Scottish Combinatorics Meeting 2019: Abstracts

Invited talks

Julia Böttcher

Spanning subgraphs of randomly perturbed graphs

(joint work with Jie Han, Yoshiharu Kohayakawa, Richard Montgomery, Olaf Parczyk and Yury Person)

Randomly perturbed graphs are obtained by adding, to a given deterministic graph satisfying a certain minimum degree condition, a certain number of random edges. I will discuss a general approach for studying the appearance of certain spanning subgraphs in this model using absorption. This approach yields simpler proofs of several known results and allows us to obtain new results concerning powers of Hamilton cycles, general spanning bounded degree subgraphs, and tree universality.

Stephen Finbow

The Firefighter problem on graphs

In 1995, Hartnell introduced the firefighter problem, where a fire breaks out at a vertex of a graph. As time passes, a firefighter protects vertices not yet on fire and the fire spreads to all the unprotected vertices that have a neighbor on fire. Various objectives have been pursued, including: how to save the maximum possible number of vertices; what structure minimizes the expected number of vertices burned; the number of firefighters needed to save a particular fraction of the vertices; for infinite graphs, find the smallest number of firefighters that can "contain" the fire in a finite number of steps. We will survey various work on the subject and focus on attempts to bound expected proportion of vertices that 2 firefighters per turn can save on planar graphs away from zero.

Daniela Kühn (University of Birmingham)

The Oberwolfach problem

(Joint with Stefan Glock, Felix Joos, Jaehoon Kim and Deryk Osthus)

The Oberwolfach problem, posed by Ringel in 1967, asks for a decomposition of the complete graph of order $2n + 1$ into edge-disjoint copies of a given 2-factor. We show that this can be achieved for all large n.

We actually prove a significantly more general result, which allows for decompositions into more general types of factors. In particular, this also resolves the Hamilton-Waterloo problem for large n.

Gary McGuire (University College Dublin)

The Number of Roots of a Linearized Polynomial

We will introduce linearized polynomials over finite fields, not assuming any previous knowledge. Then we will outline some recent results concerning methods for calculating the number of roots of a linearized polynomial in the field of coefficients. Our methods use the rank of a certain matrix arising from the coefficients. We will also outline applications to cryptography and coding theory.

Kurt Melhorn (MPI Saarbrucken)

The Complexity of a Variant of Mastermind

(joint work with Peyman Afshani, Manindra Agralwal, Benjamin Doerr, Carola Doerr, and Kasper Green Larsen)

We study a variant of the guessing game Mastermind. In this variant, the secret is a pair (z, π) which consists of a binary string z of length n and a permutation π of the integers from 1 to n.

The secret must be unveiled by asking queries; each query is a binary string x of length n. For each such query x, we are returned the length of the longest common prefix of x and z with respect to the order imposed by π. The goal is to minimize the number of queries needed to identify the secret. We prove matching upper and lower bounds for the deterministic and randomized query complexity of this game, which are $\Theta(n \log n)$ and $\Theta(n \log \log n)$, respectively.

Guus Regts (University of Amsterdam)

On the location of zeros of the independence polynomial of bounded degree graphs

(partially based on joint works with Viresh Patel and Han Peters)

In this talk I will introduce the independence polynomial (a.k.a. the partition function of the hard-core model in statistical physics) and motivate the study of the location of its zeros by applications to statistical physics and theoretical computer science.

Then I will survey some (recent) results on the location of these zeros and discuss the implications in theoretical computer science. Along the way I will indicate how the theory of complex dynamical systems has been used to settle a conjecture of Alan Sokal on the location of the zeros of the independence polynomial for bounded degree graphs. I will end with some open problems.

Nik Ruškuc (University of St Andrews)

Decidability of the WQO problem for permutations under the consecutive pattern involvement

(joint work with Matt McDevitt)

A partially or quasi-ordered set P is said to be well quasi-ordered (WQO for short) if it contains no infinite antichains and no infinite strictly descending antichains. The WQO problem for P is the decidability problem with the following specification:

INPUT: A finite set of elements $a_1, \ldots, a_n \in P$.

QUESTION: Is the downward closed subset

$$\mathrm{Av}(a_1, ..., a_n) = \{p \in P \,:\, a_i \not\leq p,\ i = 1, \ldots, n\}$$

consisting of all elements of P that avoid all a_i, well quasi-ordered?

