A structural approach for modelling performance of workflow systems

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Abstract

In this paper, we discuss a structural approach to automatic performance modelling of workflow systems. This uses a synthesis of performance evaluation process algebra (PEPA) and a pattern-based hierarchical expression scheme. Such systems are important in cases where the performance models must be updated regularly, and dynamically, based on the current state of the resources.

Keywords: Workflow systems, Performance evaluation, Patterns, Skeletons, Dynamic scheduling.

1 Introduction

Workflow systems and workflow languages are widely deployed in computing today. One example is the BPEL language, which structures a composition of Web Services into an orchestration in which simpler services are aggregated into a composite. Other examples are found in structured parallel programming where (often sequential) sub-tasks are structured into a parallel assembly. The technical agenda behind workflow description is to have a high-level, concise description of the structure of the computation which allows the workflow to be readily re-shaped in order to find a good mapping of tasks onto computing resources. The workflow designer is concerned with achieving a suitable throughput of jobs while satisfying constraints on the utilisation of components such as servers and other execution environments.

However, the composition languages which allow sub-tasks to be composed into a workflow do not usually provide a mechanism for assessing whether or not the

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new version of the workflow is likely to improve on the performance of the previous one. Furthermore, languages suitable for performance modelling such as stochastic process algebras are not usually workflow-structured and do not have linguistic apparatus to express sub-task composition. Our aim in this paper is to bridge this gap by automatically generating process algebra models from workflow descriptions and thus allowing workflow designers to compile their workflows into process algebra models suitable for performance evaluation via steady-state or transient analysis or verification via probabilistic model-checking. In the present paper we focus on the results which can be obtained by Markovian steady-state analysis of the process algebra model. We use an abstraction of real-world workflow composition languages and use Performance Evaluation Process Algebra (PEPA) [1] as our process algebra. Our examples are drawn from the domain of structured parallel programming.

2 Background

In this section we provide brief descriptions of the stochastic process algebra used and the approach to structured parallel programming based on algorithmic skeletons. For further details the reader should consult [1] and [2][3].

2.1 Performance Evaluation Process Algebra (PEPA)

PEPA is a Markovian process algebra containing *components* which are synchronised on timed *activities*.

$$S ::= (\alpha, r).S \mid S + S \mid C_S \qquad \text{(prefix, choice and component name)}$$
$$P ::= P \underset{L}{\bowtie} P \mid P/L \mid C \qquad \text{(parallel, hiding and component name)}$$

Here S denotes a *sequential component* and P denotes a *model component* which executes in parallel. C stands for a constant which denotes either a sequential component or a model component as introduced by a definition. C_S stands for constants which denote sequential components. Activity names (α) are used in CSP-style parallel composition with synchronisation ($\underset{L}{\bowtie}$) and in hiding sets (L). The duration of an activity is quantified by an exponentially-distributed random variable (r). The choice operator ($+$) enables the activities of its two operands. The first activity to complete distinguishes one of them: the other is discarded. The system will behave as the derivative resulting from the evolution of the chosen component.

The language definition is expressed in [1] via a small-step structured operational semantics which maps PEPA models onto Continuous-Time Markov Chains (CTMCs). The PEPA process algebra benefits from formal semantic descriptions of different characters which are appropriate for different uses. A denotational semantics for the language maps PEPA models to elements of metric spaces [4]. A continuous-space semantics maps PEPA models to a system of ordinary differential equations (ODEs) [5], admitting different solution procedures. We use the Markovian semantics in the present paper.

In a PEPA model if we define the stochastic process $X(t)$, such that $X(t) = C_i$ indicates that the system behaves as component C_i at time t, then $X(t)$ is a Markov process. For finite state PEPA models whose derivation graph is strongly connected,

(ergodic Markov process) the equilibrium distribution of the model, $\mathbf{\Pi}$, is found by solving the matrix equation

$$\mathbf{\Pi} \boldsymbol{Q} = \mathbf{0}$$

subject to the normalisation condition

$$\sum \mathbf{\Pi}(C_i) = 1$$

Generating the CTMC underlying a PEPA model, and finding its steady state probability vector is rarely the objective of PEPA modelling. The objective is to construct a *reward structure* over the state space of the CTMC, to be used in conjunction with the steady state probability vector to derive performance measures.

2.2 The notion of algorithmic skeletons

The skeletal approach to the design of parallel programming systems proposes that the complexity of parallel programming be contained by restricting the mechanisms through which parallelism can be introduced to a small number of architecture independent constructs, originally known as "algorithmic skeletons". Each skeleton specification captures the logical behaviour of a commonly occurring pattern of parallel computation, while packaging and hiding the details of its concrete implementation. Provision of a skeleton based programming methodology simultaneously raises the level of abstraction at which the programmer operates and provides the scheduling system with crucial information on the temporal and structural interaction patterns exhibited by the application. Responsibility for exploiting this information passes from programmer to compiler and/or run-time system. To obtain such detailed information from an equivalent *ad-hoc* message passing program is impossible in the general case. In this paper, we show how the structural information can be used to construct PEPA performance models of the application.

