IVR: Feedback control
Proportional and Proportional Integral Control

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Overview

- Feedback control
- Proportional error control
- Proportional Integral control
- Proportional Derivative control
- Next time PID
Control paradigms

Open loop control

- Inverse model
- Motor command
- Robot in environment
- Disturbances?

Feed-forward control

- Inverse model
- Motor command
- Robot in environment
- Measurement ← Disturbances

Feedback control

- Control Law
- Motor command
- Robot in environment
- Measurement ← Disturbances
Feedback control

- Simple and direct, does not require a model of the process
- Robust in the face of unknown and unpredictable disturbances
- Requires sensors capable of measuring output
- Tuning required: Low gain is slow, high gain is unstable
- Delays in feedback loop may interfere with the control law
Servo control (Proportional error control)

Simple dynamic example
\( (C = V_B, B = \frac{M}{k_2} R, A = k_1) \):

\[
V_B = \frac{M}{k_2} R \frac{ds}{dt} + k_1 s
\]

Control law:

\[
V_B = K \left( s_{\text{goal}} - s \right)
\]

So now have new process:

\[
K \left( s_{\text{goal}} - s \right) = \frac{M}{k_2} R \frac{ds}{dt} + k_1 s
\]

With steady state:

\[
s_{\infty} = \frac{Ks_{\text{goal}}}{K + k_1}
\]

And half-life:

\[
\tau_{\frac{1}{2}} = 0.7 \frac{MR}{(K + k_1) k_2}
\]
Steady state error

\[ s_{\text{goal}} - s_\infty = \frac{k_1}{K + k_1} s_{\text{goal}} \]

- \( k_1 \) is determined by the motor physics: \( e = k_1 s \)
- Large \( K \) brings the state close to desired, but we cannot put an infinite voltage into the motor!
- For any sensible \( K \) the system will undershoot the target velocity.
Proportional (P) Control

- Convenient, simple, powerful (fast and proportional reaction to errors)
- No need for modelling (sign of $K$ must be known and the order of magnitude)
- Problem: Steady state error
- Other problems: May lead to oscillations about the goal state (later!)
- From now on $K$ will be called $K_p$
Proportional Integral (PI) Control

If we could estimate this error we could add it to the control signal:

\[ V_B = K_p \left( s_{\text{goal}} - s \right) + \varepsilon \]

The best way to estimate it is to integrate the error over time:

\[ \varepsilon = \int \left( s_{\text{goal}} - s \right) dt \]

Obtain new control law:

\[ V_B = K_p \left( s_{\text{goal}} - s \right) + K_i \int \left( s_{\text{goal}} - s \right) dt \]

Basically, this sums some fraction of the error until the error is reduced to zero.

With careful choice of \( K_p \) and \( K_i \) this can eliminate the steady state error.
If the system has to hold a load against gravity, it requires a constant torque

Similar to steady state error in the first order system

But P controller cannot do this without error, as torque is proportional to error (so, needs some error)

If we knew the load $L$ could use

$$T = L - K_p \theta$$

But in practice this is not often possible
Proportional integral (PI) control

As before, we integrate the error over time, i.e.

\[ L = K_i \int_0^t \left( \theta_{\text{goal}} - \theta(t') \right) dt' \]

Effectively this gradually increases \( L \) until it produces enough torque to compensate for the load

PI copes with load droop
Towards PID control

- Proportional control reduces large errors: Fast and powerful
- Proportional integral: Precise and delicate, deals with remaining errors if they accumulate
- What else might happen?
  - Large errors may accumulate as well such that PI can overshoot
  - The state can oscillate about the goal state

Outlook:

- Solution of the oscillation (ringing) problem: Dampen oscillations by “artificial friction” which will be provided by derivative control → PD
- We will have three modes of feedback control: P, PI, PD and finally arrive at PID control is the combination of P, PI, PD
References

For the research on robot juggling see: