Logic and Automata, Homework 2

- *(2 marks)* Write an MSO formula $\phi(X_1, \ldots, X_n)$ saying that a run of an automaton over ω-words given by $X_1, \ldots, X_n$ (i.e., X_i is the set of positions in which the run in the state q_i, where $q_1, \ldots, q_n$ is the set of all states) satisfies a Muller acceptance condition $\mathcal{F} \subseteq 2^{\{q_1, \ldots, q_n\}}$.

- *(3 marks)* Write an FO formula over unranked trees that defines the "document" (depth-first) ordering. For example, in a tree with domain $\{\epsilon, 1, 2, 3, 11, 12, 121, 122, 13, 21, 31, 32\}$ the document ordering is
$$\epsilon < 1 < 11 < 12 < 121 < 122 < 13 < 2 < 21 < 3 < 31 < 32.$$

The basic predicates are child and the next-sibling relations $<_{\text{ch}}$ and $<_{\text{ns}}$ as well as their transitive closures $<_{\text{ch}}^*$ (the descendant relation) and $<_{\text{ns}}^*$ (the "younger sibling" relation: $x <_{\text{ns}}^* y$ means that x and y are children of the same node and y is younger).