On LTE-WiFi Coexistence and Inter-Operator Spectrum Sharing in Unlicensed Bands: Altruism, Cooperation and Fairness

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ABSTRACT

The coexistence of LTE-Unlicensed (LTE-U) and WiFi in unlicensed spectrum is studied in the context of airtime sharing. We consider core problem where a set of LTE-U cells from different operators share the same channel as a co-located WiFi access point (AP). We assume that LTE-U cells utilize Listen-Before-Talk (LBT) as the default channel access mechanism. Principally, we deal with the following question: how should an operator's LTE-U cell adjust its contention window in order to provide a fair coexistence both with WiFi and co-located LTE-U cells of other operators? We consider that LTE-U cells behave altruistically both among themselves and to WiFi. Cooperation of LTE-U cells is studied using a coalition formation game framework which is based on the well-known Shapley value. We define a payoff configuration scheme in the coalition game which involves altruism. We prove that the coalitional game is always zero-monotonic, and Shapley value is also max-min fair. We compare airtime sharing performance of Shapley value with weighted proportional fairness via numerical results and show that Shapley value provides much better fairness than proportional fairness as determined by entropy and Jain's index metrics while having roughly equal average airtime.

CCS Concepts

•Theory of computation → Network games;

Keywords

LTE-U; WiFi; Listen-Before-Talk; Altruism; NTU Coalitional Games; NTU Shapley Value; Fairness; Axiomatic Fairness; Proportional Fairness; Max-min Fairness; Jain's Index; Entropy;

1. INTRODUCTION

To cope with the exponential increase in the data traffic, the operation of LTE is expected to extend into unlicensed bands, an approach known as LTE-Unlicensed (LTE-U). LTE-U is suitable for LTE small cells due to the regulatory requirements that limit the maximum transmit power in unlicensed bands. Therefore, LTE-U will not only improve the spectrum efficiency of the unlicensed band, but also enhance the network capacity as LTE small cells will be able to offload data plane traffic from licensed to unlicensed bands.

When operating on the same channel as WiFi, LTE-U imposes new challenges such as LTE-WiFi interference and inter-operator interference [1]. The problem of LTE-WiFi coexistence has been evaluated in [2, 3] where it has been shown in [2] that the WiFi terminal throughput could drop by 70% and even 100%, depending on the scenario, if LTE-WiFi interference is not mitigated. The authors in [3] show that the impact of LTE-U on the performance of WiFi depends on the channel bandwidth, center frequency and MIMO configuration and can be heavily degraded for some scenarios. This degradation in the performance of WiFi is due to the difference in the MAC layers of both technologies. LTE is a scheduling based technology and is designed for the exclusive use of the channel. On the other hand, WiFi is a contention based technology which adopts a carrier sense multiple access with collision avoidance (CSMA/CA) mechanism where it would listen first to the channel before transmitting and hence would risk experiencing starvation in presence of LTE if no additional measures are taken. In this direction, Listen-Before-Talk (LBT) has emerged as the coexistence mechanism that would be default in newer 3GPP standards; with the LBT mechanism, which we also assume in our work, LTE-U cells sense the occupancy of unlicensed spectrum prior to accessing it.

As a "newcomer", an LTE-U cell will potentially be "harmful" to a WiFi cell. Moreover, in common scenarios where multiple different physical rates are used (to suit different channel conditions), airtime fairness is considered to be a better notion of fairness than throughput fairness even in a homogeneous WiFi network setting. We consider the notion of fair airtime sharing in LTE-U and WiFi coexistence scenario. with particular emphasis on the problem of inter-operator interference that arises because multiple LTE operators may operate co-located LTE small cells using the unlicensed spectrum. Unlike licensed frequency bands, unlicensed bands are free to use by anyone as long as the regulatory requirements (e.g., maximum transmit power) are met. In [4], the authors show that there is an upper limit on the number of operators that would want

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to operate in the unlicensed bands and propose a repeated game for multi-operators to share the unlicensed frequency bands. In fact, inter-operator interference results from the selfish behavior of the operators and could result in a degradation in the spectral efficiency if not managed. Therefore, we consider coordination among different LTE-U operators in order to share the unlicensed band in an altruistic way, as compared to a selfish one.

Altruism, preferring others to oneself when doing a good deed, is, according to the moralists, giving precedence to the common interests of the community over one's own interests; according to this mindset, it is devoting oneself to the lives of others in complete forgetfulness of all concerns of one's own, it is self-annihilation in the interests of others. Digitizing such a moral behavior is of course challenging. However, we shall try to overcome it by assigning a real value determining the level of altruism of a particular player to some other player which is a common approach in an altruistic game. Experimental and behavioral economics demonstrate that people altruistically sacrifice their own profit to punish unfair decisions by others [5]. Moreover, altruistic behavior is seen to be a long-term net utility optimization [6–8].

Cooperation is usually imagined as a behavior by which the agents are potentially better off. In a setting where strategic decision making exists, the cooperation is studied in the framework of coalitional games. If a coalitional game is a transferable utility (TU) game, it is supposed that the utility is freely transferable from one player to another. This is, in particular, possible in the presence of "ideal money", i.e. commodity whose utility is directly proportional to quantity, and independent of any other assets, which a player may have. In general, unfortunately, the situation is not so simple – players' utility for money may be not linear, it may depend on other assets of players, or, in some cases, side payments may even be forbidden. In such situations, it is better to represent each coalition's possibilities not by a single number, but rather by a set of all payoff vectors, which the coalition can obtain for its members. We then speak about coalition games with non-transferable utility (NTU games, for short).

