# Logic and Automata, Assignment 2

1. *(10 marks)* Construct Büchi automata accepting $\omega$-words satisfying the following LTL formulae:
   (a) $\mathbf{G}\ \mathbf{F}\ \mathbf{X}\ a$ (that is, $\mathbf{G}\ (\mathbf{F}\ (\mathbf{X}\ a)))$;
   (b) $\mathbf{G}\ \mathbf{F}\ (a\ \mathbf{U}\ b)$.

2. *(10 marks)* Consider the *validity* problem for LTL: Given an LTL formula $\phi$, is $\phi$ true in *all* $\omega$-words.

   Give an exponential-time algorithm for solving this problem.

3. *(10 marks)* Consider two properties of infinite binary trees:
   (a) There is a path starting from the root on which infinitely many nodes are labeled $a$;
   (b) On every path starting from the root, there is a node labeled $a$.

   Construct tree automata (with Muller acceptance conditions) for these properties.

4. *(Bonus problem for extra 5 marks)* Consider the Vardi-Wolper translation from LTL into Büchi automata. Note that all the operators of LTL make perfect sense over the usual, finite, words. A slight extension of the Vardi-Wolper construction produces for each LTL formula $\phi$ an NFA $A_\phi$ accepting words $w \in \Sigma^*$ that satisfy $\phi$.

   Describe this extension of the Vardi-Wolper construction.

   You do *not* have to give all the details of Vardi-Wolper; you only need to say which components (set of states, initial states, final states, transition function) change compared to what we've seen in class, and how.