1. *(10 marks)* Construct Büchi automata accepting \( \omega \)-words satisfying the following LTL formulae:

   (a) \( G F X a \) (that is, \( G (F (X a)) \));
   (b) \( G F (a U b) \).

2. *(10 marks)* Consider the *validity* problem for LTL: Given an LTL formula \( \phi \), is \( \phi \) true in all \( \omega \)-words.

   Give an exponential-time algorithm for solving this problem.

3. *(10 marks)* Consider two properties of infinite binary trees:

   (a) There is a path starting from the root on which infinitely many nodes are labeled \( a \);
   (b) On every path starting from the root, there is a node labeled \( a \).

   Construct tree automata (with Muller acceptance conditions) for these properties.

4. *(Bonus problem for extra 5 marks)* Consider the Vardi-Wolper translation from LTL into Büchi automata. Note that all the operators of LTL make perfect sense over the usual, finite, words.

   A slight extension of the Vardi-Wolper construction produces for each LTL formula \( \phi \) an NFA \( A_\phi \) accepting words \( w \in \Sigma^* \) that satisfy \( \phi \).

   Describe this extension of the Vardi-Wolper construction.

   You do *not* have to give all the details of Vardi-Wolper; you only need to say which components (set of states, initial states, final states, transition function) change compared to what we’ve seen in class, and how.