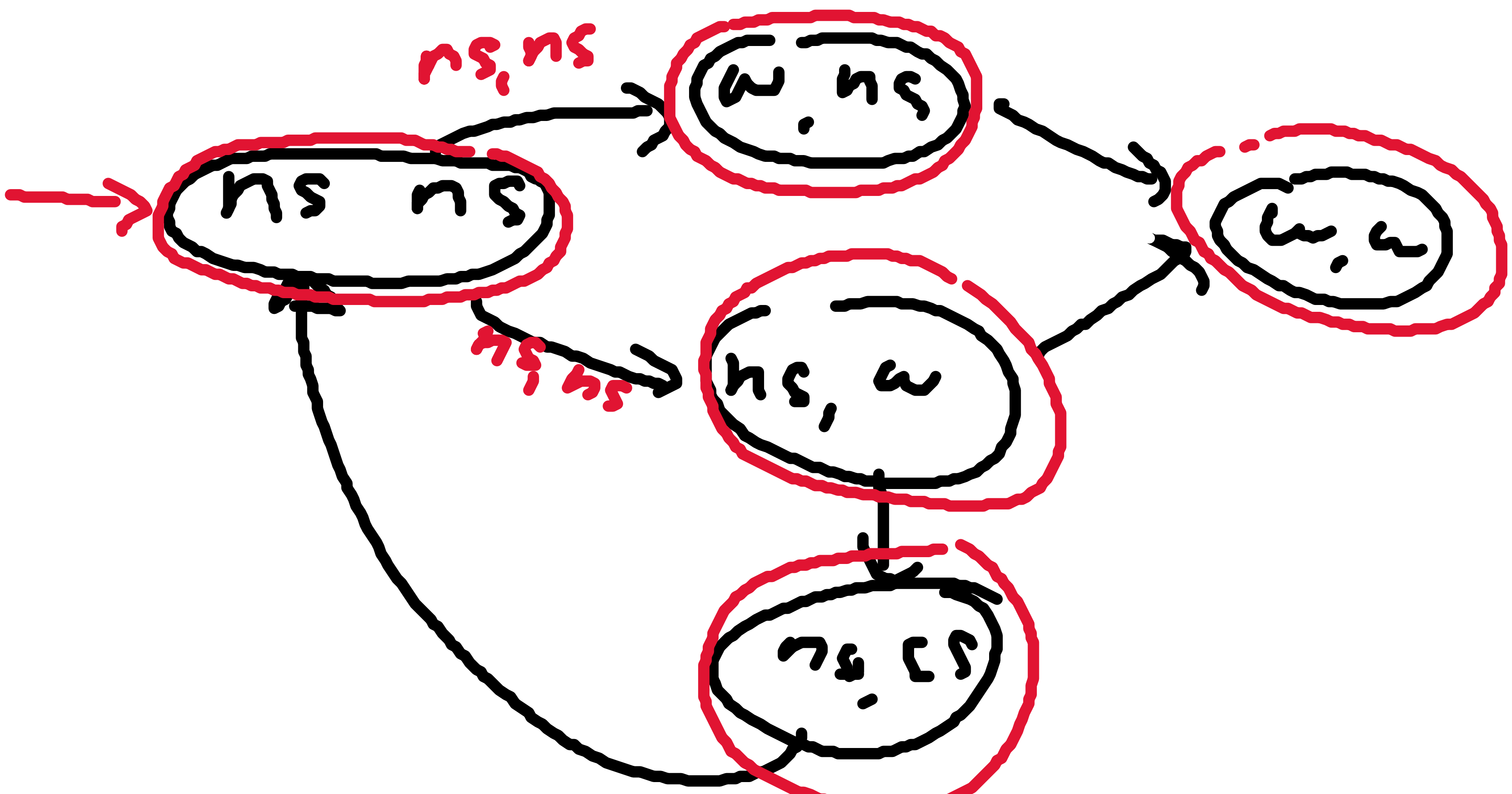
Büchi: L is ω-regular $\iff$ L is definable in MSO

$\varphi \in$ MSO $\longrightarrow A_\varphi$

$w \in \Sigma^\omega$ $\quad \mathfrak{M}_w \models \varphi \iff A_\varphi$ accepts w



