IVR: PID control

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informatics

Overview

- Proportional error + Proportional Integral + Proportional Derivative = PID
- Effects of the gain factors
- Non-stationary targets
- PID tuning

Combining controllers

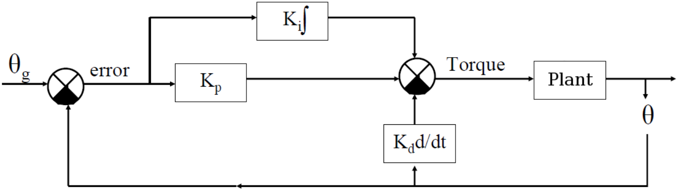
Control signals can chosen:

- Proportional to error (P)
- Proportional to error Integral (I)
- Proportional to error Derivative (D)

How do the different modes of the set-point interact?

Combine as PID control (Three mode control)

Black sectors denote negation



$$T = K_p\left(\theta_{\text{goal}} - \theta\right) + K_i \int_0^t \left(\theta_{\text{goal}} - \theta\left(t'\right)\right) dt' + K_d \frac{d\theta}{dt}$$

- PID controllers are used in by far the most continuous feedback control applications
- How to choose K_p, K_i, K_d?

Characterising the behaviour of a control system (SASO)

1. Stability: Returns to set point after (small) perturbations
2. Accuracy (Steady-state error): the difference between the steady-state output and the desired output.
3. Settling time: time it takes for the system to converge to its steady state

   Rise time: time it takes for the plant output to rise beyond 90% of the desired level for the first time

   - May be long in the case of on-going oscillations
   - Rise time and settling time replace half time (which was meaningful only for non-oscillatory exponential convergence)
4. Overshoot: how much the the peak level is higher than the steady state, normalised against the steady state.

Gingham Zhong: PID Controller Tuning: A Short Tutorial
saba.kntu.ac.ir/eecd/pcl/download/PIDtutorial.pdf

PID control example

Second-order system

$$T(t) = J\frac{d^2\theta}{dt^2} + F\frac{d\theta}{dt}$$

Controller

$$K_p\left(\theta_{\text{goal}} - \theta(t)\right) + K_i\int\left(\theta_{\text{goal}} - \theta(t)\right)dt + K_d\frac{d\theta}{dt} = T(t)$$

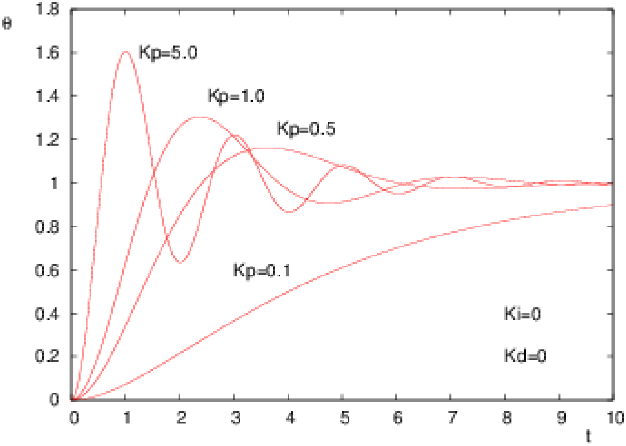
Numerical solution with:

$A_1 = \frac{K_d - F}{J}$, $A_0 = -\frac{K_p}{J}$, $C = \frac{K_i}{J}\int_0^t\left(\theta_{\text{goal}} - \theta(t)\right)dt + \frac{K_p}{J}\theta_{\text{goal}}$

$$\frac{d^2\theta}{dt^2} = A_1\frac{d\theta}{dt} + A_0\theta + C$$

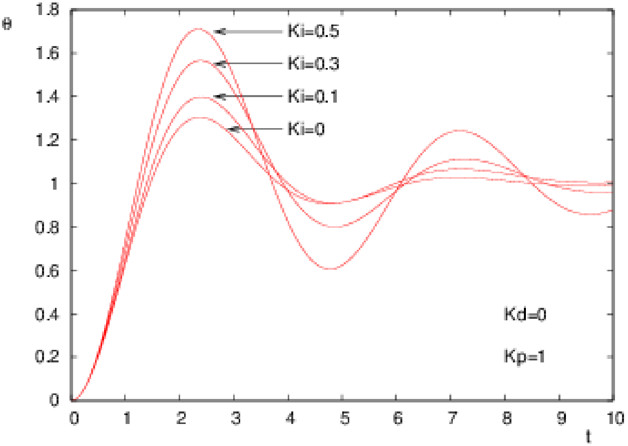
PID control example

1. Behaviour at $K_p = K_i = K_d = 0$: Choose J and F such that the system is overdamped (see 2nd order diff-eq. example)
2. Start with proportional control: $K_i = K_d = 0$, $\theta_{\text{goal}} = 1$



