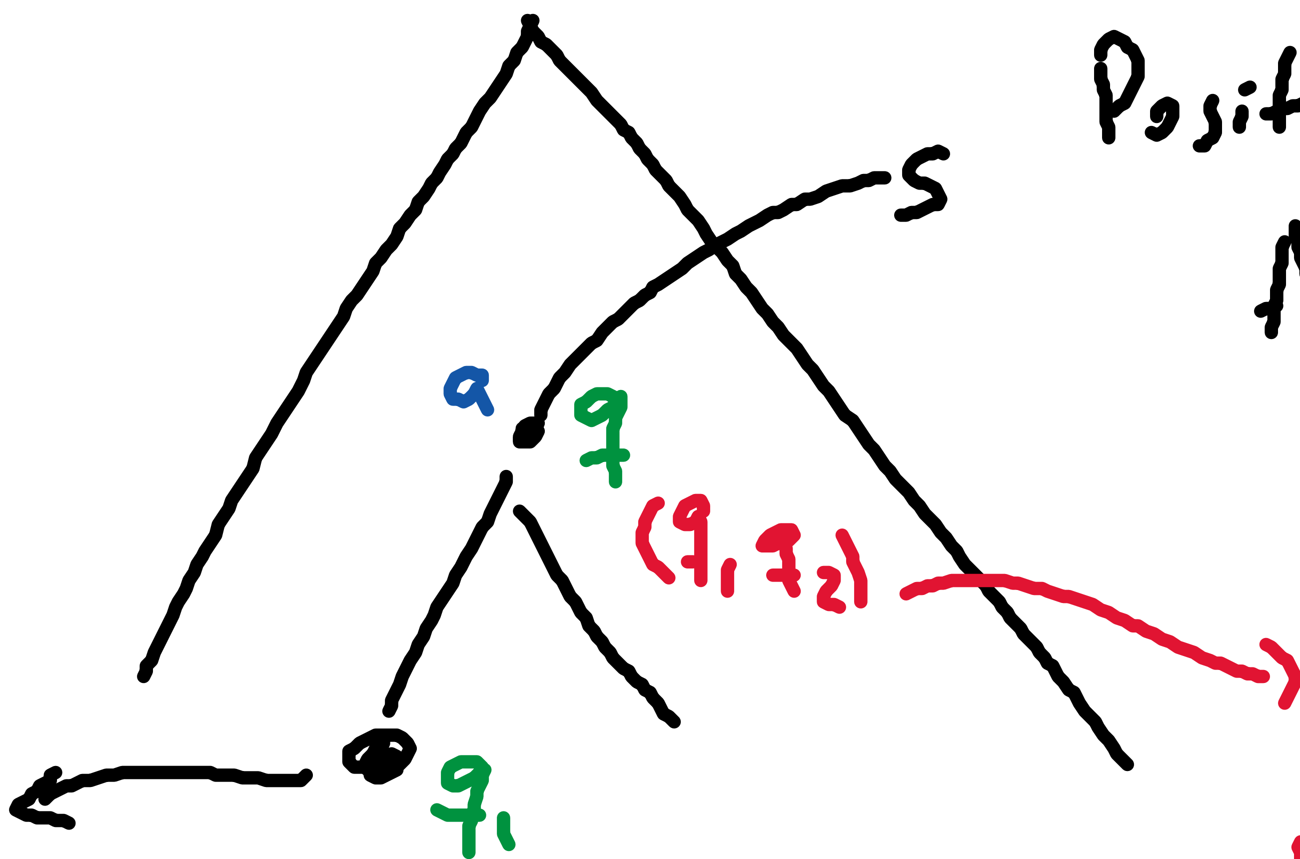
Rabin's Complementation

For every TA $\mathcal{A}$ over infinite trees, one can effectively construct an automaton $\bar{\mathcal{A}}$ that accepts the complement of the language accepted by $\mathcal{A}$

1. Games / winning strategies
2. Parity acceptance condition
3. Determinacy $\Longrightarrow$ complementation
4. Finite games $\Longrightarrow$ decidability of nonemptiness