3 Expressing workflow systems with skeletons

To automate generation of performance models, a given workflow system must be first expressed in a form which captures its essence. In this paper, we adopt a pattern-based approach, which is based on the notion of algorithmic skeletons—a system that was designed to enrich, and simplify, structural development of distributed and parallel applications (see Section 2.2). To facilitate a thorough treatment of the automation, we will focus on the following three basic skeletons.[6] The interface we present here is designed for performance model construction, hence the parameterization of rates. It might be employed directly by the human performance modeller, or generated from an application oriented interface to the same skeletons.

Pipeline skeleton A pipeline skeleton arranges a set of components sequentially, so that data units entering the pipeline are processed in each of these components in turn (in the order the components are specified) before the final result leaves the pipeline. In our approach to expressing workflow systems, we will use the following construct to specify a pipeline:

[6] Extensions to these basic forms are available in the tool: `http://groups.inf.ed.ac.uk/enhance/`.

`pipe`(<number of components>);

The components contained within a skeleton construct could be either skeleton components (which results in hierarchical nesting), or task components (where the data units are processed). In the latter case, a task component is specified with the following construct:

`task`(<component name>, <rate>);

Here <rate> is the rate at which each of the data units entering the task component is processed—used while modelling the task's computational performance.

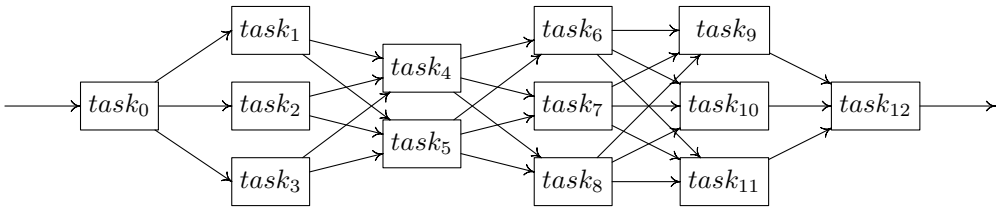
Deal skeleton A deal skeleton replicates a given task component in parallel, so that data units entering the deal could be processed by one of the replicated components, before the final result leaves the deal. The task component which receives a given data unit is chosen by a round-robin data distribution policy. We use the following construct to specify a deal:

`deal`(<number of replications>, <component name>, <rate>);

Farm skeleton A farm skeleton is similar to the deal: a given task component is replicated in parallel so that data units entering the farm are processed by one of the components. The difference, however, is that the process of choosing the task component that should receive a given data unit is unpredictable, being dynamically demand-driven upon completion of earlier computations. We use the following construct to specify a farm:

`farm`(<number of replications>, <component name>, <rate>);

We shall now illustrate the usage of these constructs by expressing a concrete example. Imagine a workflow system similar to the one shown below:



Here, we have a six stage [7] pipeline at the highest level; some of these stages are task components (for example, stages 1 and 6), while some are hierarchical skeleton nestings (say, for example, stage 2 is a Farm, while stage 3 is a Deal).

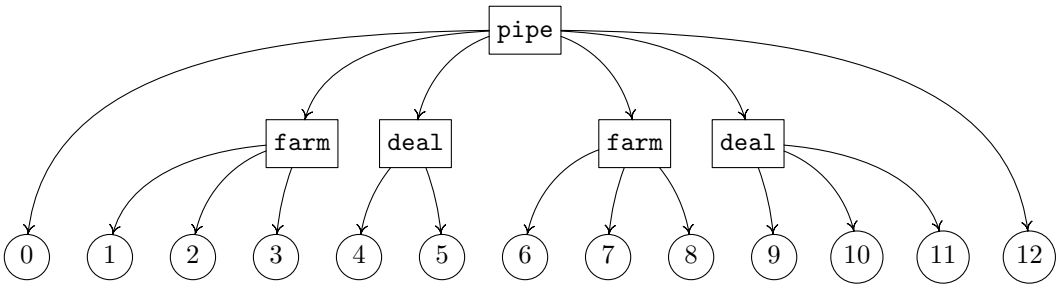
The skeleton based expression of the above workflow system is shown on the right. This description is hierarchical—each subtree is described depth-first, left-to-right across the same subtree level. We proceed to the next subtree in the same level only after all the previous subtrees have been described completely; i.e. there are no skeleton nestings with insufficient task assignments.

```
pipe(6);
task("task", 1.0);
farm(3, "task", 3.0);
deal(2, "task", 2.0);
farm(3, "task", 3.0);
deal(3, "task", 3.0);
task("task", 1.0);
```

It may be easier to view the whole exercise as a step-wise refinement of the system description, where we conceive the system at the highest level and proceed

[7] The components of a pipeline are frequently referred to as "stages".

with refinements until the lowest level descriptions consist of task components only.



In light of the discussions to follow, it would be prudent to mention here that, for every workflow description, a hierarchical tree data structure is maintained by the model generation tool, internally. We shall refer to this data structure as the *skeleton hierarchy tree* (shown above for our example workflow).

