Thm The complement of every ω-regular language is accepted by a nondet. Büchi automaton

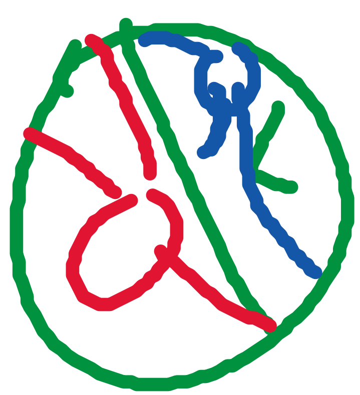
L – ω-regular language

Construct a partition $C_1 \quad C_n$ of Σ^ω s.t

(1) each C_i is regular

(2) if $C_i C_j^\omega \cap L \neq \emptyset \Rightarrow C_i C_j^* \subseteq L$

n^2 sets $C_i C_j^\omega$



$\Rightarrow \bar{L}$ is a union of sets $C_i C_j^\omega$ (ω-regular)

$\bar{L}$ is ω-regular

$X \subseteq \Sigma^*$ $\quad X^\omega = \{x_1 x_2 x_3 \ldots \mid x_i \in X\}$

$Y \subseteq \Sigma^*$ $\quad Y \cdot X^\omega = \{y x_1 x_2 \ldots \mid x_i \in X, y \in Y\}$

Lemma If $C_1 \ldots C_n$ form a partition of Σ^*

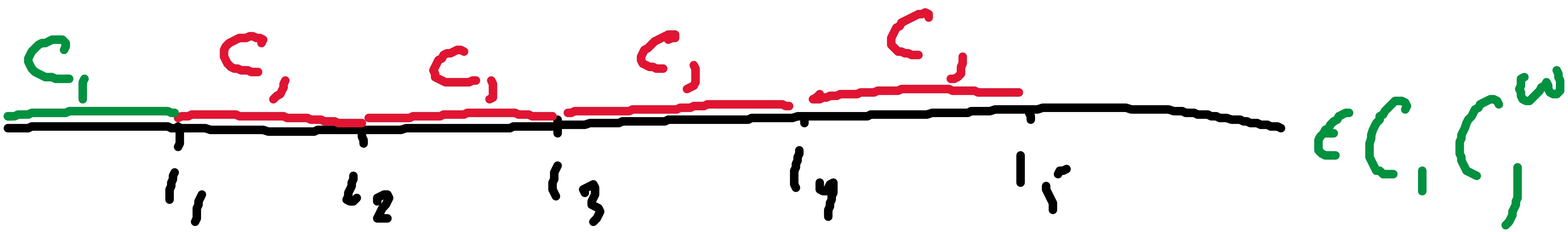
then $\bigcup C_i C_j^\omega = \Sigma^\omega$

Partition ordered pairs (k, l), $k < l$ into n sets

(k, ℓ), $k < \ell$ are partitioned into n classes

There is an infinite subset i_1, i_2, i_3

$i_1 < i_2 < i_3 <$ s.t. all pairs (i_p, i_r) belong to the same class j



Ramsey Theorem

Let X be an infinite set ordered by $<$

Let $A_1 \ldots A_m$ be a partition of the set of ordered tuples $(x_1 \ldots x_n)$, $x_1 < \ldots < x_n$

Then there is an infinite set $Y \subseteq X$ s.t. all tuples $(y_1 \ldots y_n)$, $y_i \in Y$, $y_1 < \ldots < y_n$ belong to one class of the partition

L is accepted by $A = (Q, q_0, \delta, F)$

$q, q' \in Q$ $\quad$ $A[q, q']$ be the NFA $(Q, q, \delta, \{q'\})$

$s \sim_1 s'$ if s and s' "agree on all $A(q, q')$" $\quad$ q $\quad$ q'

$s \sim_2 s'$ if $\forall q\, q'$ $\quad$ there is a run on s that starts in q, ends in q' and goes thru a state in F

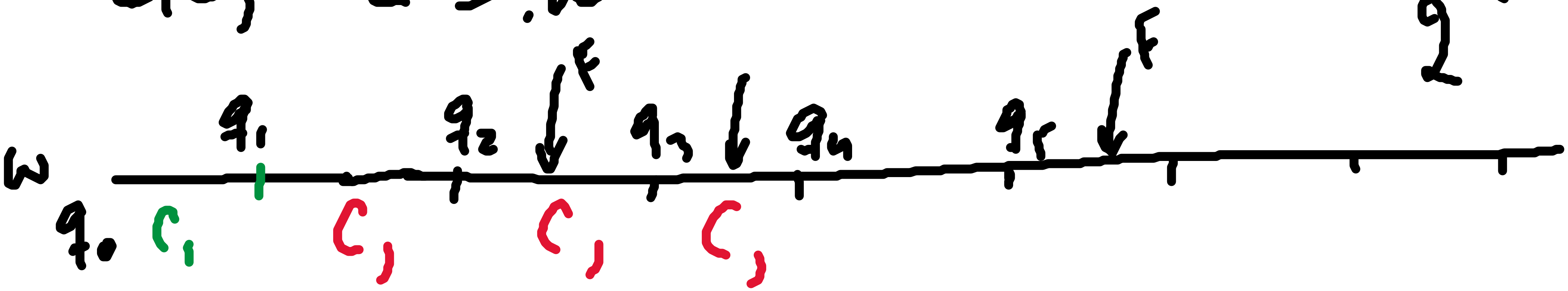
$\Updownarrow$

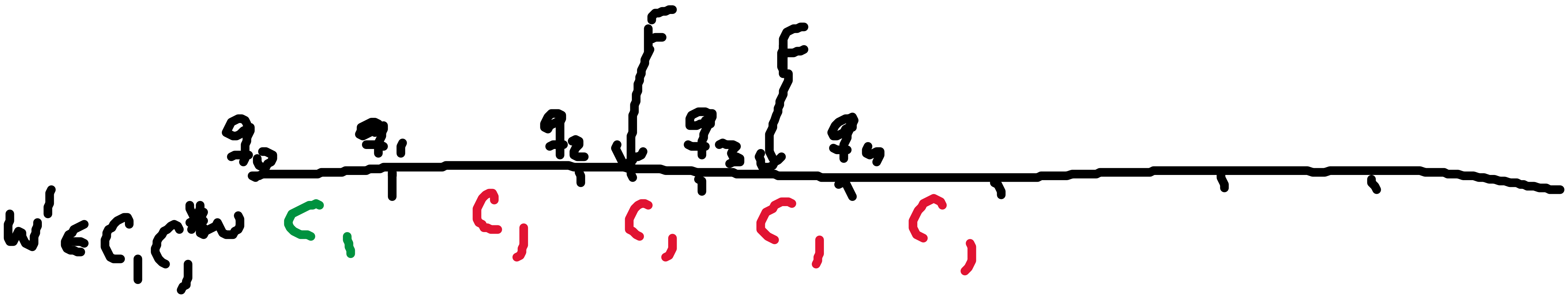
same is true for s'

$s \sim s' \iff s \sim_1 s'$ and $s \sim_2 s'$

$C_1 C_j^{\omega} \cap L \ni w$

$2^{O(n \log n)}$





$Inf(\rho) \cap F \neq \emptyset \Rightarrow w' \in L$

Hence $C_1 C_j^{\omega} \subseteq L$

Complexity

A has $\underline{n}$ states then $\bar{A}$ has $2^{O(n^2)}$ states