Shapley value [9] is widely considered as a powerful solution concept which involves "fairness" when distributing the utility among the members of grand coalition (grand coalition corresponds to the case in which every player is willing to join the game). In an NTU game, finding Shapley value might be challenging or even the game may not posses one. So, the following question arises: if it is allowed, how to design a payoff configuration in order to guarantee the existence of Shapley value. In [10], a protocol for forming coalitions is proposed. It is studied in the context of NTU games and is proven that weighted utility transfers are feasible when everybody cooperates, then the expected sub-game perfect equilibrium payoff allocation anticipated before any implemented game is the Shapley value. We utilize this protocol in our analysis and design the payoff configuration such that Shapley value always exists.

In a nutshell, in this paper, we aim at achieving airtime fairness between LTE-U and WiFi AP and airtime fairness among different LTE-U cells through cooperation and altruism. This is achieved by adjusting the contention window of an LTE-U cell according to the Shapley value. We put forward a scheme for sharing the airtime using a dynamic model where LTE-U cells calculate their contention window cooperatively by taking into account each others' channel characteristics. In a slotted basis, each cell updates its own "altruistic weight" and calculates its contention window taking into account the altruistic weights of the other cells. To the best of our knowledge, this is the first work to apply altruistic modelling and Shapley value for a network resource allocation problem.

1.1 Our Contributions

First, we consider the multi-cell case and show how the airtime fraction of a particular LTE-U cell can be found. We then introduce a framework about the altruistic gain of the WiFi AP which is based on the traffic loads of LTE-U cells and the traffic per user of the WiFi AP.

Second, we model the interaction among LTE-U cells as a coalitional game which is based on so called non-transferable utility (NTU). The motivation is to cooperate in order to access the shared channel in a fair way. Fairness is sought to be attained thanks to the altruism in NTU Shapley value. The payoff of a particular player is chosen to be the airtime fraction. Each player calculates separately a weighted combination of logarithms of airtime fractions of all players, in a similar way like in proportional fairness scheme. We call the weights as altruistic weights which we choose to be the reciprocal of normalized raw throughput[1]. Then, we show how it corresponds to Shapley value that possesses several properties satisfying some degree of axiomatic fairness [9]. Thus, we attempt to involve altruism in Shapley value in order to increase fairness. We prove that considered coalitional game is always zero-monotonic which is a property for ensuring that the Shapley value always exists and also for having all players cooperate and be part of a grand coalition. Besides, we prove that the Shapley value provides maxmin fairness.

Third, we compare Shapley value and weighted proportional fairness in which the weights are set by an altruistic scheme. We measure the fairness quantitatively using the measures of airtime fraction, Jain's index and entropy [11]. In numerical results, we demonstrate that Shapley value provides better fairness than proportional fairness while resulting in similar airtime.

2. SYSTEM MODEL

We consider an environment where WiFi APs are already deployed and pre-assigned channels in which they serve their WiFi stations. LTE-U cells come to the medium which can be interpreted as "newcomers". Even though there is a flexibility in determining the channel access mechanism in unlicensed bands, there are regulatory restrictions in some regions of the world which are driving the use of Listen-Before-Talk (LBT) as the default channel access mechanism in newer 3GPP standards for LTE operation in unlicensed bands. As per ETSI, the LBT channel access mechanism can be designed both as Frame Based Equipment (FBE) where the transmit/receive structure is not directly demand-driven but has fixed timing; and Load Based Equipment (LBE) where the transmit/receive structure is not fixed in time but demand-driven. We assume that load based LBT is implemented in an LTE-U cell. LBE-LBT (henceforth referred to as LBT, unless explicitly stated otherwise) simply works as follows [12]: an initial clear channel assessment (CCA) of at least 20 μs is performed prior to a new transmission. If the equipment finds the channel to be clear, it may transmit immediately. In case the medium is sensed to be already occupied, the transmission is deferred and an extended CCA (ECCA) is performed until the channel is found to be idle. In an ECCA check,

[1] Throughput of an LTE-U cell can be given by $R = \alpha r$ where $0 \leq \alpha \leq 1$ is airtime fraction, and r is called as raw throughput.

the operating channel is observed for the duration of a random factor N_{CIS} multiplied by the CCA observation time. N_{CIS} defines the number of clear idle slots that need to be observed before initiation of the transmission. The value of N_{CIS} is randomly selected as $N_{\text{CIS}} \in [1, Q]$ every time an extended CCA is required and the value stored in a counter. The value of Q is selected by the manufacturer in the range of $[4, 32]$. The counter is decremented every time a CCA slot is deemed to be unoccupied. When the counter reaches zero, the equipment may transmit.

With the above background, our problem in this work is

> *how to adjust the contention window adaptively in order to provide a fair coexistence of LTE-U cells with a co-located WiFi cell.*

Moreover, we consider a multi LTE-U operator case as illustrated in Figure 1. Briefly, we assume that

1. LBT is default in LTE-U cells;
2. an LTE-U cell can adapt its contention window in order to provide a fair channel access with a WiFi AP;
3. LTE-U cells of different operators can cooperate which means that inter-operator cooperation is possible;
4. WiFi APs are assumed to coordinate implicitly through distributed coordination function (DCF).

We suppose that *LTE-U cells and a particular WiFi AP operates on the same amount of bandwidth in shared single unlicensed spectrum channel*. The throughput of a particular cell is thus determined by only the airtime which is the total amount of time during which the cell accesses the channel.