The skeleton hierarchy tree encapsulates most of the information provided in the description (the overall structural and component details of the system). Additional information is derived from this tree automatically, when needed (for example, the data dependency graph connecting the tasks). We shall now discuss these in detail.

4 Generation of performance models

Generation of PEPA performance models, from a given description of a workflow system, can be divided into three phases. In the first phase, the directed acyclic graph (which represents data dependency between task components) is derived from the skeleton hierarchy tree. This graph is then used in the following phases. In the second phase, the process definitions for each of the task components are determined. Finally, in the third phase, the overall system is modelled by combining the task components, and skeletal components, based on their hierarchical organisation. The final phase is important because it completes the performance model by establishing the synchronisation sets, which will be used by the model solver while synchronising task components at different levels of composition.

4.1 Determination of the directed acyclic graph

Let the directed acyclic graph $\mathcal{G}(\mathcal{T}, \mathcal{E})$, where $\mathcal{T}$ is the set of task components and $\mathcal{E}$ is the set of directed edges connecting task components, represent the data dependency graph which corresponds to the skeleton hierarchy tree (see figure in the previous page). To derive such a graph from a given skeleton hierarchy tree, we use recursive preorder tree traversal algorithms, described as follows:

To every task in $\mathcal{T}$, assign a unique index i, where $0 \leq i < |\mathcal{T}|$. We will use the notation t_i to mean: "task component with index i," or sometimes, "task i". To concretely implement the graph $\mathcal{G}$, associate with every task, t_i, two ordered sets of task indices: (1) the source list $\mathcal{S}_i$, which gives the set of tasks in $\mathcal{T}$ from which task i can receive data; (2) the destination list $\mathcal{D}_i$, which gives the set of tasks to which task i can send data. They are formally defined as follows:

$$\mathcal{S}_i = \{j : (t_j, t_i) \in \mathcal{E}, i \neq j\} \text{ and } \mathcal{D}_i = \{j : (t_i, t_j) \in \mathcal{E}, i \neq j\}.$$

Algorithm 1 $GS(node)$: Generate source lists from the skeleton hierarchy tree

```
n := node.nchildren // Number of children nodes
node.slist := parent.slist // Inherit source list from parent node
node.stype := parent.stype // Inherit source pattern of interaction
for i := 0 to (n − 1) do
        GS(node.child_i) // Recursively generate children source lists
if node.type is task then // Node is a task component
        v := {x : where x = node.index}
        if parent.type is deal or farm then // Parent is a replicable skeleton
                temp_node := v
        else
                parent.slist := v
                parent.stype := pipe // Update source pattern
else if node.type is pipe then // Node is a pipeline skeleton
        if parent.type deal or farm then // Pipeline within replicable
                temp_node := node.slist
        else
                parent.slist := node.slist
                parent.stype := node.type // Update source pattern
else if node.type deal or farm then // Node is a replicable skeleton
        m := ⋃_{i=0}^{n−1} temp_i // Merge temporary lists on children
        if parent.type deal or farm then // Replicable within replicable
                temp_node := m
        else
                parent.slist := m
                parent.stype := node.type // Update source pattern
```

When it is clear from context which task we are referring to, we may choose to drop the subscripts in $\mathcal{S}_i$ and $\mathcal{D}_i$. It is important to note here that these sets only list all the possible predecessors (or successors) of a task—the effective set with which the task eventually communicates is determined from these sets by applying the corresponding source (or destination) *pattern of interaction*, which we will be discussing shortly. For example, given a pipeline containing two consecutive deals, the general case is that all tasks in the first deal will be in the source list for the second deal. However, if the deals are of the same size, then in fact, a task in the second deal will only ever receive data from the task in the first deal which has the same intra-deal sibling rank (the effective set is therefore a *singleton* set).

It can be observed that these two sets, in combination with the task set $\mathcal{T}$, completely define the directed acyclic graph $\mathcal{G}$. We therefore use recursive tree traversal algorithms to generate these sets from the skeleton hierarchy tree. In Algorithm 1, we show the manner in which the source lists are derived from the skeleton hierarchy tree; a similar algorithm is used to derive the destination lists.

4.2 Modelling the task components

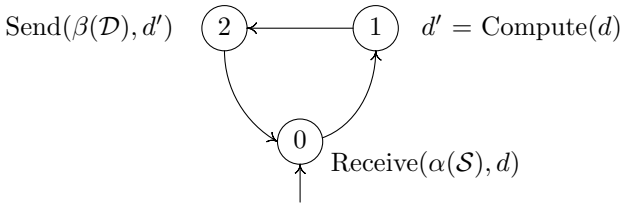
In general, a task component in a workflow system is a process which repeatedly undergoes the transitions: *receive* → *compute* → *send*. Depending on the higher-level

structure containing the task, these three basic activities are specialised accordingly. In some cases, for example, some of these activities are skipped (as in producer tasks, which does not perform receive activities; or consumer tasks, where send activities are never performed).