This problem has received a lot of attention for finite graphs under the induced subgraph ordering, but a solution remains elusive. One would expect that resolving this question should be easier in the more basic cases of words and permutations under various involvement orderings. Indeed, the WQO problem for the poset A^* of all words over a finite alphabet A under the subword (subsequence) ordering is trivially decidable, since A^* itself is WQO by the classical theorem of Higman (1952). The problem is also decidable for A^* under the factor (contiguous subsequence) ordering, which was proved by Atminas, Lozin and Moshkov (2013), using finite state automata. On the other hand, for the poset of permutations under the pattern involvement ordering, the problem remains open, and apparently out of reach for the moment. This pushes the WQO problem for permutations under the consecutive pattern involvement into the foreground. In this talk I will report on a recent positive solution to the problem, obtained jointly with my PhD student Matt McDevitt.

1 Contributed talks

David Bevan (University of Strathclyde)

The local structure of semi-sparse permutations

Benjamin Bumpus (University of Glasgow)

The Width of Minimum Cost Tree Decompositions

(joint work with Kitty Meeks and Puck Rombach)

Tree decompositions have been very successfully employed in the design of parameterized graph algorithms. Typically an upper bound on the running time of such algorithms depends on the width of the decomposition provided: i.e the size of its largest bag. For this reason much effort has been directed towards finding tree decompositions with minimum width. However, this is not the right way of constructing an 'algorithmically best' tree decomposition because the width of a tree decomposition which minimizes the running time of some algorithm is not always minimum. The intuition behind this phenomenon is that it is sometimes better to allow a few large bags in order to accommodate many small bags. This talk will address progress related to the question:

"is the width of an 'algorithmically best' tree decomposition bounded with respect to treewidth?"

Padraig Condon (University of Birmingham)

Resilience of random graphs with respect to Hamiltonicity

(joint work with Alberto Espuny Díaz, António Girão, Jaehoon Kim, Daniela Kühn and Deryk Osthus)

The local resilience of a graph G with respect to a property P measures how much one has to change G locally in order to destroy P. We prove 'resilience' versions of several classical results for the graph models $G_{n,p}$ and $G_{n,d}$.

For example, we prove a resilience version of Dirac's theorem in the setting of random regular graphs. More precisely, we show that, whenever d is sufficiently large compared to $\epsilon > 0$, a.a.s. the following holds: let G' be any subgraph of the random n-vertex d-regular graph $G_{n,d}$ with minimum degree at least $(1/2+\epsilon)d$. Then G' is Hamiltonian. This proves a conjecture of Ben-Shimon, Krivelevich and Sudakov.

We also prove a resilience version of Pósa's Hamiltonicity condition for the binomial random graph $G_{n,p}$ and show that a natural guess for a resilient version of Chvátal's theorem for $G_{n,p}$ fails to be true.

Matthew Coulson (University of Birmingham)

On the largest component of the critical random digraph

We consider the largest component of the random digraph $D(n,p)$ inside the critical window $p = n^{-1} + \lambda n^{-4/3}$. We show that the largest component $\mathcal{C}_1$ has size of order $n^{1/3}$ in this range. In particular we give explicit bounds on the probabilities that $|\mathcal{C}_1|n^{-1/3}$ is very large or very small that are analogous to those given by Nachmias and Peres for $G(n,p)$.

Konrad Dabrowski (Durham University)

Graph Isomorphism for (H_1, H_2)-free Graphs: An Almost Complete Dichotomy

(joint work with Marthe Bonamy, Matthew Johnson and Daniël Paulusma)

The Graph Isomorphism problem, which is that of deciding whether two given graphs are isomorphic, is a central problem in algorithmic graph theory. Babai recently proved that the problem can be solved in quasi-polynomial time, but it is not known if this can be improved to polynomial-time on general graphs.

We consider the Graph Isomorphism problem restricted to classes of graphs characterized by two forbidden induced subgraphs H_1 and H_2. By combining old and new results, Schweitzer settled the computational complexity (polynomial-time solvable or GI-complete) of this problem restricted to (H_1, H_2)-free graphs for all but a finite number of pairs (H_1, H_2), but without explicitly giving the number of open cases. Grohe and Schweitzer proved that Graph Isomorphism is polynomial-time solvable on graph classes of bounded clique-width. By combining previously known results for Graph Isomorphism with known results for boundedness of clique-width, we reduce the number of open cases to 14. By proving a number of new polynomial-time and GI-completeness results, we then further reduce this number to seven.

