omega_T = (D, \prec_{ch}, \prec_{ns}, (Pa_a)_{a \in \Sigma}) \)

Trees are ordered

\( M'_T = (D, \prec_{fc}, \prec_{ns}, (Pa_a)_{a \in \Sigma}) \)

first child