As noted in the concluding paragraphs of the previous section, when a task communicates with other tasks, the tasks with which the communications are performed are based on an effective subset of $\mathcal{S}$ (or $\mathcal{D}$), determined by the pattern of interaction. We define this *pattern of interaction* as a function over the source (or destination) list, which chooses task indices from the corresponding list, thus establishing the effective subset for the current communication. In essence, this function outlines for each task how the task should interact with the remaining tasks in the skeleton hierarchy tree: the source pattern, therefore, defines how data should be received; the destination pattern, how data should be sent.

The patterns of interaction for a given task correspond to the location of the task within the skeleton hierarchy tree. As we can see in Algorithm 1, the source pattern (`stype`), is set with respect to the skeleton components containing the task; the destination pattern is set similarly.

If we respectively represent the source and destination patterns of interaction with α and β, we can summarise a task as follows:



From this abstract representation, it is clear that the PEPA process definition of a task component is determined by the subsets $\alpha(\mathcal{S})$ and $\beta(\mathcal{D})$; and the relationship between α and β, as required by the transition *receive* $\rightarrow$ *compute* $\rightarrow$ *send*. Since tasks can have different α and β, we have to determine process definition templates for all the possible pattern combinations. Let us represent, for brevity, such combinations with $\{\alpha(\mathcal{S}) \rightarrow t_i \rightarrow \beta(\mathcal{D})\}$; meaning, "task i receives data based on the source pattern α; and sends data based on the destination pattern β". When either of the patterns are not defined (as discussed at the beginning of this section), we represent this with a $*$ (as in $\{* \rightarrow t_i \rightarrow \beta(\mathcal{D})\}$ for a producer task).

Furthermore, since enumerating all the cases can be quite involving, we shall condense the case investigations further by making some observations on the relationship between the different patterns of interaction (based on the definition of the skeleton constructs, see Section 3). These observations are: (a) the *Deal* pattern function is a special case of the *Farm*, where non-determinism in the *Farm* is removed by enforcing a round-robin data distribution policy. (b) the *Pipeline* pattern function is a special case of the *Deal*, where the source (or destination) list is a *singleton* set. Based on the later observation, discussion of cases involving the *Pipeline* will be ignored, since it is covered in the cases with *Deal*. We will, however, cover the combinations of *Deal* and *Farm* pattern functions.

Case $\{* \rightarrow t_i \rightarrow Deal(\mathcal{D})\}$ In this case, t_i is a producer task. This task produces

data units, which are then sent to one of the tasks in $\mathcal{D}$, chosen according to the round-robin policy. The corresponding PEPA process definition, where $n = |\mathcal{D}|$ and μ is the rate of the computational activity λ, is expressed as follows:

$$t_i \stackrel{def}{=} (\lambda, \mu).(move_{i0}, \top).(\lambda, \mu).(move_{i1}, \top). \cdots .(\lambda, \mu).(move_{i(n-1)}, \top).t_i;$$

Here $move_{ij}$ represents communication of data from task i to the jth task in $\mathcal{D}$. We choose this notation instead of, say $send_{ij}$, because these activities will be used again later when we define the synchronisation sets. If the latter notation was adopted, we are required to define a system for matching up corresponding $send_{ij}$ and $receive_{ji}$ pairs (which is, in fact, unnecessary).

Case $\{Deal(\mathcal{S}) \rightarrow t_i \rightarrow *\}$ In this case, t_i is a consumer task. The task receives data units from one of the tasks in $\mathcal{S}$, which it then consumes. By following a notation similar to the previous case, we have the following process definition:

$$t_i \stackrel{def}{=} (move_{0i}, \top).(\lambda, \mu).(move_{1i}, \top).(\lambda, \mu). \cdots .(move_{(n-1)i}, \top).(\lambda, \mu).t_i;$$

Here $n = |\mathcal{S}|$, and μ is the rate at which the task consumes each data unit. Based on arguments similar to the one used in the previous case, we use $move_{ji}$ to represent communication of data from the jth task in $\mathcal{S}$ to task i.

Case $\{Deal(\mathcal{S}) \rightarrow t_i \rightarrow Deal(\mathcal{D})\}$ In this case, t_i is an intermediate task: data units received from one of the tasks in $\mathcal{S}$ is processed, and the result is sent to one of the tasks in $\mathcal{D}$. In each instance of the send and receive communications, the effective task is chosen independently based on the round-robin distribution policy.

When the cardinalities of the source and destination lists are the same, $p = |\mathcal{S}| = |\mathcal{D}|$, the process definition is simple, as shown below:

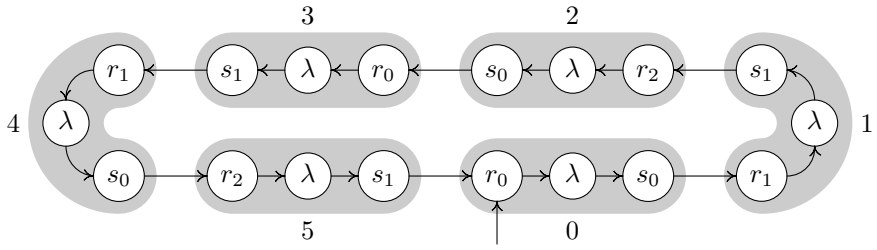
$$\begin{aligned} t_i \stackrel{def}{=} & (move_{0i}, \top).(\lambda, \mu).(move_{i0}, \top). \\ & (move_{1i}, \top).(\lambda, \mu).(move_{i1}, \top). \\ & \cdots \\ & (move_{(p-1)i}, \top).(\lambda, \mu).(move_{i(p-1)}, \top).t_i; \end{aligned}$$