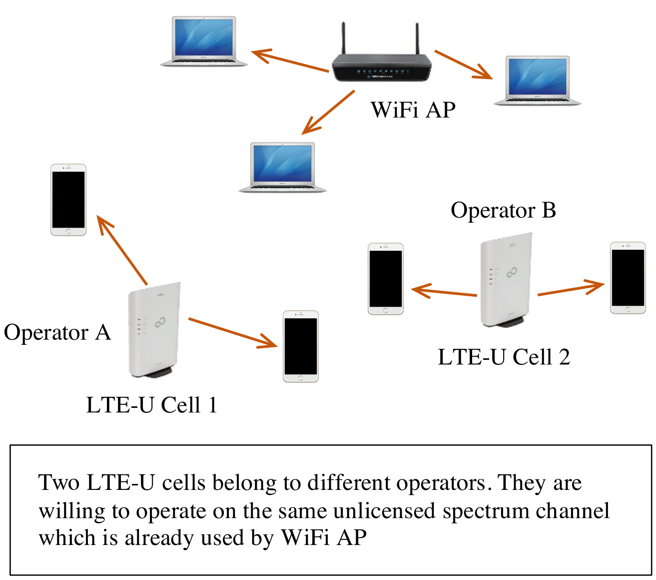

Figure 1: System model.

2.1 LTE-U Transmission Mode and WiFi

LTE-U can operate both frequency division (FDD) and time division (TDD) multiplexing modes. In unlicensed spectrum, supplemental downlink and TDD access is preferred. As default, LTE-U is based on OFDMA which gives LTE additional diversity in the time and frequency domain that Wi-Fi lacks, since Wi-Fi bandwidth is assigned to a single user at any time. However, in this work, we assume that an LTE-U cell transmits in unlicensed spectrum using supplemental downlink in FDD mode to one or more UEs using all or a subset of subcarriers in the corresponding unlicensed spectrum channel.

2.2 Inter-Operator Cooperation

The operators cooperate in order to increase their airtime in the shared channel as well as aim to "behave" fairly to the WiFi AP. The LTE-U cells' network constitute a bidirectional graph which represents the signalling arising due to the formation of coalitions of these cells. The coalitions of LTE-U cells are determined in a slotted basis. At the beginning of every slot, the LTE-U cells determine contention windows cooperatively, then transmission is performed until the end of the slot.

The cost of cooperation among LTE-U cells is due to the information exchanged among them with respect to raw throughputs and number of UEs in each cell. Obviously, the cost of cooperation increases when the number of players within a coalition increases. However, the frequency of adaptation of contention window is crucial. In this work, we assume that the impact of cooperation cost is limited since the frequency of reforming coalition is not high.

3. FORMAL PROBLEM DESCRIPTION

The study below considers a set of LTE-U cells denoted by $\mathcal{N} = \{1, \ldots, |\mathcal{N}|\}$, and a WiFi cell denoted by w operating on the same unlicensed spectrum channel. We denote by n_i the number of UEs that are served by LTE-U cell i.

The *interference channel* model, known also as the many-to-many channel is assumed. It consists of point-to-point links which are close enough to produce mutual interference because of sharing the same channel. We consider a block slow fading channel model such that channel realizations remain constant during a period. The channels statistically experience some fading distribution and we represent by G_{ij} the channel gain between i and j. The transmit power of cell i is denoted by P_i and the signal-to-noise ratio (SNR) between LTE-U cell i and UE j that it serves can be given by $P_i G_{ij}/N_0$ where N_0 is the power spectral density of additive white Gaussian noise. The throughput of the link between LTE-U cell i and its UE is a function of SNR and can be given by

$$R_{ij} = \alpha_i B \log(1 + \gamma_i P_i G_{ij}/BN_0) = \alpha_i r_{ij} \tag{1}$$

where α_i is the fraction of airtime used by cell $i \in \mathcal{N}$ during the corresponding slot, B is the channel bandwidth, and γ_i is a coefficient related to the modulation scheme. We call r_{ij} as *raw throughput*.

We denote by n_w the *number of WiFi stations* in the WiFi cell. We denote by CW_i and CW_w the contention window of LTE-U cell i, and contention window of WiFi cell, both operating on the same unlicensed channel, respectively. The contention window of the WiFi cell has an exponential backoff scheme with a maximum of c_S^w retries.

3.1 Contention Window of an LTE-U Cell and Airtime

In this work, we consider that the contention window of an LTE-U cell does not follow any exponential backoff scheme but rather, we calculate its size in order to provide a fair channel access with the WiFi AP. The LTE-U cells update their contention window in every slot.

