LOGIC & AUTOMATA — HOMEWORK 1

In all question, assume that the finite alphabet is $\Sigma = \{a, b\}$.

1. Write a first-order sentence that defines the language $(a + b)^+ \cdot a$ (1 mark)
2. Write an MSO sentence that defines the following regular language:

$$\big((aa)^*(bb)^*\big)^* \qquad \text{(2 marks)}$$

3. What is the regular language defined by the MSO sentence

$$\forall X \forall Y \ \big(\forall x(X(x) \wedge Y(x) \rightarrow P_a(x))\big)$$

Can you express it in first-order logic? If so, provide an equivalent first-order sentence. (2 marks)