

Closure properties

- union
- intersection
- complement

Decidability

- nonemptiness
- universality

Closure under union

$$A^1 = (Q^1, q_0^1, F^1, \delta^1)$$

$$A^2 = (Q^2, q_0^2, F^2, \delta^2)$$

$$Q^1 \cap Q^2 = \emptyset$$

$$A^1 \cup A^2 = (Q^1 \cup Q^2, q_0^1 \cup q_0^2, F^1 \cup F^2, \delta: \delta^1 \cup \delta^2)$$

Intersection

finite words \rightarrow product construction

$$A^1 = (Q^1, q_0^1, F^1, \delta^1)$$

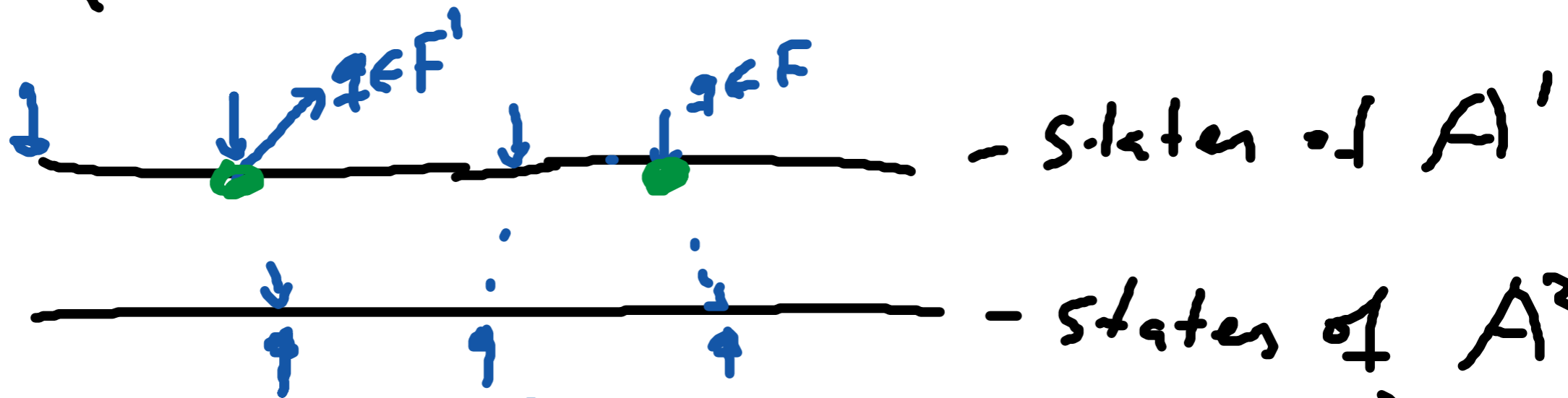
$$A^2 = (\dots)$$

$$A^1 \times A^2 = (Q^1 \times Q^2, q_0^1 \times q_0^2, F^1 \times F^2, \delta^1 \times \delta^2)$$

$$\delta((q_1, q_2), a) = \left\{ (q_1', q_2') \mid \begin{array}{l} q_1' \in \delta^1(q_1, a) \\ q_2' \in \delta^2(q_2, a) \end{array} \right\}$$

$$A' \times A^2 = (Q, Q_0, F, \delta)$$

$$Q = Q^1 \times Q^2 \times \{1, 2\}$$



$$Q^0 = Q_0^1 \times Q_0^2 \times \{1\}$$

$$F = F^1 \times Q^2 \times \{1\}$$

$$q_1' \in \delta^1(q_1, a) \quad q_2' \in \delta^2(q_2, a)$$

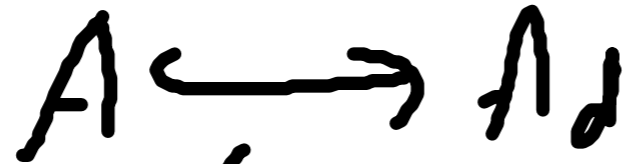
$$(q_1' q_2' 1) \in \delta((q_1, q_2, 1), a)$$

$$(q_1' q_2' 2) \in \delta((q_1, q_2, 1), a) \quad q_1 \in F^1$$

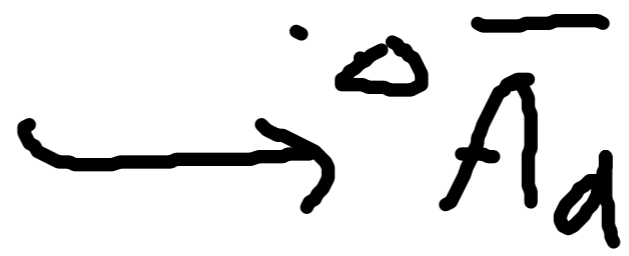
$$(q_1' q_2' 1) \in \delta((q_1, q_2, 2), a) \quad q_2 \in F^2$$

Complementation

finite



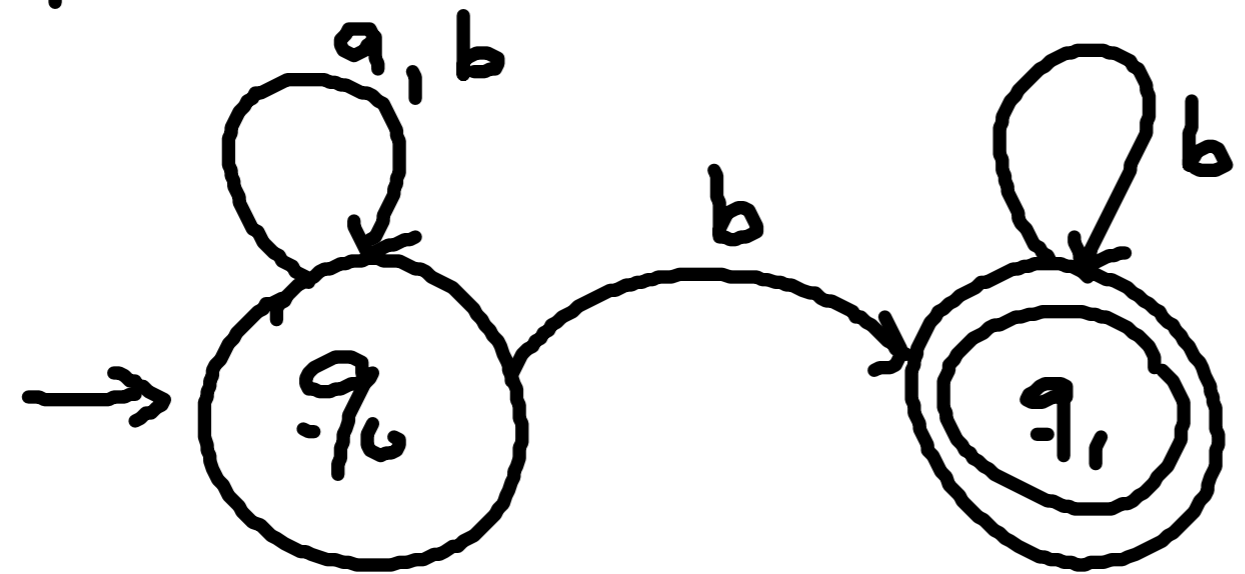
det



changing acc / rej states

powerset constr

A



$\underbrace{\quad}_{L_w(A)}$ bb . b
 finitely many a's.

Fact Deterministic Buchi automata are strictly weaker than Buchi aut

\exists language L that is not accepted by a det. BA
 $L = w$ -words with finitely many a 's