Let us now calculate the contention window of an LTE-U Cell. We define the following probabilities:

- q_w: collision probability of a WiFi station
- q_i^L: collision probability of LTE-U cell i
- p_w: stationary probability of a WiFi station
- p_i^L: stationary probability of LTE-U cell i

Since the contention window of an LTE-U cell is constant during a slot, we have the following:

$$p_i^L = \frac{2}{CW_i + 1}, \quad \forall i \in \mathcal{N}, \tag{2}$$

$$p_w = \frac{2(1-2q_w)}{(1-2q_w)(CW_w+1) + q_w CW_w(1-(2q_w)^{c_S^w})} \tag{3}$$

These are based on the well-known Bianchi model [13]. The collision probabilities can be calculated as following:

$$q_i^L = 1-(1-p_w)^{n_w}\prod\nolimits_{j\in\mathcal{N}\setminus i}(1-p_j^L), \quad \forall i \in \mathcal{N} \tag{4}$$

and

$$q_w = 1-(1-p_w)^{n_w-1}\prod\nolimits_{j\in\mathcal{N}}(1-p_j^L). \tag{5}$$

The airtime fraction of LTE-U cell i can be calculated as following:

$$\alpha_i = p_i^L \prod\nolimits_{j\in\mathcal{N}\setminus i}(1-p_j^L)(1-p_w), \quad \forall i \in \mathcal{N}. \tag{6}$$

On the other hand, the airtime fraction of the WiFi cell can be given by

$$\alpha_w = 1 - \sum_{i\in\mathcal{N}} \alpha_i. \tag{7}$$

In the following, we aim at finding stationary probabilities of LTE-U cells such that the airtime fraction is optimized under some constraints, and then, adjust contention window accordingly.

3.2 Minimal Airtime Fraction of WiFi Cell

We represent by $f_w \in [0,1]$ *minimal airtime fraction* that the WiFi cell w operates on the channel. Note that this is the airtime fraction of the WiFi cell which comprises the total usage fraction scattered throughout a slot. Statistically, it can be understood as the probability of channel usage per slot. On the other hand, total *maximal airtime fraction of all LTE-U cells* is given by $1-f_w$.

The value of f_w is determined by the LTE-U cells. Each LTE-U cell assigns a value which we call as *altruistic gain* and denote by $g_i \in [0,1]$. The latter means that LTE-U cell i renounce the airtime fraction by g_i to WiFi AP. Formally, f_w is given by

$$f_w = \sum_{i\in\mathcal{N}} g_i \le 1, \text{ such that } g_i \le \frac{1}{|\mathcal{N}|}, \quad \forall i \in \mathcal{N}. \tag{8}$$

We set upper-bound $1/|\mathcal{N}|$ for g_i such that when every LTE-U cell is fully altruistic, then $f_w = \sum_{i\in\mathcal{N}} 1/|\mathcal{N}| = 1$ corresponds to the extreme case where every LTE-U does not operate on the channel. Moreover, we assume that the altruistic gains are calculated using a data-based model taking into account the number of UEs in each LTE-U cell and number of stations in the WiFi cell. Formally, g_i is calculated as follows:

$$g_i = \left(1-\kappa\frac{n_i}{n_w}\right)^+ \frac{1}{|\mathcal{N}|}, \tag{9}$$

$$\left(1-\kappa\frac{n_i}{n_w}\right)^+ = \max\left(1-\kappa\frac{n_i}{n_w}, 0\right), \tag{10}$$

where $0 \le \kappa \le 1$ is parameter which can be interpreted as a calibrating parameter of altruistic gains and we call it as *altruism modulator*. It plays a significant role in determining the airtime fraction of the WiFi cell. When $\kappa = 1$, and if $n_i > n_w, \forall i \in \mathcal{N}$, then we come up with the case where $f_w = 0$. In the other extreme case when $\kappa = 0$, then it means that all airtime will be used by the WiFi cell.

Note that if LTE-U cell i has zero UEs, then the gain that WiFi cell achieves is $1/|\mathcal{N}|$ in airtime fraction. In the extreme case where there is no any UE that any LTE-U cell serves, then $f_w = 1$ since for every i, $g_i = 1/|\mathcal{N}|$. Based on the proposed formulation, an LTE-U cell needs to know active WiFi devices. We assume that an LTE-U cell can learn the number of active WiFi devices based on their corresponding MAC addresses during the sensing period.

4. THE GAME: ALTRUISTIC COOPERATION MODEL

We are modelling the interaction among the cells as a cooperative game which is based on so called *non-transferable utility* (NTU). Non-transferable utility is the concept capturing the fact that the utility of a coalition is not dividable among the members of the corresponding coalition, but rather it is given by its set of players and the sets of outcomes that are feasible for each subset coalition of players.

4.1 The NTU Game Formulation

Let $\mathcal{N}$ be the set of the players (i.e., LTE-U cells). Non-empty subsets of $\mathcal{N}$ are called as *coalitions*, i.e. $\mathcal{S} \subseteq \mathcal{N}$. A non-transferable utility game on $\mathcal{N}$ is a correspondence V *–the coalitional function* – that assigns to each coalition $\mathcal{S}$ a subset $V(\mathcal{S}) \in \mathfrak{R}^{\mathcal{S}}$ where $\mathfrak{R}^{\mathcal{S}}$ is the set of all functions from $\mathcal{S}$ to $\mathfrak{R}$. An element $\alpha = (\alpha_i)_{i\in\mathcal{S}}$ where $\alpha \in V(\mathcal{S})$ is interpreted as follows: there exists an outcome that is feasible for the coalition $\mathcal{S}$ whose payoff to player i is α_i, $\forall i \in \mathcal{S}$. Moreover, V satisfies the following properties [10]:

1. For each $\mathcal{S} \subseteq \mathcal{N}$, the set $V(\mathcal{S})$ is non-empty, closed, convex, *comprehensive* (i.e., if $\alpha \in V(\mathcal{S})$ and $\alpha' \le \alpha$, then $\alpha' \in V(\mathcal{S})$) and *bounded from above*, (i.e., for each $\alpha \in \mathfrak{R}^{\mathcal{S}}$, the set $\{\alpha' \in V(\mathcal{S}) : \alpha' \ge \alpha\}$ is compact).

