Infinite, or ω-words over alphabet Σ

is a sequence

$a_0 a_1 a_2$ st each $a_i \in \Sigma$

Can view as a map $w \; \mathbb{N} \to \Sigma$

$w(i)$ is the ith position in the ω-word

Set of all ω-words is denoted by Σ^ω

Automata for ω-words

$A = (Q, Q_0, \delta, Acc)$

states, initial states, transition, acceptance

(Q, Q_0, δ) - just as for NFAs $\quad \delta\ \Sigma \times Q \to 2^Q$

Run of A on $w = a_0 a_1$ $\quad \rho\ \mathbb{N} \to Q$

1 $\rho(0) \in Q_0$

2 $\rho(i+1) \in \delta(\rho(i), a_i)$

$$\begin{array}{cccc} a_0 & a_1 & a_2 & a_3 \\ \hline q_0 & q_1 & q_2 & q_3 \end{array}$$

Buchi Automata

Acc $\quad F \subseteq Q$ set of final (accepting) states

$a_0 a_1 a_2$

$q_0 \, q_1 \, q \, q_3 \, q_4 \, q_5 \quad q \ldots q \quad q_{75} \quad q$

nonaccepting $\notin F$

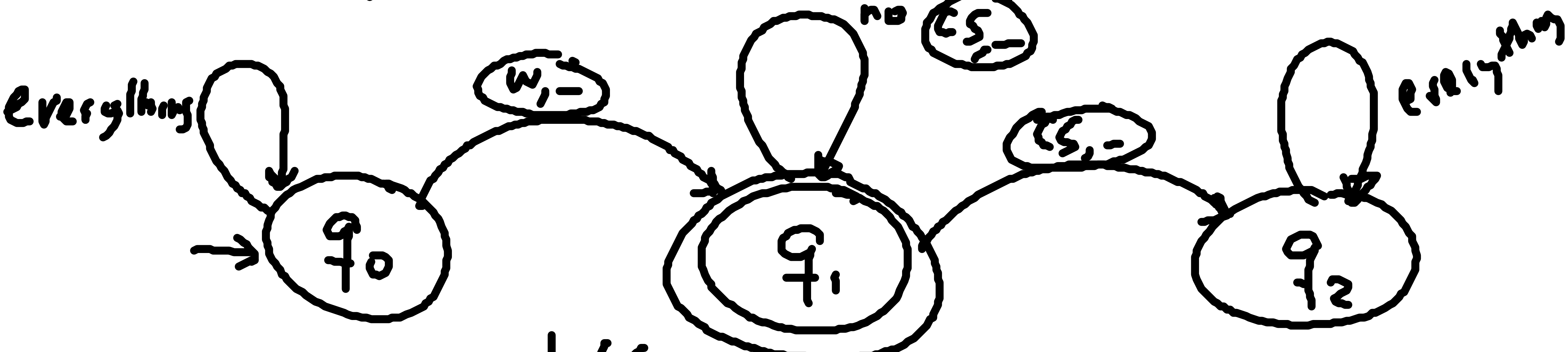
in F

For a run ρ, $Inf(\rho)$ is the set of states that occur infinitely often in ρ

A run ρ is accepting if $Inf(\rho) \cap F \neq \emptyset$

Liveness if (w,_) then eventually (cs,_)

$\neg$ $\exists$ moment when we see (w,_) and from that point on we never see (cs,_)



1 $q_0 \; q_0 \; q_0$ $\quad Inf(\rho) = \{q_0\}$ ✓

2 $q_0 \; q_0 \; q_1 \; q_1 \; q_1 \; q_2 \; q_2$ $\quad Inf(\rho_2) = \{q_2\}$ ✓

3 $q_0 \; q_0 \; q_1 \; q_1 \; q_1$ $\quad Inf(\rho) = \{q_1\}$ ✓

ω-regular languages =
accepted by nondeterministic Buchi automata

(Q, Q_0, F, δ)

- closure prop
- decision proc
- determinization
- reg expr