Closure of a formula

$Cl(a) = \{a\}$ $\quad Cl(\neg a) = \{\neg a\}$

$Cl(\varphi \vee \psi) = \{\varphi \vee \psi\} \cup Cl(\varphi) \cup Cl(\psi)$

$Cl(X\varphi) = \{X\varphi\} \cup Cl(\varphi)$

$Cl(\varphi U \psi) = \{\varphi U \psi\} \cup Cl(\varphi) \cup Cl(\psi)$

R

$G \neg a \leftrightarrow \neg F a \leftrightarrow \neg T U a \leftrightarrow F R \neg a$

$Cl(G \neg a) = \{G \neg a, F, \neg a\}$ $\quad Cl(T) = \{T\}$

$Cl(F) = \{F\}$

Construction of A_φ

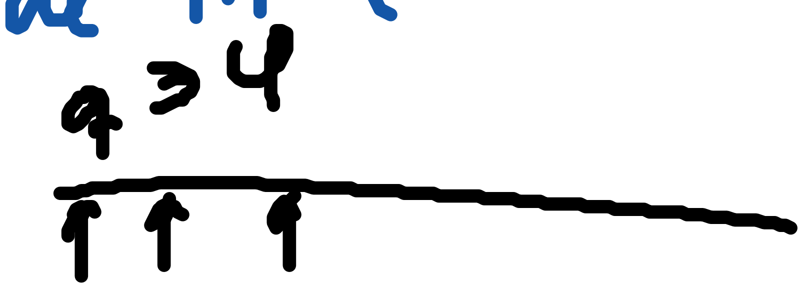
- States are consistent subsets of $Cl(\varphi)$
- Transition ensures that they are locally consistent

  if $X\varphi$ is true in i, then φ is true in $i+1$
- Acceptance condition ensures global consistency

  if $\varphi U \psi$ is true in i, ψ is true in $j \geq i$

In a successful run, if α is in the state assigned to i, then α is true in i

- initial states contain φ



Consistency

$X \subseteq Cl(\varphi)$ is consistent if

1. if $\alpha \in X$ then $\neg\alpha \notin X$ and
   if $\neg\alpha \in X$ then $\alpha \notin X$
2. if $\alpha \vee \beta \in X$ then $\alpha \in X$ or $\beta \in X$
3. if $\alpha \wedge \beta \in X$ then $\alpha \in X$ and $\beta \in X$
4. $F \notin X$

Transition $\Sigma = 2^{AP}$

$q' \in \delta(q, \bar{a})$ $\qquad$ $q, q' \subseteq Cl(\varphi)$

$\bar{a} \subseteq AP$

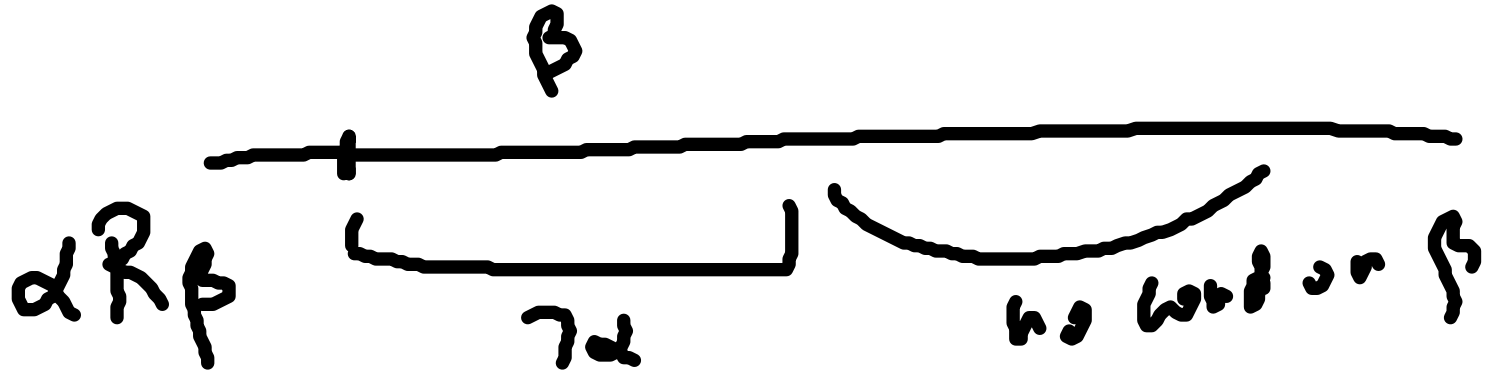
1 if $a \in q$ then $a \in \bar{a}$

2 if $\neg a \in q$ then $a \notin \bar{a}$

3 If $X\alpha \in q$ then $\alpha \in q'$

4. If $\alpha U \beta \in q$ then either $\beta \in q$ or $\alpha \in q$ and $\alpha U \beta \in q'$

5 If $\alpha R \beta \in q$ then $\beta \in q$ and ($\alpha \in q$ or $\alpha R \beta \in q'$)



$\alpha R \beta$ never promises that α will be true

BUT $\alpha U \beta$ promises that β will be true!

The run as defined so-far does not ensure that if $\alpha U \beta \in q$ and q is assigned to position i, then $\beta \in q'$ assigned to $j \geq i$

Potentially bad situation

$\alpha U \beta \in cl(\varphi)$, a run ρ s.t.

from some point on, $\alpha U \beta$ occurs without β ever occurring

$$F_{\alpha U \beta} = \{ q \mid \text{1. both } \alpha U \beta \text{ and } \beta \text{ are in } q \text{ or 2. } \alpha U \beta \notin q \}$$

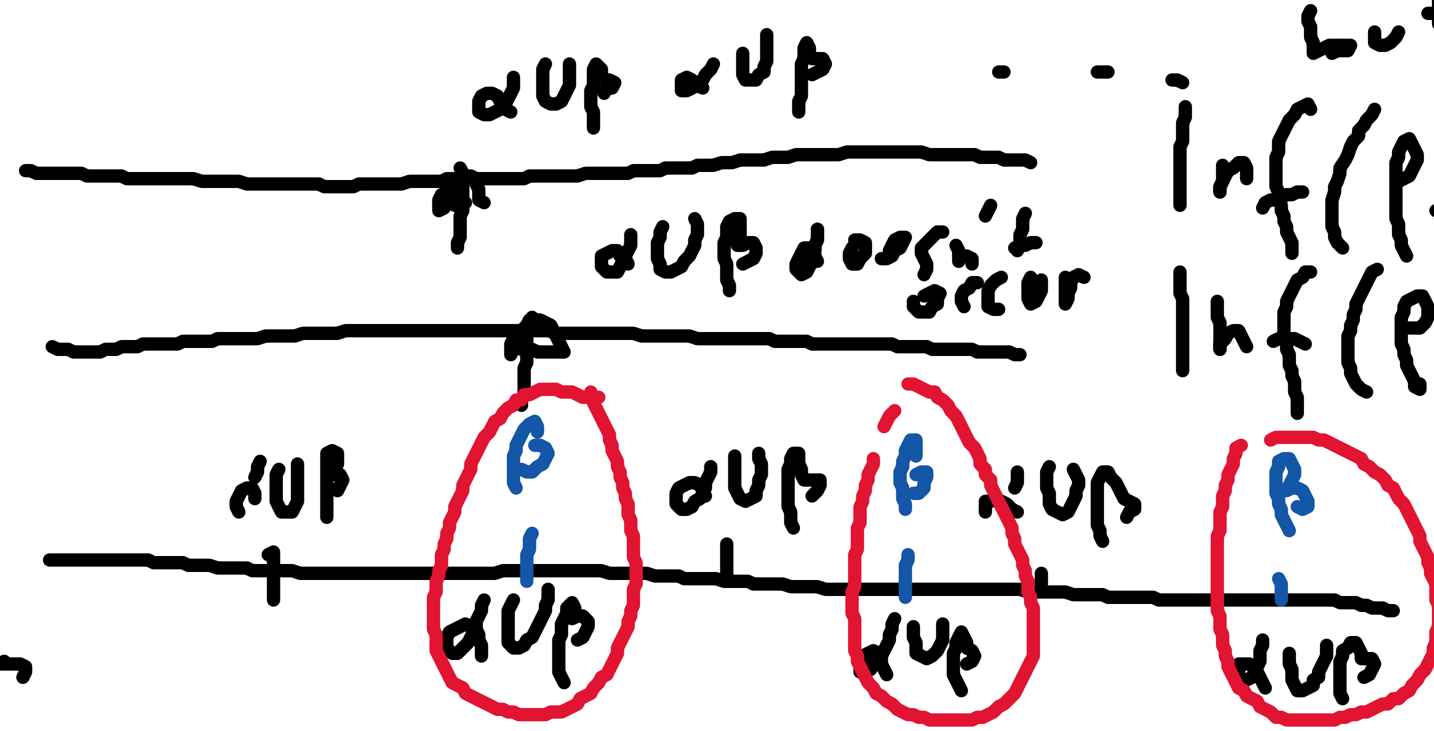
but never see β

Bad case ρ:

Good case$_1$

Good case$_2$

$\alpha U \beta$ occurs inf often



$Inf(\rho) \cap F_{\alpha U \beta} = \emptyset$

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A run ρ fullfills each $\alpha U \beta$ promise (i.e. if $\alpha \underline{U} \beta$ is in a state in position i, then β is in a state in pos $j \geq i$)

$$\Longleftrightarrow$$

$$\mathrm{Inf}(\rho) \cap F_{\alpha U \beta} \neq \emptyset$$

$\alpha_1 U \beta_1, \alpha_2 U \beta_2 . \quad \alpha_k U \beta_k \in cl(\varphi)$

Run ρ s.t. $\forall i, \ \mathrm{Inf}(\rho) \cap F_{\alpha_i U \beta_i} \neq \emptyset$

Multi-Buchi automaton

$(Q, Q_0, \delta, F_1 \quad F_k)$

a run ρ is accepting iff $\mathrm{Inf}(\rho) \cap F_i \neq \emptyset \ \forall i$

$\Downarrow$

Buchi aut

#states $\leq 2^{|\varphi|}$ for Buchi aut, #states $\leq 2^{|\varphi|} O(|\varphi|)$

$= 2^{O(|\varphi|)}$