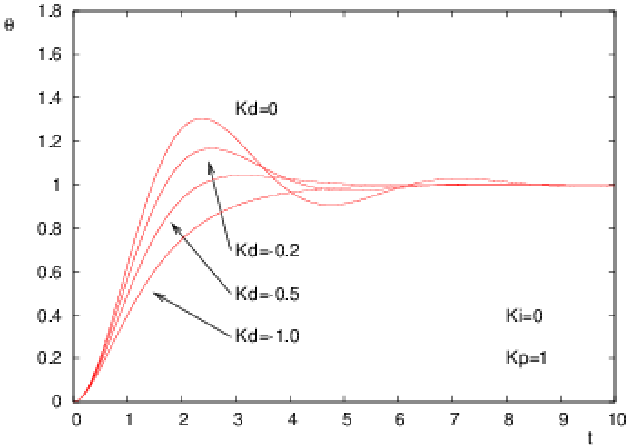
PID control example

3. Choose a reasonable rise time, e.g. $K_p = 1$, now vary K_i: Similar effect as increasing K_p



PID control example

4. K_i did not help, so set $K_i = 0$ and $K_p = 1$, and vary K_d: Reduces oscillations, rise time still low (e.g. at $K_d = 0.5$)



Note, that the negative sign of K_d is for consistency with the equation two slides back. Considered as artificial friction, the used differential term is clearly damping.

PID control example: Evaluation

- Often PD (proportional + derivative) control is sufficient
- Integral term needed only if steady state error is expected or if the system is noisy
- Consider priorities when determining overshoot:
  - When catching a ball: fast rise time is essential (could set goal state to a lower value and make sure that the ball arrives at the overshoot)
  - When moving towards a position near an obstacle: slow rise time, overdamped movement $\rightarrow$ no overshoot
  - In many other cases: Adjust K_d to realise critical damping

Effects of increasing a parameter independently

Parameter	Rise time	Overshoot	Settling time	Steady-state error	Stability
K_p	Decrease	Increase	Small change	Decrease	Degrade
K_i	Decrease	Increase	Increase	Decrease signific-antly	Degrade
K_d	Minor increase	Decrease	Minor change	No effect in theory	Improve (if K_d is small)

http://en.wikipedia.org/wiki/PID_controller (except red entries)

Typical steps for designing a PID controller

- Determine what characteristics of the controlled system need to be improved
- Use K_p to decrease the rise time.
- Use K_i to eliminate the steady-state error.
- Use K_d to reduce the overshoot and settling time.

This works in many cases

JFYI: Ziegler-Nichols tuning rule (reaction curve method)

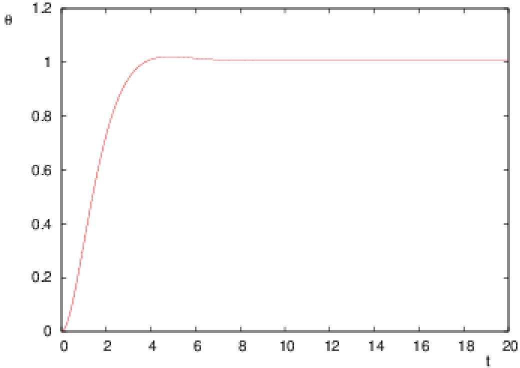
- Practical control method, i.e. the controlled system is accessed experimentally

1. Set I and D gains to zero.
2. Check sign of gain (say positive)
3. Increase P gain (from zero) until until output starts to oscillate $\rightarrow$'ultimate gain' K_u and oscillation period T_u
4. Use K_u and T_u to set K_p, K_i and K_d based on heuristic values: $K_p = 0.65 K_u$, $K_i = 2K_p/T_u$ and $K_d = K_p T_u/8$

- May create some overshoot
- Stable to disturbances
- Not very good in tracking tasks
- Not equally good in all applications

Ziegler-Nichols rule tested

- Second order system ($F = 0.5$, $J = 0.5$)
- $K_u \approx 1$: one full oscillation period visible $\Rightarrow T_u = 5$
- $\Rightarrow K_p = 0.65$, $K_d = 1.25$, $K_i = 0.26$ (scaled by time step!)