2 players Automaton / Pathfinder



Position for ~~at~~ automaton

Aut chooses a pair

$(q_1, q_2) \in \delta(q_1, a)$

position for Path

Pathfinder chooses direction 0 or 1

new position for Auth

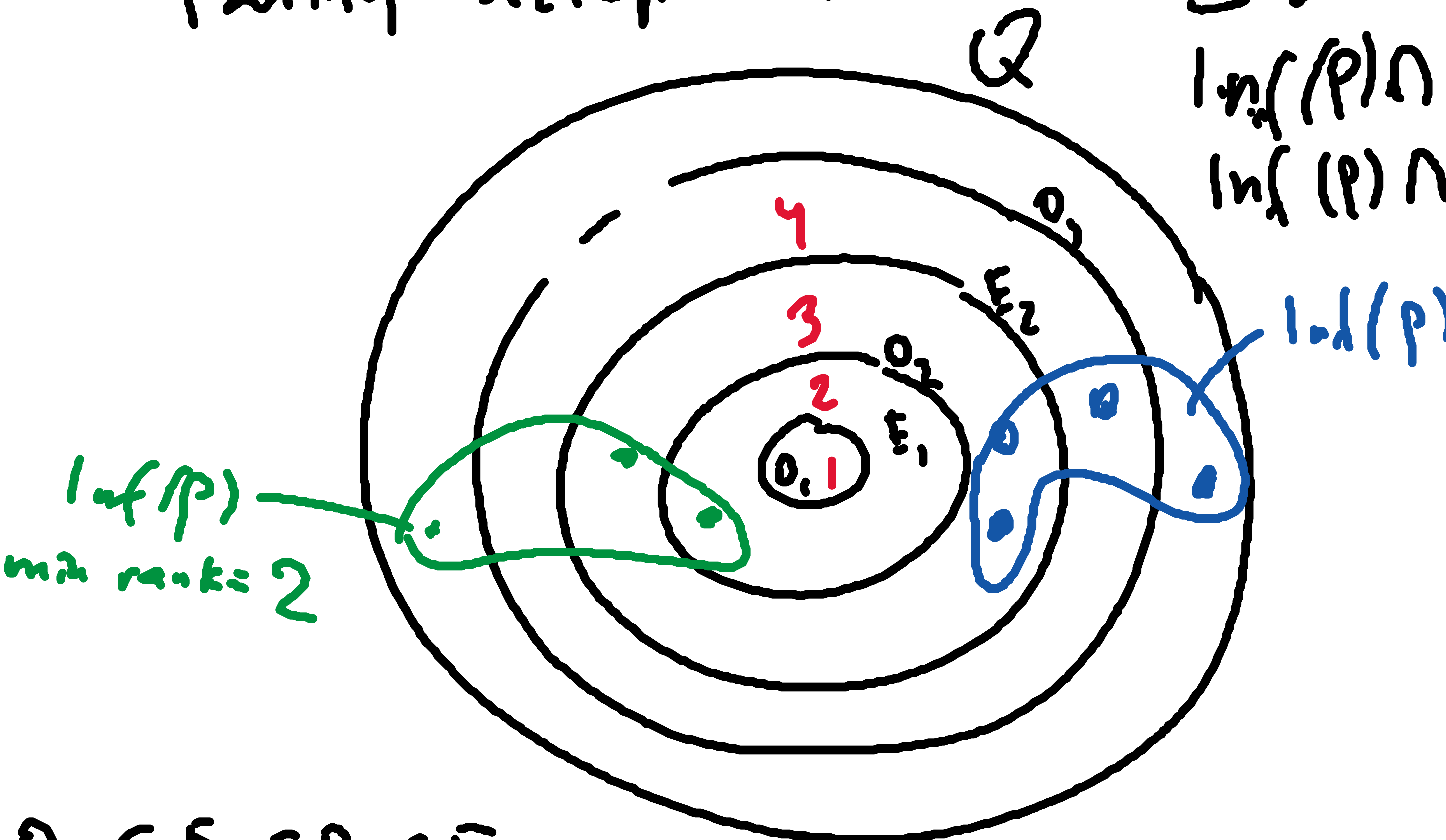
Aut wins if for each infinite branch, the set of states seen along this branch satisfies Acc

Parity acceptance

$\exists\, i$

$\mathrm{Inf}(\rho) \cap E_i \neq \emptyset$

$\mathrm{Inf}(\rho) \cap O_i = \emptyset$



$O_1 \subset E_1 \subset O_2 \subset E_2 \subset$

min rank in $\mathrm{Inf}(\rho)$

Parity TA accept all regular tree languages

$\rho: q_0 q_1 q_2 \;.\; \in Q^\omega$ $\qquad \mathrm{Inf}(\rho) = F$

1 2 3 4 - states $\qquad F = \{2, 4\}$

initial

3 1 4 2 4 4 2 4 4

2341 2413 2431 2314 3142 $31\overline{24}$ $31\overline{24}$ $31\overline{42}$ $31\overline{24}$ $31\overline{24}$

