Rabin acceptance condition is a set of pairs of states \( R = \{(U_i, V_i), (U_k, V_k)\}\). 

\( U_i, V_i \leq \varnothing \)

A run \( p \) satisfies \( R \) if for each \( i \),

\( \text{Inf}(p) \cap U_i \neq \varnothing \) and \( \text{Inf}(p) \cap V_i = \varnothing \)

Rabin automaton \( (Q, Q_0, \delta, R) \)

Büchi automaton with accepting states \( F = \text{Rabin aut} \) \( R = \{(F, \varnothing)\} \)
Muller acceptance condition $F \subseteq 2^Q$

$p$ is accepting if $\inf \{ f(p) \} \subseteq F$

$(Q, Q_0, \delta, F)$ - Muller automata

Let $A$ be a det. Muller automaton $(Q, q_0, \delta, F)$

$A = (Q, q_0, \delta, F) \quad \overline{F} = 2^Q - F$

$L_w(\overline{A}) = \Sigma^\omega - L_w(\hat{A})$
$A_m = (Q, q_0, \delta, F)$ s.t. $Q \times \Sigma \rightarrow 2^Q$

need $A_b = (Q', q'_0, \delta', F)$ s.t. $L_w(A_m) = L_w(A_b)$

$P$ is accepting, $L_w(P) = G \subseteq Q$, $G = \langle q_{i_1}, f_1, \ldots, f_k \rangle$

- only states from $G$ appear
- all appear infinitely often

$Q - G$ only appear here
Theorem (Sadri & 1988)

For every nonded Buchi automaton $A$ with $n$ states, there is (and can be constructed) an equivalent deterministic Rabin automaton with

- $O(n)$ $(2n)$ pairs in the acc condition
- $n^0(n)$ states

powered const $\rightarrow 2^n$

Sadri $\rightarrow 2^{O(n/\log n)}$

Corollary: w-reg languages are closed under complement
Theorem (Büchi.)

A language \( L \subseteq \Sigma^* \) is \( \omega \)-regular \( \iff \) it is definable in MSO

\( \varphi_{\text{MSO}} \xrightarrow{\text{algorithm}} A \varphi \)

\( N = a_0a_1a_2 \quad \rightarrow \quad \Omega^N_5 = (N_1 <, (Pa)_{a \in \Sigma} ) \)

\( \Rightarrow \) (easy direction) code Büchi automata

\( \exists X_0 X_{n-1} \)

\( \Leftarrow \) (hard) exactly the same

\( \Rightarrow \) (hard) exactly the same

all condition

\( \forall x \exists y (y > x \land X_1(y)) \)

q1eF
Corollary: Satisfiability of MSO over \( w \)-words is decidable

\[ \varphi \in \text{MSO} \rightarrow A\varphi \]

\( \varphi \) has a model \( \iff L_w(A\varphi) \neq \emptyset \) \( \rightarrow \) decidable

- in polytime
- in log space

\[ \exists \text{path from } q_0 \text{ to } q \in F \]

\[ \exists \text{path from } q \text{ to } q \]