TAGUNGSBERICHT

KATEGORIEN

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Die Tagung stand wieder unter der Leitung der Herren:

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Formal Spaces

M. P. Fourman

We recall some aspects of the theory of locales which may be developed in an arbitrary topos with NNO. A complete Heyting algebra (locale, Isbell, Benabou; frame, Dowker u. Papert) is an $\wedge$ over $\bigvee$ distributive complete lattice. A morphism of cHa is a map preserving $\top \wedge \bigvee$. A point of a cHa Ω is a morphism $\varphi : \Omega \to P(1)$. The functor O taking a space to its opens has a right adjoint S taking a cHa Ω to its space of points topologised by opens

$$U^* = \{\varphi : \Omega \to P(1) \mid \varphi(U) = 1\} \quad \text{where } U \in \Omega$$

$$cHa^{op} \underset{O}{\overset{S}{\rightleftarrows}} Top \qquad O \dashv S$$

The functors O and S define an equivalence between their images, the full subcategories Pts (cHa with enough points) and Sob (sober spaces) respectively. Pointless cHa exist in profusion, even classically. Some cHa which classically have enough points may not do so intuitioniscally.

Models of any propositional geometric theory T are classified by (sheaves for the canonical topology on) a locale J(T) the formal opens of the space of T-models. Particular propositional theories $R, 2^N$ and N^N describe (respectively) a Dedekind real, an element of Cantor space and an element of Baire space. The spaces of points of $J(R)$ $J(2^N)$ and $J(N^N)$ are just the Dedekind reals Cantor space and Baire space with their usual topologies.

Comparing formal and actual opens leads us to consider certain higher-order principles:

We say R is <u>locally compact</u> iff whenever a rational interval (p,q) is covered by a family U of opens and p<r<s<q then some finite (indexed by a natural number = Kuratowski finite) sub-family of U covers (r,s).

<u>Compactness of 2^N</u> is equivalent to the "Fan Theorem"

$$\forall A\in P(2^{<N})\ (\forall\alpha\in 2^N.\ \ \exists n\in N.\ \ \alpha 1 n\in A \rightarrow \exists k.\ \ \forall\alpha.\ \exists n<k.\ \ \alpha 1 n\in A)$$

<u>Bar Induction</u> is the principle allowing us from

	$\forall\alpha\in N^N.\ \exists n\in N.\ \alpha 1 n\in A$	"A is a bar"
and	$\forall a\in N^{<N}.\ \forall k\in N\ (a\in A \rightarrow a^\wedge k\in A)$	"A is monotone"
and	$\forall a\in N^{<N}\ (\forall k\in N.\ a^\wedge k\in A \rightarrow a\in A)$	"A is inductive"
to deduce	$< > \in A$	"the empty sequence is in A"

(where $A\in P(N^{<N})$ is a set of finite sequences).

<u>Theorem</u> (Comparsion of Formal and Actual opens)

1) $O(R) \cong J(R)$ iff R is locally compact
2) $O(2^n) \cong J(2^N)$iff 2^N is compact (Fan theorem)
3) $O(N^N) \cong J(N^N)$iff Bar Induction is valid.

Classically our three principles are provable so formal and actual opens coincide. Fourman and Hyland (Sheaf models for Analysis, proc.Durham Symposium) have shown that they may fail in certain topoi. In these topoi formal and actual opens fail to coincide.