When $|\mathcal{S}| \neq |\mathcal{D}|$, however, there exists no immediate correspondence between the source and destination tasks. It is therefore necessary to resolve this mismatch until we find a repeatable sequence of activities. If we define *periodicity* as the number of distinct *receive* $\rightarrow$ *compute* $\rightarrow$ *send* transitions after which the repetition ensues, it is easy to see that the periodicity is the *least common multiple* of $|\mathcal{S}|$ and $|\mathcal{D}|$. Based on this, we have the following process definition:

$$\begin{aligned} t_i \stackrel{def}{=} & (move_{xi}, \top).(\lambda, \mu).(move_{iy}, \top). \\ & \cdots (\text{repeat } p \text{ times, incrementing } k \text{ in every iteration}).t_i; \end{aligned}$$

where $0 \leq k < p$, $x = k \mod |\mathcal{S}|$ and $y = k \mod |\mathcal{D}|$.

When $|\mathcal{S}| = 3$ and $|\mathcal{D}| = 2$, for example, the steady-state activity sequence is



which gives the following process definition:

$$t_i \stackrel{def}{=} (move_{0i}, \top).(\lambda, \mu).(move_{i0}, \top).(move_{1i}, \top).(\lambda, \mu).(move_{i1}, \top).$$
$$(move_{2i}, \top).(\lambda, \mu).(move_{i0}, \top).(move_{0i}, \top).(\lambda, \mu).(move_{i1}, \top).$$
$$(move_{1i}, \top).(\lambda, \mu).(move_{i0}, \top).(move_{2i}, \top).(\lambda, \mu).(move_{i1}, \top).t_i;$$

The above three cases can be used to completely define task components with any combination of *Pipeline* and *Deal*. We shall now extend this by introducing cases which account for the non-determinism associated with a *Farm* skeleton.

Case $\{* \rightarrow t_i \rightarrow Farm(\mathcal{D})\}$ In this case, the task is a producer. The main difference, however, is the non-determinism, which we capture with the *choice*(+) operator of PEPA. This is shown in the following process definition:

$$t_i \stackrel{def}{=} (\lambda, \mu).t_i';$$
$$t_i' \stackrel{def}{=} (move_{i0}, \top).t_i + (move_{i1}, \top).t_i + \cdots + (move_{i(n-1)}, \top).t_i;$$

where $n = |\mathcal{D}|$. After a data unit has been produced, it is sent to any one of the tasks in $\mathcal{D}$, which brings back the task to the data production state.

Case $\{Farm(\mathcal{S}) \rightarrow t_i \rightarrow *\}$ In this case, the task is a consumer. The process definition is similar to the previous case, as shown in the following definition:

$$t_i \stackrel{def}{=} (move_{0i}, \top).t_i' + (move_{1i}, \top).t_i' + \cdots + (move_{(n-1)i}, \top).t_i';$$
$$t_i' \stackrel{def}{=} (\lambda, \mu).t_i;$$

where $n = |\mathcal{S}|$. After a data unit has been received from any one of the tasks in $\mathcal{S}$, it is consumed; consequently bringing the task back to the receiving state.

Case $\{Farm(\mathcal{S}) \rightarrow t_i \rightarrow Farm(\mathcal{D})\}$ In this case, the t_i is an intermediate task. The process definition for such tasks can be achieved by combining the previous two cases, as shown in the following:

$$t_i \stackrel{def}{=} (move_{0i}, \top).(\lambda, \mu).t_i' + \cdots + (move_{(x-1)i}, \top).(\lambda, \mu).t_i';$$
$$t_i' \stackrel{def}{=} (move_{i0}, \top).t_i + (move_{i1}, \top).t_i + \cdots + (move_{i(y-1)}, \top).t_i;$$

where, $x = |\mathcal{S}|$ and $y = |\mathcal{D}|$. Data units are received from any one of the tasks in $\mathcal{S}$, processed, and the result sent to any one of the tasks in $\mathcal{D}$.

Case $\{Deal(\mathcal{S}) \rightarrow t_i \rightarrow Farm(\mathcal{D})\}$ In this case, t_i is an intermediate task. What is unique about this task is that for every data received in round-robin fashion, the result is sent non-deterministically. Hence, we have a choice composition for each of the tasks from which a data unit was received. For tasks such as t_i, we have the following process definition:

$$t_i \stackrel{def}{=} (move_{0i}, \top).(\lambda, \mu).t_i^0;$$
$$t_i^0 \stackrel{def}{=} (move_{i0}, \top).t_i^1 + (move_{i1}, \top).t_i^1 + \cdots + (move_{i(y-1)}, \top).t_i^1;$$
$$\cdots$$
$$t_i^{x-1} \stackrel{def}{=} (move_{(x-1)i}, \top).(\lambda, \mu).t_i^x;$$
$$t_i^x \stackrel{def}{=} (move_{i0}, \top).t_i + (move_{i1}, \top).t_i + \cdots + (move_{i(y-1)}, \top).t_i;$$

where $x = |\mathcal{S}| - 1$ and $y = |\mathcal{D}|$.