2. *Normalization*: For each $i \in \mathcal{N}$, the maximum of $\{\alpha : \alpha \in V(i)\}$, denoted by $\tilde{\alpha}_i$ is nonnegative and called as *individual payoff* of i.

3. *Zero-monotonicity*: For each $\mathcal{S} \subseteq \mathcal{N}$, $\alpha \in V(\mathcal{S})$ and $i \notin \mathcal{S}$, the utility vector is $(\alpha, \tilde{\alpha}_i) \in V(\mathcal{S} \cup i)$. This implies that $(\tilde{\alpha}_i)_{i\in\mathcal{S}} \in \mathfrak{R}^{\mathcal{S}}$ belongs to $V(\mathcal{S})$.

4. The boundary of $V(\mathcal{N})$ denoted by $\partial V(\mathcal{N})$ is non-level in the positive orthant meaning that at any point of $\partial V(\mathcal{N}) \cap \mathfrak{R}^{\mathcal{N}}$, there exists an outward vector with positive coordinates.

5. For each $\mathcal{S} \subset \mathcal{N}$, if $\alpha \in \partial V(\mathcal{S})$ with $\alpha_i < 0$ for $i \in \mathcal{S}' \subset \mathcal{S}$, then $\partial V(\mathcal{S})$ at α is parallel to the subspace $\mathfrak{R}^{\mathcal{S}'}$.

Note that property 5 is not relevant in airtime sharing problem since all relevant action occurs in positive orthant (i.e. $\alpha_i \geq 0, \forall i \in \mathcal{N}$).

A *transferable utility game* (TU game) on $\mathcal{N}$ is a characteristic *value function* $v : 2^{\mathcal{S}} \to \Re$ which assigns a real value $v(\mathcal{S})$ for each coalition $\mathcal{S}$, also $v(\emptyset) = 0$. We can express a TU game as a special NTU game:

$$\tilde{V}(\mathcal{S}) = \left\{ \alpha \in \Re^{\mathcal{S}} : \sum_{i \in \mathcal{S}} \alpha_i \leq v(\mathcal{S}) \right\}. \tag{11}$$

Let us denote by Π the set of all the orders of players in $\mathcal{N}$. Given $\pi \in \Pi$ and $i \in \mathcal{N}$, we define $\mathcal{P}_i^\pi$ which is the set of players coming before player i in the order π:

$$\mathcal{P}_i^\pi = \{ j \in \mathcal{N} : \pi(j) < \pi(i) \}. \tag{12}$$

4.2 The NTU Shapley Value

Consider a TU game on $\mathcal{N}$ with value function v and let $\pi \in \Pi$. The *marginal contribution* of player i under the order π can be given by $v(\mathcal{P}_i^\pi \cup i) - v(\mathcal{P}_i^\pi) \in \Re$. The *Shapley value* of a TU game is the vector $Sh(\mathcal{N}, v) \in \Re^{\mathcal{N}}$ of which ith coordinate is given by

$$Sh_i(\mathcal{N}, v) = \frac{1}{|\Pi|} \sum_{\pi \in \Pi} \left[v(\mathcal{P}_i^\pi \cup i) - v(\mathcal{P}_i^\pi) \right].$$

Let $\lambda = (\lambda_i)_{i \in \mathcal{N}} \in \Re^{\mathcal{N}}$ and for each $i \in \mathcal{N}$, $\lambda_i > 0$. Let us define the following:

$$v^\lambda(\mathcal{S}) := \max \left\{ \sum_{i \in \mathcal{S}} \lambda_i \alpha_i : \alpha \in V(\mathcal{S}) \right\}.$$

A vector $\alpha \in V(\mathcal{N})$ is a NTU Shapley value of V if there exists a vector λ such that

$$\forall \mathcal{S} \subseteq \mathcal{N}, \exists \alpha \in V(\mathcal{S}) \text{ such that } \sum_{i \in \mathcal{S}} \lambda_i \alpha_i = v^\lambda(\mathcal{S}). \tag{13}$$

$$\lambda_i \alpha_i = Sh_i(\mathcal{N}, v^\lambda), \quad \forall i \in \mathcal{N}. \tag{14}$$

4.3 Coalition Formation Mechanism for Finding the NTU Shapley Value

The mechanism is proposed in [10] and we refer to it for detailed reading. Let us define a rule as a function δ which assigns a vector $\delta(\mathcal{S}) \in V(\mathcal{S})$ to each coalition $\mathcal{S}$. Formally, a rule is a "payoff configuration" which should be interpreted as an index that indicates payoff allocations when a particular coalition is formed. The mechanism $\mathbb{M}(\pi)$ works as following:

1. An order of the players π is randomly chosen;

2. When the turn comes to player i, it is faced with coalition $\mathcal{S} \subset \mathcal{P}_i^\pi$ with a specific rule δ, and a set of players $\mathcal{E} = \mathcal{P}_i^\pi \setminus \mathcal{S}$ having chosen to stay out of coalition $\mathcal{S}$;

3. Players in $\mathcal{S}$, $\mathcal{E}$ and $\mathcal{N} \setminus \mathcal{P}_i^\pi$ are called *active players*, *passive players* and *candidates*, respectively;

4. Then, player i must either agree to join the coalition (in which case, player i becomes an active player and the turn passes to candidate $i+1$) or disagree, proposing both a new rule δ' and a new coalition $\mathcal{S}' \subset \mathcal{P}_i^\pi \cup i$ which includes himself and all the members of the old coalition (i.e., $\mathcal{S} \cup i \subset \mathcal{S}'$);