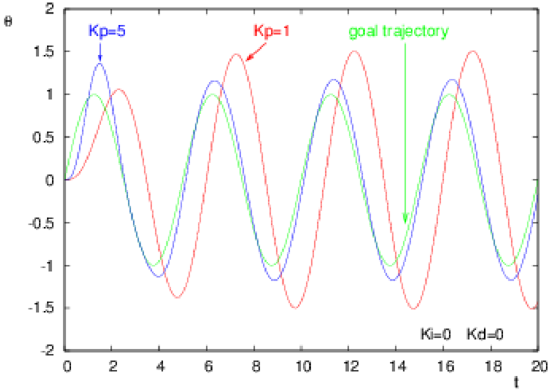
- Performance similar as tuned by hand (Z-N rule works best for first-order systems!)

Limitations of PID control

- PID control is usually the best controller with no model of the process (PID can be used on top of model-based control)
- Does not provide optimal control
- Is only reactive, may be slow, and needs errors to be able to react (combine with feed-forward control or forward models)
- D-term may suffer from intrinsic or measurement noise ($\Rightarrow$ use low-pass filter)
- D-term (error derivative) and I-term may suffer from sudden set-point changes ($\Rightarrow$ use set-point ramping)
- Is tuned to a particular working regime ($\Rightarrow$ gain scheduling)
- Is linear and (anti)symmetric (e.g. usually a heating system does not involve symmetric cooling)

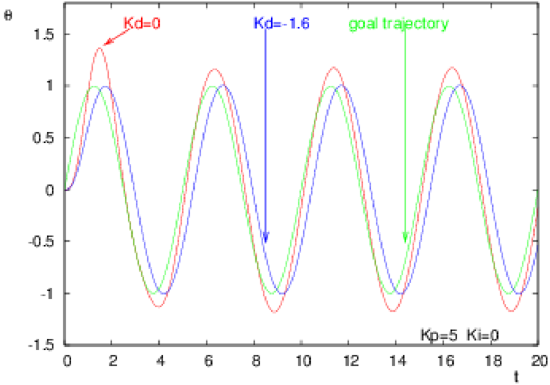
Control with changing set points

- In tracking tasks the set point changes continuously: goal trajectory
- Rise time and settling time appear as delays (phase shifts) which depend on the rate of change of the goal trajectory
- Delays can be reduced by high-gain proportional control



Control with changing set points

- Differential feedback can reduce the overshoot, but tends to increases phase shift
- Integral feedback will not improve the situation
- Solution: Forward models



Changing set-points: Possible improvements

- Set-point switching: Change the set point in a ramp-like way
- Initialising the integral term at a suitable value
- Disabling the integral function until the state has entered the controllable region
- Limiting the time period over which the integral error is calculated
- Preventing the integral term from accumulating above or below pre-determined bounds
- For a constant set-point, the D-term can be either $\frac{d}{dt}\theta$ or $\frac{d}{dt}\left(\theta_{\text{goal}} - \theta(t)\right)$, now the second form should be considered

http://en.wikipedia.org/wiki/PID_controller

More on control (JFYI)

- Non-linear control: How to deal with complex systems?
  - Linearisation, gain-scheduling, Lyapunov stability, sliding-mode control. ...
- Robust control: Uncertain parameters or disturbances?
  - H_∞ control: Stabilisation (and other desired properties) with guaranteed performance, ...
- Adaptive control: Changing environments?
  - System identification, model-based control, self-tuning, ...
- Distributed control: Communication, negotiation and all of the above
- Intelligent control: How to deal with all the remaining cases?
  - Use AI, learning, evolutionary algorithms etc.

Further reading on control theory and applications

Most standard control textbooks discuss PID control, e.g.: Andrew D. Lewis: A Mathematical Approach to Classical Control. 2003. www.mast.queensu.ca/~andrew/teaching/math332/notes.shtml

igor.chudov.com/manuals/Servo-Tuning/PID-without-a-PhD.pdf