Model of a program P   Buchi automaton A_P
(all states are accepting)

Specifications   Safety (never in a bad state)

Liveness (if in "wait" then eventually enter CS)

Are expressible in MSO

$$\forall x \bigvee_{a=(w,-)} P_a(x) \rightarrow \exists y \left(y > x \wedge \bigvee_{b=(CS,-)} P_b(y)\right)$$

Program P $\quad A_P$

Specification $\varphi \in MSO$

$P \models \varphi$? $\rightarrow$ every possible behaviour of P satisfies φ

If $P \not\models \varphi$ $\rightarrow$ an example of a behaviour of P that violates φ.

$$\boxed{L_\omega(A_P \times A_{\neg\varphi}) \neq \emptyset}$$

$A_P \times A_{\neg\varphi}$ $\Longrightarrow$ hopeless for MSO

model of the program $\nearrow$ A_P

$\psi \in MSO \longrightarrow A_\psi$

A_ψ $\to$ Muller $\to$ determ $\to$ into Buchi $\to$ $A_{\neg\psi}$

$2^{O(n\log n)}$ $\Longleftarrow$ Muller

$=$ $n^{O(n)}$

$\Omega\left(2^{2^{2^{2}}}\right)$ neg pos depth

$s \in \Sigma^*$ $\quad A$ is an NFA

$s \in L(A)$ $\quad O(p(|A|) \cdot |s|)$ $\quad p$ is a polynomial

$s \in \Sigma^*$ $\quad \varphi \in MSO$

$\mathfrak{M}_s \models \varphi$ $\quad$ Assume $\exists k$ s.t we can do it

in $$O\Big(\left.2^{2^{2^{\cdot^{\cdot^{2^{|\varphi|}}}}}}\right\}k \; |s|\Big)$$

Then (Frick/Grohe) $\quad P = NP$

Logic LTL (linear-time temporal logic)

Theorem (Kamp '69) LTL = First Order logic

Theorem (Vardi/Wolper '86) Every LTL formula φ can be converted into an equivalent Buchi automaton of size $2^{O(|\varphi|)}$

AP - atomic propositions (ns_1, w_1, cs_1
$ns_2\ w_2\ cs_2$)

$\Sigma = 2^{AP}$ Σ^ω

Syntax of LTL (future LTL)

$$\varphi, \psi ::= a \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \neg \varphi \mid X\varphi \mid \varphi U \psi$$

$a \in \Sigma$ "next" φ φ "until" ψ

Semantics $w \in \Sigma^\omega,\ i \in \mathbb{N}$

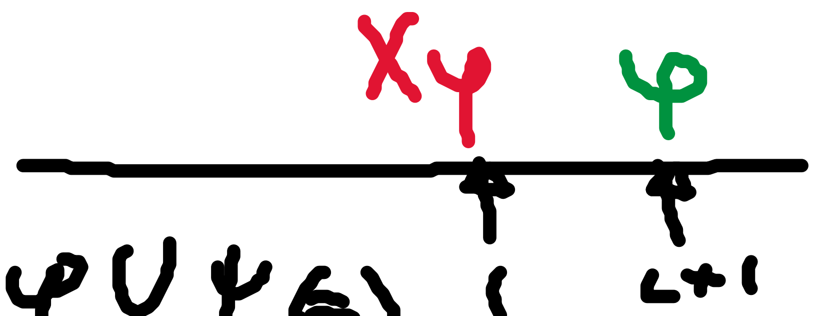
$(w, i) \models \varphi$

$\varphi, \psi ::= a \mid \varphi \vee \psi \mid \neg \varphi \mid X\varphi \mid \varphi \, U \, \psi$

$(w, i) \models a$ if $a \in$ label of the ith position

$(w, i) \models \varphi \vee \psi \iff (w, i) \models \varphi$ or $(w, i) \models \psi$

$(w, i) \models \neg \varphi \iff (w, i) \not\models \varphi$



$(w, i) \models \varphi \, U \, \psi \iff$
$\exists j \geq i$ s.t.
$(w, j) \models \psi$ and
$(w, k) \models \varphi$ for all $i \leq k < j$