Case $\{Farm(\mathcal{S}) \rightarrow t_i \rightarrow Deal(\mathcal{D})\}$ This case is similar to the previous, except for the reversal in the placement of the choice composition. We therefore have the following process definition:

$$t_i \stackrel{def}{=} (move_{0i}, \top).t_i^0 + (move_{1i}, \top).t_i^0 + \cdots + (move_{(x-1)i}, \top).t_i^0;$$
$$t_i^0 \stackrel{def}{=} (\lambda, \mu).(move_{i0}, \top).t_i^1;$$
$$\cdots$$
$$t_i^{y-1} \stackrel{def}{=} (move_{0i}, \top).t_i^y + (move_{1i}, \top).t_i^y + \cdots + (move_{(x-1)i}, \top).t_i^y;$$
$$t_i^y \stackrel{def}{=} (\lambda, \mu).(move_{iy}, \top).t_i;$$

where $x = |\mathcal{S}|$ and $y = |\mathcal{D}| - 1$.

	*	Pipe	Deal	Farm
*	0	1	2	3
Pipe	4	5	6	7
Deal	8	9	10	11
Farm	12	13	14	15

In Algorithm 2, we incorporate all the above cases for generating the corresponding process definition of the tasks in the skeleton hierarchy tree. The table on the righthand side summarises the combination number of the patterns—*Pipeline*, *Deal* and *Farm*—that are used in the algorithm. The columns list destination patterns; whereas, the rows enumerate source patterns. In this algorithm, the expression $\mathcal{S}(j)$ gives the mth task index in $\mathcal{S}$, where $m = j \mod |\mathcal{S}|$; the corresponding expression for the destination list, $\mathcal{D}(j)$, is defined similarly.

We use the interface `Output:` to emit segments of the generated process definition. For every invocation to this interface, all the characters immediately following this, until the end of line, are emitted as part of the process definition. Also note that the order in which `Output:` is invoked is significant to the validity of the generated process definition.

To generate all the process definitions of all the tasks in the skeleton hierarchy tree, we traverse the hierarchy tree and invoke Algorithm 2 for all the nodes which is a `task` node. Once this is done, we have completed the second phase of performance model generation. We therefore proceed with the final phase where we define the synchronisation sets. Before we proceed, it will be worth recalling that the $move_{ij}$ and $move_{ji}$ activities, which correspond to the communications between tasks, will be used while defining these synchronisation sets.

4.3 Modelling the system

All the process definitions generated at the end of the second phase only model the performance of each task component, independently of the others. Since the workflow system is a cooperative manifestation of these tasks, these tasks must be