Inf. many times in 2nd from the end

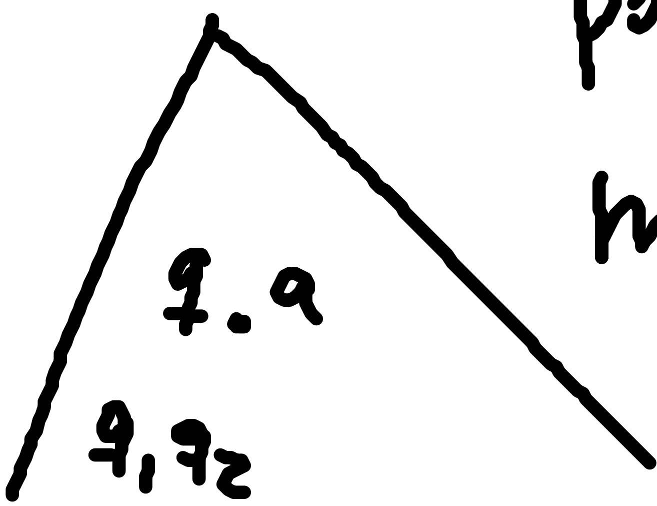
LAR

For each $i < |Q|$, if the set of the last i elements $\notin F$ $\Rightarrow$ odd rank

otherwise $\rightarrow$ even rank

Suppose that T is not accepted by A

Then Path has a memoryless winning strategy on T



position $\subseteq Q \times \Sigma \times Q \times Q$

memoryless strategy

$f(s, \text{position}) \to \{0, 1\}$

$s \in \{0,1\}^*$

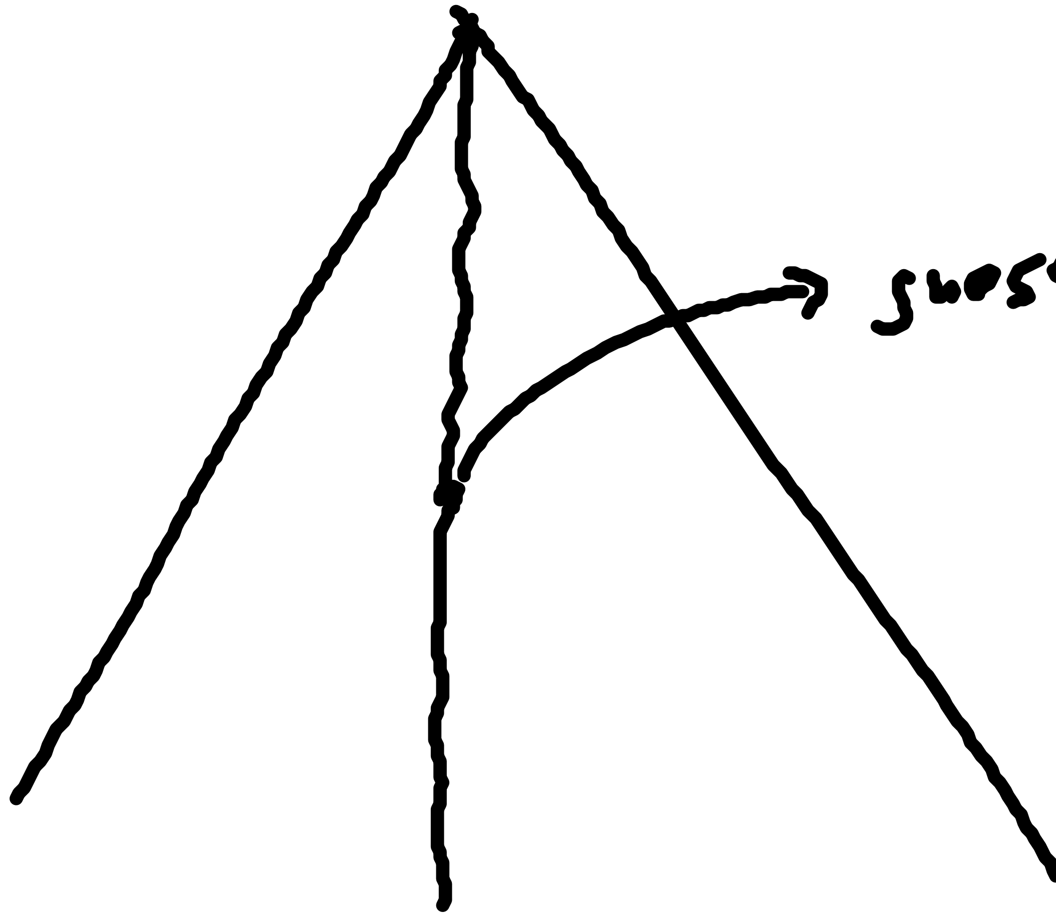
T_f tree labeled by "local instructions"

