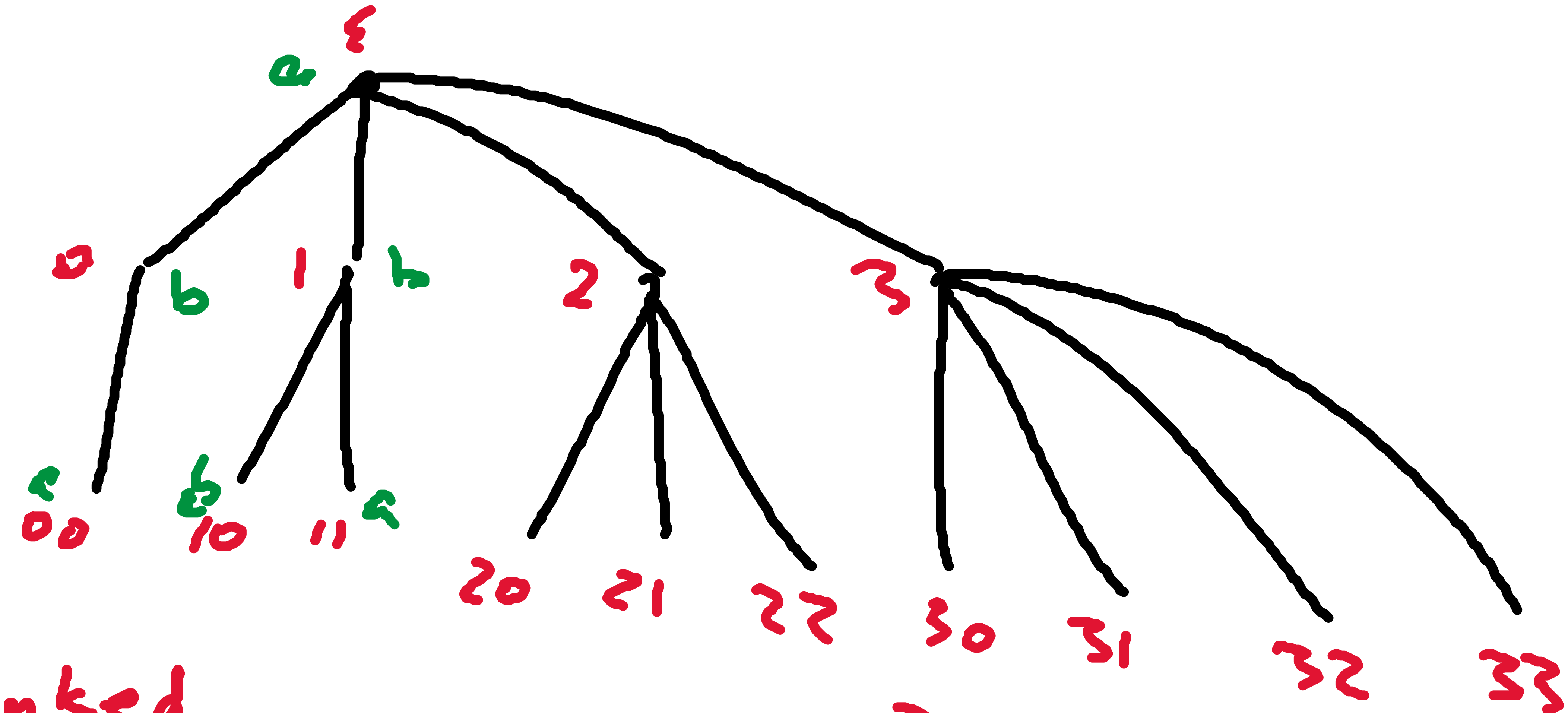


Unranked

Tree domain is a subset D of $\mathbb{N}^*$
such that
1. D is prefix-closed $s \in D,\ s' \preceq s \Rightarrow s' \in D$
2. if $s \cdot (i+1) \in D \Rightarrow s \cdot i \in D$