Grahame Erskine (Open University)

Upper embeddings of symmetric configurations with block size 3

A *symmetric configuration* of triples is an incidence structure consisting of v points and v blocks; each block contains 3 points and each point is in 3 blocks, with each pair of points represented in at most one block. The *associated graph* of the configuration is a 6-regular graph with the points as vertices; two points are adjacent if they participate in a triple. An *upper embedding* of the configuration is an embedding of the associated graph into an orientable surface of suitable genus, such that the triangles representing the blocks are faces of the embedding and there is exactly one further face. Necessary and sufficient conditions for such an embedding to be possible can be given in elementary combinatorial terms; we will focus on those configurations for which these conditions are known to be satisfied. Investigation of these conditions for small values of v leads to some natural conjectures and open questions which we will discuss.

Rachel Kirsch (London School of Economics)

Many cliques with few edges

The problem of maximizing the number of cliques has been studied within several classes of graphs. For example, among graphs on n vertices with clique number at most r, the Turán graph $T_r(n)$ maximizes the number of copies of K_t for each size t. Among graphs on m edges, the colex graph $\mathcal{C}(m)$ maximizes the number of K_t's for each size t.

In recent years, much progress has been made on the problem of maximizing the number of cliques among graphs with n vertices and maximum degree at most r. In this talk, we discuss the edge analogue of this problem: which graphs with m edges and maximum degree at most r have the maximum number of cliques? We prove in some cases that the extremal graphs contain as many disjoint copies of K_{r+1} as can fit, with the leftovers in another component. These remaining edges form a colex graph.

Alexander Levine (Heriot-Watt University)

Counting elements of Thompson's group F

Thompson's group F has generated interest since its discovery for its exotic properties, and among its many realisations it can be viewed as a set of pairs of trees together with an appropriate composition. We give a brief introduction to Thompson's group F, using the pairs of trees, and use these pairs of trees to create a bijective correspondence between elements of F and pairs of finite sequences of non-negative integers. We then discuss our attempts to count certain classes of elements of F, using these sequences.

Sofiat Olaosebikan (University of Glasgow)

Strongly Stable Matchings in the Student Project Allocation Problem with Ties

(Joint work with David Manlove)

The Student-Project Allocation problem with lecturer preferences over Students (SPA-S) involves assigning students to projects based on student preferences over projects, lecturer preferences over students, and the maximum number of students that each project and lecturer can accommodate. This classical model assumes that preference lists are strictly ordered.

In this work, we study a variant of SPA-S where ties are allowed in the preference lists of students and lecturers, which we refer to as the Student-Project Allocation problem with lecturer preferences over Students with Ties (SPA-ST). In this context, what we seek is a stable matching of students to projects. However, when the preference lists involve ties, three different stability concepts naturally arise; namely, weak stability, strong stability and super-stability. In this talk, I will focus on the concept of strong stability in SPA-ST. Further, I will describe a polynomial-time algorithm to find a strongly stable matching or to report that no such matching exists, given an instance of SPA-ST. Our algorithm runs in $O(m^2)$ time, where m is the total length of the students' preference lists.

Elena Zamaraev (University of Warwick)

On the number of 2-threshold functions

We consider 2-threshold functions over a 2-dimensional integer grid of a fixed size $m \times n$, that is the functions which can be represented as the conjunction of two threshold functions.

The asymptotic of the number of threshold functions is known to be $\frac{6}{\pi^2}(mn)^2+O(mn^2)$[1]. It immediately gives an upper bound on the number of 2-threshold functions which is $\frac{18}{\pi^4}(mn)^4+O(m^2n^4)$.

In this work we provide an asymptotic formula for the number of 2-threshold functions. To achieve this goal we establish a one-to-one correspondence between almost all 2-threshold functions and pairs of integer segments with specific properties. We expect this bijection to be useful in algorithmic studies of 2-threshold functions.

References

[1] P. Haukkanen, J. K. Merikoski. *Asymptotics of the number of threshold functions on a two-dimensional rectangular grid.* Discrete Applied Mathematics, 161, 2013: 13-18

Luca Zanetti (University of Cambridge)

Hermitian Laplacians, Cheeger inequalities, and 2-variable linear equations

(joint work with Huan Li (Fudan) and He Sun (Edinburgh))

We consider the following problem: we are given a system of linear equations in two variables modulo k, and we want to find an assignment to the variables so that the number of satisfied equations is maximised.

We prove a Cheeger-type inequality that relates the maximum number of satisfiable equations to the smallest eigenvalue of the Laplacian of a graph whose edges are signed by complex roots of unity. Using a framework introduced by Trevisan, we harness this Cheeger-type inequality to obtain an algorithm that, given a system where a $(1-\epsilon)$-fraction of equations can be satisfied, returns an assignment which satisfies a $(1-O(k)\sqrt{\epsilon})$-fraction.

Dr Mary Cryan, Organising Committee, School of Informatics, Edinburgh.