IVR: Feedback control
Proportional and Proportional Integral Control

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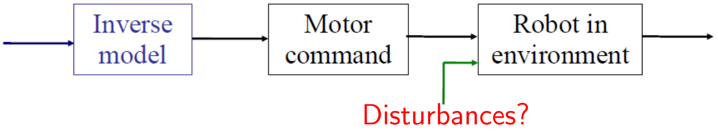
informatics

Overview

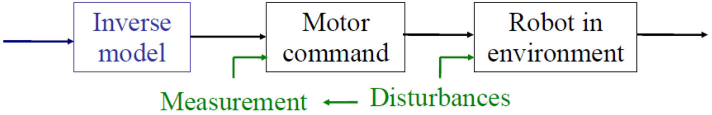
- Feedback control
- Proportional error control
- Proportional Integral control
- Proportional Derivative control
- Next time PID

Control paradigms

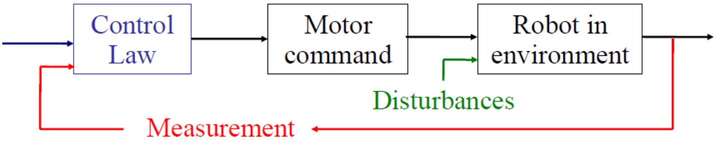
Open loop control



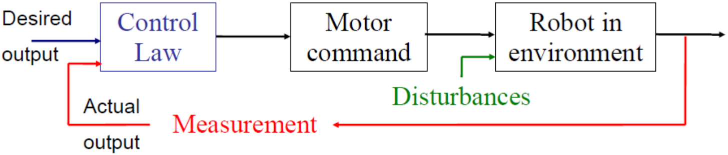
Feed-forward control



Feedback control



Feedback control



- Simple and direct, does not require a model of the process
- Robust in the face of unknown and unpredictable disturbances
- Requires sensors capable of measuring output
- Tuning required: Low gain is slow, high gain is unstable
- Delays in feedback loop may interfere with the control law

Servo control (Proportional error control)

Simple dynamic example
($C = V_B$, $B = \frac{M}{k_2}R$, $A = k_1$):

$$V_B = \frac{M}{k_2}R\frac{ds}{dt} + k_1 s$$

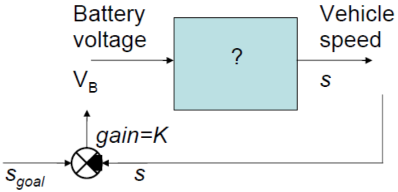
Control law:

$$V_B = K\left(s_{\text{goal}} - s\right)$$

So now have new process:

$$K\left(s_{\text{goal}} - s\right) = \frac{M}{k_2}R\frac{ds}{dt} + k_1 s$$

$$Ks_{\text{goal}} = \frac{M}{k_2}R\frac{ds}{dt} + (K + k_1)\,s$$



With steady state:

$$s_\infty = \frac{Ks_{\text{goal}}}{K + k_1}$$

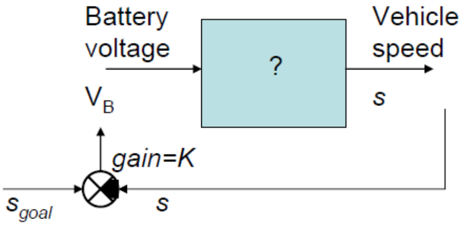
And half-life:

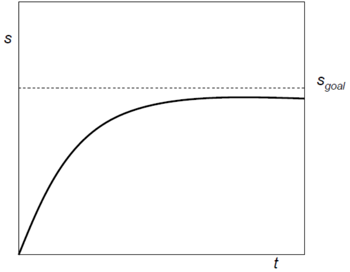
$$\tau_{\frac{1}{2}} = 0.7\frac{MR}{(K + k_1)\,k_2}$$

Steady state error

$$s_{\text{goal}} - s_{\infty} = \frac{k_1}{K + k_1} s_{\text{goal}}$$

- k_1 is determined by the motor physics: $e = k_1 s$
- Large K brings the state close to desired, but we cannot put an infinite voltage into the motor!
- For any sensible K the system will undershoot the target velocity.





Proportional (P) Control

- Convenient, simple, powerful (fast and proportional reaction to errors)
- No need for modelling (sign of K must be known and the order of magnitude)
- Problem: Steady state error
- Other problems: May lead to oscillations about the goal state (later!)
- From now on K will be called K_p

Proportional Integral (PI) Control

If we could estimate this error we could add it to the control signal:

$$V_B = K_p\left(s_{\text{goal}} - s\right) + \varepsilon$$

The best way to estimate it is to integrate the error over time:

$$\varepsilon = \int \left(s_{\text{goal}} - s\right) dt$$

Obtain new control law:

$$V_B = K_p\left(s_{\text{goal}} - s\right) + K_i \int \left(s_{\text{goal}} - s\right) dt$$

Basically, this sums some fraction of the error until the error is reduced to zero.

With careful choice of K_p and K_i this can eliminate the steady state error.

Back to PID control: Steady state error – Load droop

- If the system has to hold a load against gravity, it requires a constant torque
- Similar to steady state error in the first order system
- But P controller cannot do this without error, as torque is proportional to error (so, needs some error)
- If we knew the load L could use

$$T = L - K_p \theta$$

- But in practice this is not often possible

Proportional integral (PI) control

As before, we integrate the error over time, i.e.

$$L = K_i \int_0^t \left(\theta_{\text{goal}} - \theta\left(t'\right)\right) dt'$$

Effectively this gradually increases L until it produces enough torque to compensate for the load

PI copes with load droop

Towards PID control

- Proportional control reduces large errors: Fast and powerful
- Proportional integral: Precise and delicate, deals with remaining errors if they accumulate
- What else might happen?
  - Large errors may accumulate as well such that PI can overshoot
  - The state can oscillate about the the goal state

Outlook:

- Solution of the oscillation (ringing) problem: Dampen oscillations by “artificial friction” which will be provided by derivative control $\rightarrow$ PD
- We will have three modes of feedback control: P, PI, PD and finally arrive at PID control is the combination of P, PI, PD

References

For the research on robot juggling see:

- Rizzi, A.A. & Koditschek, D.E. (1994) Further progress in robot juggling: Solvable mirror laws. *IEEE International Conference on Robotics and Automation*, p. 2935-2940
- S. Schaal and C.G. Atkeson. Open loop stable control strategies for robot juggling. In *Proc. IEEE Conf. Robotics and Automation*, pages 913 –918, Atlanta, Georgia, 1993.