$g \quad Q \times \Sigma \times Q \times Q \to \{0,1\}$

\# of such g

$2^{|Q|^3 |\Sigma|}$

To check if Path has a winning strategy, guess T_f

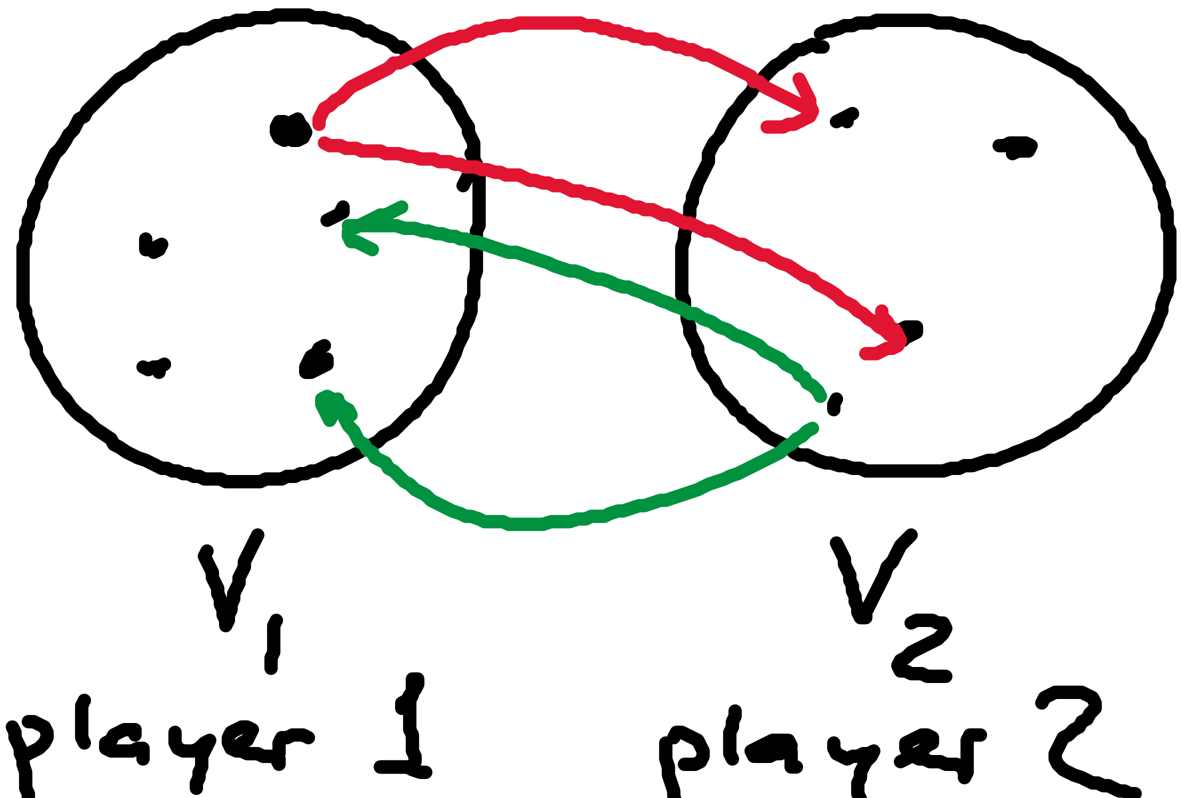


guess local inst g

For each inf. branch t,

Muller automaton $\Longleftarrow$ for each assignment of states q along this branch, local instructions applied to these states violate Acc

Finite games



$c: V_1 \cup V_2 \to C$

play leads to $w \in C^\omega$

Acc over C^ω

A) Acc is a parity condition
Then it is decidable whether player 1 wins
NP

B) Consider input-free automata   Winning = accepting some tree

W. Thomas "Logic Languages Automata"

+ Handbook of Formal Languages

LNCS 2500