Algorithm 2 $GP(i)$: Generate process definition for task i

`Output:` $t_i \stackrel{def}{=}$
$l :=$`lcm`$(|\mathcal{S}|, |\mathcal{D}|)$ // Least common multiple, or sum if either is zero
if case is 1 or 2 **then** // Predecessor $*$, successor *Pipe* or *Deal*
    for $0 \leq j < l$ **do** `Output:` $(\lambda, \mu).(move_{i,\mathcal{D}(j)}, \top)$.
else if case is 4 or 8 **then** // Predecessor *Pipe* or *Deal*, successor $*$
    for $0 \leq j < l$ **do** `Output:` $(move_{\mathcal{S}(j),i}, \top).(\lambda, \mu)$.
else if case is 5, 6, 9 or 10 **then** // Predecessor and successor *Pipe* or *Deal*
    for $0 \leq j < l$ **do** `Output:` $(move_{\mathcal{S}(j),i}, \top).(\lambda, \mu).(move_{i,\mathcal{D}(j)}, \top)$.
else if case is 3 **then** // Predecessor $*$, successor *Farm*
    `Output:` $(\lambda, \mu).t'_i; t'_i \stackrel{def}{=} (move_{i,\mathcal{D}(0)}, \top).t_i$
    for $1 \leq j < |\mathcal{D}| - 1$ **do**
        `Output:` $+(move_{i,\mathcal{D}(j)}, \top)$.
        if $(|\mathcal{D}| > 1) \wedge (j < |\mathcal{D}| - 1)$ **then** `Output:` t_i
else if case is 12 **then** // Predecessor *Farm*, successor $*$
    `Output:` $(move_{\mathcal{S}(0),i}, \top).t'_i$
    for $1 \leq j < |\mathcal{S}|$ **do**
        `Output:` $+(move_{\mathcal{S}(j),i}, \top).t'_i$
    `Output:` $; t'_i \stackrel{def}{=} (\lambda, \mu)$.
else if case is 7 or 11 **then** // Predecessor *Pipe* or *Deal*, successor *Farm*
    for $0 \leq j < |\mathcal{S}|$ **do**
        `Output:` $(move_{\mathcal{S}(j),i}, \top).(\lambda, \mu).t^j_i; t^j_i \stackrel{def}{=} (move_{i,\mathcal{D}(0)}, \top)$.
        if $j < |\mathcal{S}| - 1$ **then** `Output:` t^{j+1}_i
        else `Output:` t_i
        for $1 \leq k < |\mathcal{D}|$ **do**
            `Output:` $+(move_{i,\mathcal{D}(k)}, \top)$.
            if $j < |\mathcal{S}| - 1$ **then** `Output:` t^{j+1}_i
            else if $j < |\mathcal{D}| - 1$ **then** `Output:` t_i
else if case is 13 or 14 **then** // Predecessor *Farm*, successor *Pipe* or *Deal*
    for $0 \leq j < |\mathcal{D}| - 1$, incrementing x in each step by 2 **do**
        `Output:` $(move_{\mathcal{S}(0),i}, \top).t^x_i$
        for $1 \leq k < |\mathcal{S}|$ **do**
            `Output:` $+(move_{\mathcal{S}(k),i}, \top).t^x_i$
        `Output:` $; t^x_i \stackrel{def}{=} (\lambda, \mu).(move_{i,\mathcal{D}(j)}, \top)$.
        if $j < |\mathcal{D}| - 1$ **then** `Output:` $t^{x+1}_i; t^{x+1}_i \stackrel{def}{=}$
else if case is 15 **then** // Both predecessor and successor *Farm*
    `Output:` $(move_{\mathcal{S}(0),i}, \top).(\lambda, \mu).t'_i$
    for $1 \leq j < |\mathcal{S}|$ **do**
        `Output:` $+(move_{\mathcal{S}(j),i}, \top).(\lambda, \mu).t'_i$
    `Output:` $; t'_i \stackrel{def}{=} (move_{i,\mathcal{D}(0)}, \top)$.
    for $1 \leq j < |\mathcal{D}|$ **do**
        `Output:` $t_i + (move_{i,\mathcal{D}(j)}, \top)$.
`Output:` t_i;

Algorithm 3 $GM(node, nchild)$: Generate system model

```
i := node.index
r := node.rank // Node rank among siblings
if node.type is task then
    Output: t_i
    if r < nchild − 1 then
        if parent.type ≠ (deal or farm) then
            Output ⋈_L where L = {move_{i,j} : j = D_i(k), 0 ≤ k < |D_i|}
        else
            Output: ||
else
    Output: (
    for 0 ≤ j < node.nchild do
        GM(child_j, node.nchild)// Recursively model subtree
    Output: )
    if r < nchild − 1 then
        if parent.type ≠ (deal or farm) then
            Output ⋈_L where
            L = {move_{x,y} : y = D_i(j), x = S_y(k), 0 ≤ j < |D_i|, 0 ≤ k < |S_y|}
        else
            Output: ||
```

synchronised accordingly with respect to the level of hierarchical composition. This is done in the final phase of model generation, which we shall now discuss.

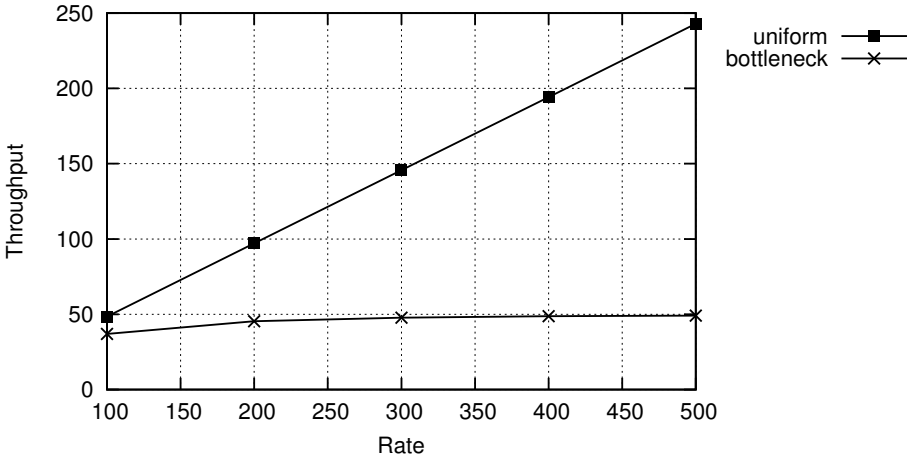
At every level of the skeleton hierarchy tree, each subtree corresponds to a closed sub-system where only the boundary task components on either side interacts with their adjacent sibling subtrees. The task components which are inside this sub-system (the intermediate components) are synchronised with other task components within the same sub-system—there is no cross boundary synchronisation. Hence, the final phase of model generation proceeds by defining synchronisation sets between adjacent sub-trees in each level of the hierarchy tree; which are refined repeatedly until all the tasks components are synchronised.