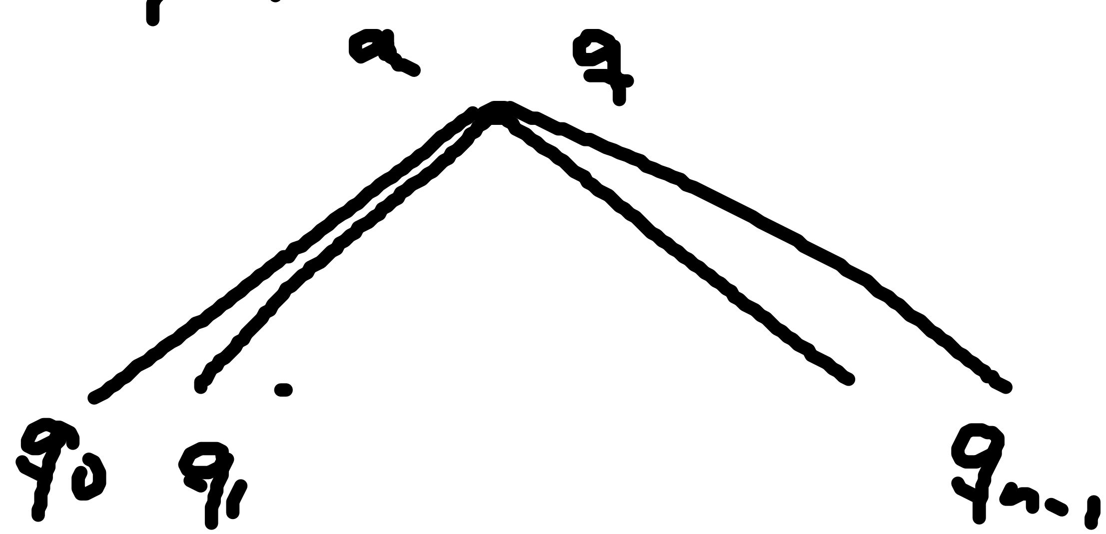
Unranked Tree (D, λ) s.t. D is a tree domain, $\lambda\ D \to \Sigma$

Unranked tree automata

$A = (Q, F, \delta)$

$\delta \quad Q \times \Sigma \longrightarrow 2^{Q^*}$, $\delta(q,a) \leq Q^*$

each $\delta(q,a)$ is a regular language



$q_0 \quad q_{n-1} \in \delta(q,a)$

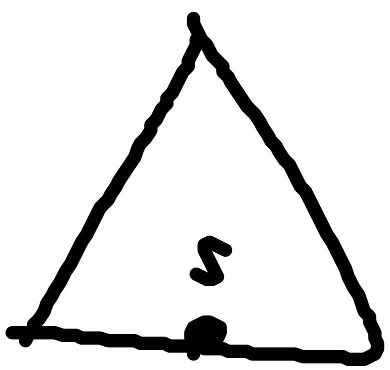
A run is a mapping $\rho : D \to Q$

on a tree (D, λ)

such that

for each node s with children $s.0, \ldots, s(n-1)$

$$\delta(\rho(s), \lambda(s)) \ni \rho(s0) \cdots \rho(s(n-1))$$



$\varepsilon \in \delta(\rho(s), \lambda(s))$ if s is a leaf

A run is accepting if $\rho(\varepsilon) \in F$

Extended DTDs d over labeling alphabet Σ is a DTD d' over an alphabet $\Sigma' \supseteq \Sigma$ together with a mapping $\mu \; \Sigma' \to \Sigma$

~~root → new, used~~
~~new → car*~~
~~used → car*~~
~~car → price, model, was? mil?~~

$\Sigma' = \{$root, new, used, car_n, car_o, was, mil

$car_n, car_o \xrightarrow{\mu} car$

root → new, old
new → car_n^*
old → car_o^*

car_n → pr, model, was
car_o → pr, model, mil

A tree $T^{=(D,\lambda)}$ conforms to an extended DTD
over Σ over Σ'

if there is a tree $T' = (D, \lambda')$ that
conforms to d' s.t

for each node s, $\lambda(s) = \mu(\lambda'(s))$

Extended DTD = unranked tree automata

(1967, rediscovered 1999, 2000, 2001,)

<u>Theorem</u> A set of unranked trees is definable in MSO $\Longleftrightarrow$ it is regular, i.e. given by an unranked tree automaton

Unranked trees as structures

$T = (D, \lambda)$

$$\mathfrak{M}_T = (D, <_{ch}, <_{ns}, (P_a)_{a \in \Sigma})$$

$s \cdot i <_{ns} s \cdot (i+1)$

Trees are ordered

$$\mathfrak{M}'_T = (D, <_{fc}, <_{ns}, (P_a)_{a \in \Sigma})$$

fc - first child