Rabin acceptance condition is a set of pairs of states $\mathcal{R} = \{(U_1, V_1), \ldots (U_k, V_k)\}$

$U_i, V_i \subseteq Q$

A run ρ satisfies $\mathcal{R}$ if for each i,

$$\mathrm{Inf}(\rho) \cap U_i \neq \emptyset \text{ and } \mathrm{Inf}(\rho) \cap V_i = \emptyset$$

Rabin automaton $(Q, Q_0, \delta, \mathcal{R})$

Buchi aut with acc states F = Rabin aut $\mathcal{R} = \{(F, \emptyset)\}$

Muller acceptance condition $\mathcal{F} \subseteq 2^Q$

ρ is accepting if $\mathrm{Inf}(\rho) \in \mathcal{F}$

$(Q, Q_0, \delta, \mathcal{F})$ – Muller automata

Let A be a det. Muller automaton $(Q, q_0, \delta, \mathcal{F})$

$\delta \quad Q \times \Sigma \to Q$

$\overline{A} = (Q, q_0, \delta, \overline{\mathcal{F}}) \quad \overline{\mathcal{F}} = 2^Q - \mathcal{F}$

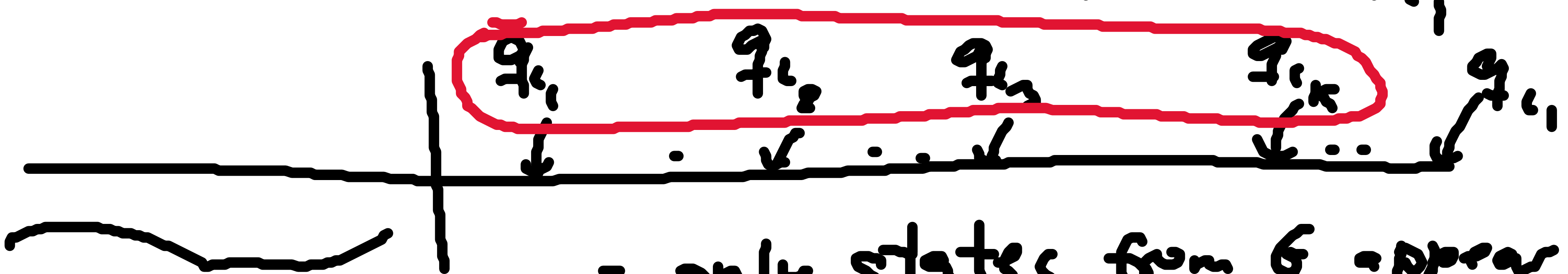
$L_\omega(\bar{A}) = \Sigma^\omega - L_\omega(\bar{A})$

$A_m = (Q, Q_0, \delta, \mathcal{F})$ $\qquad \delta: Q \times \Sigma \to 2^Q$

$\mathcal{F} \subseteq 2^Q$

need $A_b = (Q', Q_0', \delta', F)$ st $L_w(A_m) = L_w(A_b)$

ρ is accepting, $\mathrm{Inf}(\rho) = G \subseteq Q$, $G = \{q_{i_1}, \ldots q_{i_k}\}$



states from $Q - G$ only appear here

- only states from G appear
- all appear infinitely often

q_{i_1} $\qquad q_{i_k}$

Theorem (Safra ≈ 1988)

For every nondet Buchi automaton A with n states, there is (and can be constructed) an equivalent det Rabin automaton with

- $O(n)$ ($2n$) pairs in the acc condition
- $n^{O(n)}$ states

powerset const → 2^n Safra → $2^{O(n \log n)}$

Corollary ω-reg languages are closed under complement

Theorem (Büchi)

A language $L \subseteq \Sigma^\omega$ is ω-regular $\iff$ it is definable in MSO

$\varphi \in MSO \xrightarrow[\text{algorithm}]{} A_\varphi$

$w = a_0 a_1 a_2 \longrightarrow \mathfrak{M}_S = (\mathbb{N}, <, (P_a)_{a \in \Sigma})$

$\Rightarrow$ (easy direction) code Büchi automata

$\exists X_0 \quad X_{n-1}$

acc. condition

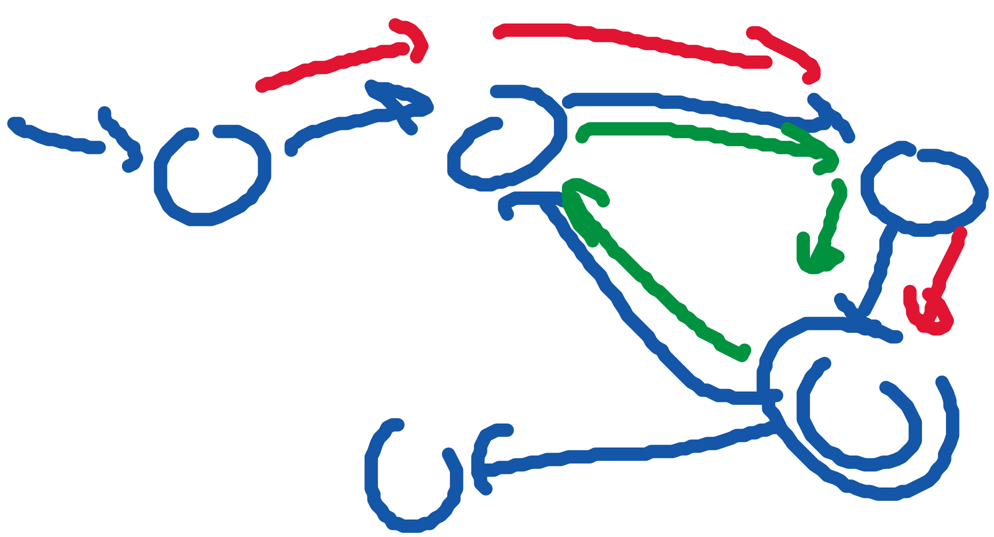
$\bigvee_{q_i \in F} \forall x \exists y \, (y > x \wedge X_i(y))$

$\Leftarrow$ (hard)
EXACTLY THE SAME

Corollary Satisfiability of MSO over ω-words is decidable

$$\varphi \in MSO \longrightarrow A_\varphi$$

$$\varphi \text{ has a model} \iff L_\omega(A_\varphi) \neq \emptyset$$ $\rightarrow$ decidable in polytime in logspace



$\exists$ path from q_0 to $q \in F$

$\exists$ path from q to q