IVR: Feedback control
Towards PID control

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informatics

Overview

- Overshoot and oscillations in control systems
- Second order differential equations as models for systems with oscillations
- Proportional Derivative control to complement Proportional error control and Proportional Integral control towards PID control

Towards PID control

- Proportional control reduces large errors
- Proportional integral control deals with remaining errors if they accumulate
- What else might happen?
  - Large errors may accumulate as well such that PI can overshoot
  - The state can oscillate about the the goal state
    - due to inertia
    - due to delays

Imagine trying to move a robot to some zero position with the simple control law:

- If $x_t < -\delta$ meters, move forward at 1 m/sec
- If $x_t > \delta$ meters, move backward at 1 m/sec
- If $-\delta < x_t < \delta$, then stop

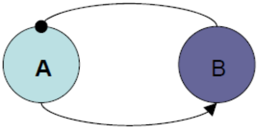
What happens if there is a delay in feedback that exceeds 2δ seconds?

Oscillations are not all bad: Central Pattern Generators

Many movements in animals, and robots, are rhythmic, e.g. walking

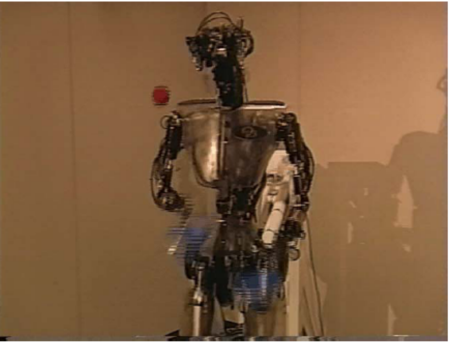
Rather than explicitly controlling position, can exploit an oscillatory process, e.g.

If A is tonically active, it will excite B
When B becomes active it inhibits A
When A is inhibited, it stops exciting B
When B is inactive it stops inhibiting A



Useful not only for control of periodic repeated movements but possibly also as elements for complex movement sequences.

CPGs in Robotics



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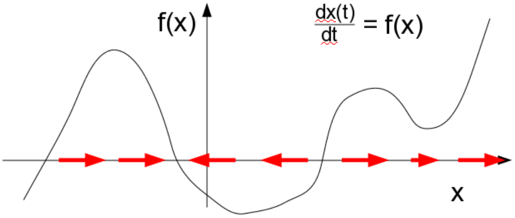
Auke Ijspeert: Bastian

Oscillations in Feedback Systems

- In general, a time-lag in a feedback loop will result in overshoot and oscillation.
- Sometimes we want oscillation, e.g. CPG
- Depending on the dynamics, the oscillation could fade out, continue or increase.
- Note that integration introduces a time delay
- Time delays are equivalent to energy storage e.g. inertia will cause similar effects.
- We should study control systems with inertia ...

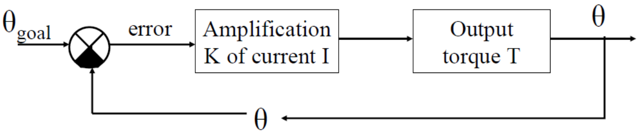
Oscillations in General Dynamical Systems



arrows indicate whether x(t) increases or decreases

- For a first order differential equation of **one variable and without delay** the present state determines uniquely the behaviour. Therefore, the system **cannot be oscillatory**.
- Second order differential equations (or systems of dim ≥ 2) can describe systems where oscillations are possible. Solutions of such differential equations are characterised by an initial velocity in addition to the initial state.

Second order system



Want to move a simple robot arm to a desired angular position θ_{goal}

For a DC motor on a robot joint

$$T = J\frac{d^2\theta}{dt^2} + F\frac{d\theta}{dt}$$

where J is the inertia of the joint and F is the joint friction

Proportional control

$$K_p\left(\theta_{\text{goal}} - \theta\right) = T$$

Second order system solution

For simplicity let $\theta_{\text{goal}} = 0$. Then the system process is

$$K_p \theta = J \frac{d^2\theta}{dt^2} + F \frac{d\theta}{dt}$$

The solution to this equation has the form (insert and check)

$$\theta = e^{-\frac{F}{2J}t} \left(c_1 e^{\frac{\omega}{2}t} + c_2 e^{-\frac{\omega}{2}t} \right)$$

where

$$\omega = \sqrt{\frac{F^2}{J^2} - \frac{4K_p}{J}}$$

which may be complex $\Longrightarrow$ oscillatory solution

c_1, c_2 can be determined from the initial state and initial velocity.