5. The members of $\mathcal{S}' \setminus i$ vote sequentially to either accept or reject this proposal; if they all vote "yes", the new coalition $\mathcal{S}'$ forms with the new rule (the proposal is accepted), and the turn passes to candidate $i+1$; if at least one member of $\mathcal{S}' \setminus i$ votes "no", then player i becomes a passive player and the turn passes to candidate $i+1$;

6. When no more candidates remain, the following coalitions are obtained: $\mathcal{S} = \mathcal{N}$ coalition of active players, $\mathcal{E} = \mathcal{N} \setminus \mathcal{S}$ set of passive players, and a rule δ for the coalition $\mathcal{S}$;

7. The final payoff, for each player $i \in \mathcal{S}$ is $\delta_i(\mathcal{S})$ and all the players $i \in \mathcal{E}$ receives their individual payoffs $\tilde{\alpha}_i$;

REMARK 1. A strategy profile is a subgame perfect equilibrium if it represents a Nash equilibrium of every subgame of the original game. Informally, this means that if (i) the players played any smaller game that consisted of only one part of the larger game and (ii) their behavior represents a Nash equilibrium of that smaller game, then their behavior is a subgame perfect equilibrium of the larger game. In [10], it is proven that mechanism $\mathbb{M}(\pi)$ provides the following result: if $V(\mathcal{N})$ is bounded by a hyperplane, then there exists a unique expected subgame perfect Nash equilibrium (SPNE) payoff in the negotiation mechanism $\mathbb{M}$, that is the NTU Shapley value. Furthermore, the strategy of a player in SPNE in the negotiation mechanism $\mathbb{M}(\pi)$ for any π is robust to deviations by coalitions of his predecessors in π.

4.4 Airtime Fraction as Payoff Configuration

We basically consider that the payoff of an LTE-U cell is exactly the value of airtime fraction. The individual payoffs are calculated using a *weighted logarithmic* scheme which can be interpreted as a version of proportional fairness with different constraints. The individual payoffs are obtained by solving the following optimization problem:

$$\begin{aligned} &\max_{\tilde{\alpha}, q_w, p^L} \sum_{i \in \mathcal{N}} u_i \log \tilde{\alpha}_i \text{ subject to} \\ &\tilde{\mathbb{C}}, \\ &0 \leq \tilde{\alpha}_i \leq (1 - f_w) \frac{1}{|\mathcal{N}|}, \quad \forall i \in \mathcal{N}, \end{aligned} \tag{15}$$

where we group the constraints related to the probabilities as follows:

$$\tilde{\mathbb{C}} \equiv \left\{ \begin{array}{l} p_w = \frac{2(1-2q_w)}{(1-2q_w)(CW_w+1)+q_w CW_w (1-(2q_w)^{c_S^W})} \\ \tilde{\alpha}_i = p_i^L \prod_{j \in \mathcal{N} \setminus i} (1 - p_j^L)(1 - p_w), \forall i \in N \\ q_w = 1 - (1 - p_w)^{n_w - 1} \prod_{j \in \mathcal{N}} (1 - p_j^L) \\ 0 \leq p_i^L \leq 1, \forall i \in N \\ 0 \leq q_w \leq 1 \end{array} \right\}. \tag{16}$$

Note that the individual payoff of i is limited by $(1 - f_w)/|\mathcal{N}|$. $1 - f_w$ corresponds to the total airtime fraction of all LTE-U cells. $1/|\mathcal{N}|$ is what an LTE-U cell can claim when it is passive. $u_i \in [0, 1]$ is the altruistic weight of LTE-U cell i. If $u_i = 0$, then it means that LTE-U cell i is completely unselfish while $u_i = 1$ corresponds to completely selfish case. Note that individual payoffs are calculated at the beginning of mechanism $\mathbb{M}(\pi)$.

REMARK 2. The weighted proportional fairness can be obtained by solving the same problem given in equation (15) but the constraints $0 \le \tilde{\alpha}_i \le (1 - f_w)/|\mathcal{N}|$, $\forall i \in \mathcal{N}$ must be changed to $0 \le \sum_{i \in \mathcal{N}} \tilde{\alpha}_i \le (1 - f_w)$.

Assume that LTE-U cell $k \in \mathcal{N} \setminus \mathcal{P}_k^{\pi}$ is a candidate. Then, it calculates the payoff configuration. If it proposes a better payoff configuration for every cell which are active players in $\mathcal{S}$ in mechanism $\mathbb{M}(\pi)$, then the new payoffs are $\alpha(\mathcal{S}') = (\alpha_1(\mathcal{S}'), \ldots, \alpha_{|\mathcal{S}'|}(\mathcal{S}'))$ where $\mathcal{S}' = \mathcal{S} \cup k$. The constraints related to probabilities of $\mathcal{S}'$ are given by

$$\mathbb{C}(\mathcal{S}') \equiv \left\{ \begin{array}{l} q_w = q_w(\mathcal{S}') = 1 - (1-p_w)^{n_w - 1} \prod_{j \in \mathcal{N}} (1 - p_j^L(\mathcal{S}')) \\ p_w = \frac{2(1-2q_w)}{(1-2q_w)(CW_w+1) + q_w CW_w (1-(2q_w)^{c_S^w})} \\ \tilde{\alpha}_i = p_i^L(\mathcal{S}') \prod_{j \in \mathcal{N} \setminus i} (1 - p_j^L(\mathcal{S}'))(1-p_w), \forall i \in \mathcal{E} \setminus k \\ 0 \le p_i^L(\mathcal{S}') \le 1, \forall i \in N \\ 0 \le q_w(\mathcal{S}') \le 1 \end{array} \right\}. \tag{17}$$