We use Algorithm 3 to perform this final phase. In this algorithm, we use depth-first preorder tree traversal again. Since the synchronisation set between two subtrees can be expressed with respect to one of these subtrees, we choose a *forward* expression approach where the synchronisation set for a subtree is determined after the sub-system which corresponds to that subtree has been synchronised. We can see this in the algorithm: whenever the node is a task, we emit that task, and generate the synchronisation sets with which this task synchronises with all its successor tasks; when the node is a skeleton component, we generate the synchronisation set by accounting for the tasks on the "send" boundary of this sub-system, which interacts with the tasks on the "receive" boundary of the succeeding sub-tree.

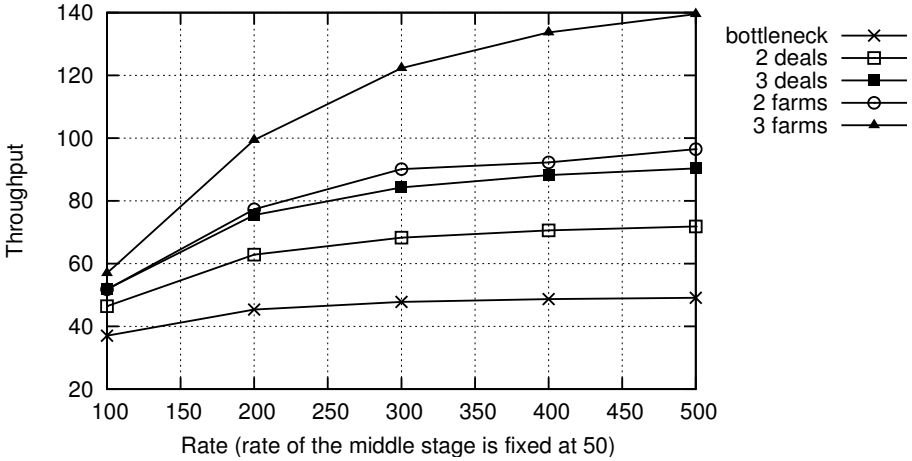
4.4 Analysis of performance results

We will now discuss a numerical analysis of performance results which demonstrates that the generated models produce results that are consistent with our intuition.

In the following analysis, we draw heavily on one practical application of the automatic model generation approach: dynamic scheduling of tasks in parallel and distributed applications [6][7]. When viewed from a higher-level abstraction, most parallel and distributed applications can be considered as workflow systems where groups of instructions form a task, which is then scheduled to different processors. In general, the highest-level structure of these systems often forms a *Pipeline*. In the following figure, we plot the predicted performance (measured in throughput) of a pipeline application with five stages [8]:



As we can see, the throughput of the pipeline increases linearly as long as the rates at which data units are processed by each of the stages increases uniformly. However, when some stages of the pipeline become a bottleneck (in the above figure, we have made the middle stage a bottleneck, with its task rate kept at a constant value of 50) it is often the case that the overall performance of the pipeline degrades, staying almost at the same level (since, the throughput is determined by the worst performing task) even when the rates of the other stages are increased. This shows that in order to improve the overall performance of *Pipeline* application, we must ensure uniform task rates.



[8] To focus our analysis on the task rates, we have set the same communication rates for all the inter-task communications. This is necessary in order to minimise the effect of the communications on the relative throughputs while we consider performance due to different task rates.

One way of ensuring uniformity of task rates is the replication of the worst performing task so that multiple tasks of the same kind can share the load. As we have discussed in Section 3, this could be done in two ways. First, we use a *Deal* where the bottleneck stage is replicated in a manner so that data is processed in round-robin fashion. Second, we use a *Farm* where the data distribution is not fixed, but probabilistic. In the figure immediately preceding this paragraph, we show the throughputs for five variations of the replication: (1) the *Pipeline* application with a middle stage bottleneck (same as shown in the previous figure); (2) the case when the middle task is replicated twice as a *Deal*, (3) thrice as a *Deal*; (4) the case when the middle task is replicated twice as a *Farm*, (5) thrice as a *Farm*.

As we can see, the performance of the pipeline improves when the bottleneck stage is replicated. We also see that the throughput depends on the number of replications in relation to the degree of deviation of the rate of the bottleneck stage from the rate of the others; i.e., the throughput increases more sharply when the combined rate of the replicated tasks is closer to the rate of the others, than it does when the uniform rate is higher than the combined rate. This can be seen in the saturation curve as we proceed towards higher uniform rates in the other stages. We also notice that the throughputs of the *Farm* based replications are higher than those of the *Deal* replications. This, we believe, is a consequence of the strict round-robin policy that is imposed on the *Deal* replications; whereas, the policy for the *Farm* replications is determined responsively based on the given rates.

5 Conclusion

In this paper, we have discussed an approach for generating PEPA based performance models for workflow systems based on a pattern-oriented hierarchical expression scheme. Such automatic approaches are important in systems where the model must be updated regularly, and dynamically, depending on the current state of the resources. We have demonstrated in a practical setting that the generated models produce throughputs that are consistent with the expected outcomes.

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