A remark on linear differential equations

Second-order system

$$\frac{d^2\theta}{dt^2} = A_1 \frac{d\theta}{dt} + A_0\theta + C$$

Define $\rho = \frac{d\theta}{dt}$ and insert

$$\frac{d\rho}{dt} = A_1 \frac{d\theta}{dt} + A_0\theta + C$$

We get an equivalent two-dimensional first-order system

$$\begin{pmatrix} \frac{d\rho}{dt} \\ \frac{d\theta}{dt} \end{pmatrix} = \begin{pmatrix} A_1 & A_0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \rho \\ \theta \end{pmatrix} + \begin{pmatrix} C \\ 0 \end{pmatrix}$$

Diagonalize:

$$R\begin{pmatrix} \frac{d\rho}{dt} \\ \frac{d\theta}{dt} \end{pmatrix} = R\begin{pmatrix} A_0 & A_1 \\ 1 & 0 \end{pmatrix} R^{-1} R\begin{pmatrix} \rho \\ \theta \end{pmatrix} + R\begin{pmatrix} C \\ 0 \end{pmatrix} \text{ such that } R\begin{pmatrix} A_0 & A_1 \\ 1 & 0 \end{pmatrix} R^{-1} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

Transform to the new coordinates, solve two simple diff-eqs with constant terms, transform back to ρ,θ, insert initial conditions, done

Over- and Under-Damping

The system behaviour depends on J, F, K_p as follows:

$$\theta = e^{-\frac{F}{2J}t}\left(c_1 e^{\frac{\omega}{2}t} + c_2 e^{-\frac{\omega}{2}t}\right) \quad \text{with} \quad \omega = \sqrt{\frac{F^2}{J^2} - \frac{4K_p}{J}}$$

- The system returns to the goal with under-damped, sinusoidal behaviour for
$$\frac{F^2}{4K_p} < J$$
- The system returns to the goal with over-damping
$$\frac{F^2}{4K_p} > J$$
- The system returns to the goal (critical damping)

$$\frac{F^2}{4K_p} = J \;\Rightarrow\; \text{solution becomes: } \theta = c\, e^{-\frac{F}{2J}t}$$

Proportional derivative (PD) control

Often, e.g. for large robots, inertia is large and friction small.

Consequently the system overshoots, reverses the error and control signal, overshoots again ...

To actively brake the motion, we want to apply negative torque when error is small and velocity high

Make

$$T = K_p \left(\theta_{\text{goal}} - \theta \right) + K_d \frac{d\theta}{dt} = J \frac{d^2\theta}{dt^2} + F \frac{d\theta}{dt}$$

This can be seen as P (or PI) control with artificial friction

$$K_p \left(\theta_{\text{goal}} - \theta \right) = J \frac{d^2\theta}{dt^2} + (F - K_d) \frac{d\theta}{dt}$$

Proportional derivative (PD) control

Example

$$T = J\frac{d^2\theta}{dt^2} + F\frac{d\theta}{dt}$$

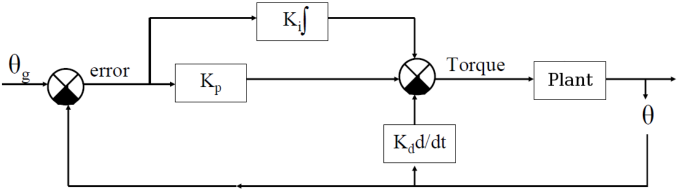
Control law (P and PD):

$$T = K_p\left(\theta_{\text{goal}} - \theta\right) + K_d\frac{d\theta}{dt}$$

- Dampens oscillations, improves stability
- Useful for large inertia & small friction (adds artificial friction)
- Derivative term should be $\frac{d}{dt}\left(\theta_{\text{goal}} - \theta\right)$, use $-K_d$ in this case
  Derivative of θ can improve stability even when θ_{goal} changes
- Possible problems:
  - Derivative is sensitive to measurement noise
  - Tends to slow down the control action
  - D-term does not contain information about the (constant) goal state $\rightarrow$ usually in combination with other control signals

Combine as PID control (Three mode control)

Black sectors denote negation



$$T = K_p\left(\theta_{\mathrm{goal}} - \theta\right) + K_i \int_0^t \left(\theta_{\mathrm{goal}} - \theta\left(t'\right)\right) dt' + K_d \frac{d\theta}{dt}$$

- How do K_p, K_i, K_d interact?
- How to choose K_p, K_i, K_d?
- PID controllers are used in more than 95% of continuous feedback control applications (as of 1995)

Summary

- Proportional error control: Fast and powerful
- Proportional Integral control: Precise and delicate
- Proportional Derivative control: Stabilising (unless the system is rather noisy)
- Usually combinations, i.e. PD, PI or PID, are used