Thus, the payoff configuration proposed by LTE-U cell k is given by

$$\begin{aligned} & \max_{\alpha, q_w, p^L} \sum_{i \in \mathcal{S}'} u_i \log \alpha_i(\mathcal{S}') \text{ subject to} \\ & \mathbb{C}(\mathcal{S}'), \\ & \alpha_i(\mathcal{S}') = p_i^L(\mathcal{S}') \prod_{j \in \mathcal{N} \setminus i} (1 - p_j^L(\mathcal{S}'))(1 - p_w), \quad \forall i \in \mathcal{S}', \\ & \sum_{i \in \mathcal{S}'} \alpha_i(\mathcal{S}') \le (1 - f_w) \frac{|\mathcal{S}'|}{|\mathcal{N}|}, \\ & \alpha_i(\mathcal{S}) \le \alpha_i(\mathcal{S}') \le 1, \quad \forall i \in \mathcal{S}, \\ & \tilde{\alpha}_k \le \alpha_k(\mathcal{S}') \le 1. \end{aligned} \tag{18}$$

$\alpha_i(\mathcal{S}) \le \alpha_i(\mathcal{S}') \le 1, \forall i \in \mathcal{S}$ ensures that every cell in active players set $\mathcal{S}$ is better off in new active player set $\mathcal{S}'$. Besides cell k is better off in $\mathcal{S}'$ than its individual payoff $\tilde{\alpha}_k$. Note that the rule $\delta(\mathcal{S})$ for any $\mathcal{S}$ represented in $\mathbb{M}(\pi)$ is exactly $\alpha(\mathcal{S})$.

4.5 Altruistic Weights

The choice of altruistic weights is flexible. However, here we shall assume that a particular player adjusts its altruistic weight according to its normalized raw throughput which is given by

$$\bar{r}_i = \frac{r_i'}{\min_{k \in \mathcal{N}} r_k'}, \tag{19}$$

where for any $i \in \mathcal{N}$, we define $r_i' = \min_{j \in \{1, \ldots, n_i\}} r_{ij}$. Thus, the altruistic weight of player i is chosen to be inversely proportional to the normalized raw throughput, i.e.

$$u_i = \frac{1}{\bar{r}_i}, \quad \forall i \in \mathcal{N}. \tag{20}$$

Note that such a choice captures the fact that the LTE-U cell shall update its altruistic weight whenever the raw throughputs are changed.

4.6 Properties of the Coalitional Game

Theoretical results about the coalitional game are stated below.

LEMMA 1. The region imposed by $\tilde{\mathbb{C}}$ and $0 \le \tilde{\alpha}_i \le (1 - f_w)/|\mathcal{N}|, \forall i \in \mathcal{N}$ is bounded and feasible. The game imposes zero-monotonicity.

LEMMA 2. $V(\mathcal{N})$ is bounded by a hyperplane, then there exists a unique expected subgame perfect Nash equilibrium payoff in mechanism $\mathbb{M}$, that is the NTU Shapley value.

LEMMA 3. The airtime fraction configuration is max-min fair.

PROOF. For proofs see appendix. □

5. DYNAMICS OF THE PROBLEM

The considered problem possesses the following discrete-time dynamics:

1. *adaptation* procedure is carried out by LTE-U cells at the beginning of a slot; adaptation procedure comprises of updating the information of raw throughputs, number of UEs in each LTE-U cell and number of stations in the WiFi cell and calculating the contention window;
2. *transmission*: LTE-U cells and the WiFi cell operate until the end of the slot.

We can consider a time diagram where adaptation and transmission are performed serially. Basically, the frequency of adaptation can be defined as $F_A = \tau_A/(\tau + \tau_A)$ where τ_A and τ stands for the time needed for adaptation procedure, and the transmission time of all LTE-U cells and WiFi cell, respectively. At the end of $\tau + \tau_A$, the adaptation procedure is repeated. Note that τ_A increases when the coalition increases. The time cost of exchanging the information increases linearly, however, the time cost for calculation of contention window is determined by the computational complexity of the optimization problem given in equation (15). We choose τ (e.g. around seconds) such that the impact of τ_A (e.g. around milliseconds) becomes negligible. Note that the channel statistics determine if we need to calculate the airtime fractions of the LTE-U cells. So, whenever the channel coefficients are unchanged, we can skip the adaptation procedure and continue on transmission. In summary, we can design an algorithm which involves the adaptation procedure and calculation of Shapley value. Each LTE-U cell possesses all related information, and they calculate separately $\{\tilde{\alpha}, q_w, p^L\}$. Then, they adapt their own contention window according to these values. Note that an algorithm, as such, is completely distributed.

6. NUMERICAL RESULTS

In this section, we obtain numerical results by calculating the airtime fraction of a particular LTE-U cell. We also measure the fairness by two quantitative fairness measures: entropy and Jain's index. Entropy reflects fairness aspects [11] considering the proportions of resources are allocated to the individuals in $\mathcal{N}$. Proportion of LTE-U cell i is $\alpha_i / \sum_{j \in \mathcal{N}} \alpha_j$, and

$$\text{Entropy} = -\sum_{i \in \mathcal{N}} \left[\frac{\alpha_i}{\sum_{j \in \mathcal{N}} \alpha_j} \log_2 \left(\frac{\alpha_i}{\sum_{j \in \mathcal{N}} \alpha_j} \right) \right].$$

Algorithm 1 Contention Window Adaptation

Initialization:
1) at the beginning of slot, each LTE-U cell i sends n_i and r'_i;
2) n_w is sensed by each LTE-U cell;
Adapting Contention Window:
3) each LTE-U cell determines its own stationary probability
4) each LTE-U cell calculates its own contention window according to stationary probability
5) each LTE-U cell operates until the end of slot and then go to step 1)

Jain's index is well-known and is defined as:

$$\text{Jain's Index} = \frac{(\sum_{i \in \mathcal{N}} \alpha_i)^2}{|\mathcal{N}| \sum_{i \in \mathcal{N}} \alpha_i^2}.$$

We compare NTU Shapley value and proportional fairness [14] with respect to average airtime fraction, average entropy and Jain's index. We choose the comparison with proportional fairness since it is widely accepted and used fair resource allocation scheme. In the figures, SV and PF stands for Shapley Value and Proportional Fairness, respectively.

In the following, actually, we seek a trade-off between fairness and performance by comparing SV and PF. In the figures, we change f_w between 0 and 1, which should be interpreted as changing parameters on which f_w depends. We assume that $n_w = 6$ and $\kappa = 0.5$, since $f_w = \sum_{i \in \mathcal{N}} \max(1 - \kappa n_i / n_w, 0)$, n_i changes between 0 and 3 for all $i \in \mathcal{N}$.

6.1 Example Scenario: Different Throughputs of LTE-U Cells

Consider that there are $|\mathcal{N}| = 4$ LTE-U cells and the following parameters are assumed for WiFi cell: $CW = 32, c_S^w = 3, n_w = 6$. Let the raw throughputs be given as $4r'_1 = 3r'_2 = 2r'_3 = r'_4$, and so $\min(r'_1, r'_2, r'_3, r'_4) = r'_1$. Therefore, $u_1 = 1, u_2 = 3/4, u_3 = 1/2$, and $u_4 = 1/4$. This example, as such, demonstrates the performance comparison of SV and PF in a scenario where the raw throughputs of the LTE-U cells are diverse — LTE-U cell 4 has the best raw throughput while LTE-U 1 has the worst one. In Figures 2 and 3, we depict the change of airtime fraction, Jain's index and entropy, respectively, with respect to f_w. The airtime fraction in case of SV is not always monotonic with respect to f_w. Note that LTE-U cell 2, 3, and 4 has a maximum for different values of f_w. Only LTE-U cell 1 is favored by PF. The other cells are better off in SV. Jain's index and entropy figures show similar behavior with respect to f_w. Note that SV is superior compared to PF in terms of fairness measures (Jain's index and Entropy). The calculation of airtime fraction in SV is a procedure involving each LTE-U cells' own objective, and that results in a better fairness.

6.2 Average Airtime Fraction, Jain's Index and Entropy

We do Monte Carlo simulations for finding average values of airtime fraction, Jain's index and entropy with respect to changing values of f_w. We assume the following parameters in WiFi: $CW_w = 32, c_S^w = 3, n_w = 6$. We also suppose that the normalized raw throughputs are produced according to uniform distribution taking values in $[1, 5]$. In Figure 4, we plot the comparison of SV and PF in terms of mean airtime fraction with respect to f_w for different numbers of LTE-U cells. We plot also the change of mean airtime fraction of a WiFi station with respect to f_w.

Figure 4 reveals the fact that both SV and PF have similar properties. The monotonicity is not guaranteed for changing values of f_w. The curves corresponding to $|\mathcal{N}| = 5$ show that mean airtime fraction is not monotonic. On the other hand, the difference between the curves of SV and PF is not so high even though the curves related to PF are slightly better than those of SV. On the other hand, as it can be expected, a WiFi station is better off when f_w increases. SV provides slightly better mean airtime fraction for a WiFi station.

As per mean entropy and average Jain's index, in Figure 5, we compare SV and PF for increasing numbers of LTE-U cells. SV demonstrates better performance for both measures. When f_w is small, SV and PF are near each other both for mean entropy and Jain's index. Besides, the gap in mean Jain's index increases when f_w is high. It means that PF is less fair when the LTE-U cells operate less on the channel.

In Figure 6, we compare SV and PF in terms of mean airtime fraction with respect to κ for different numbers of LTE-U cells. We plot also the change of mean airtime fraction of a WiFi station with respect to κ. We keep the same parameters for WiFi as above. Moreover, we generate integers randomly according to uniform distribution taking values in $[3, 10]$ to obtain values of n_i. It is obvious that when κ increases mean airtime fraction of an LTE-U cell increases and of a WiFi station decreases. However, the figure implies the logarithmic dependence on κ which may be interpreted that dramatic impact of κ occurs in the lower values. For example, for $|\mathcal{N}| = 3$ and $\kappa \approx 0.3$, mean airtime fraction of an LTE-U cell matches that of a WiFi station.

7. RELATED WORK

In order to coexist fairly with WiFi in the unlicensed band, several approaches for LTE-U have been investigated in the literature. The suggested techniques can be divided into two different categories according to the adopted methodology: (i) LBT based which involves LTE carrier sensing the unlicensed channel before any transmission and (ii) duty cycling based where LTE adopts the almost-blank-subframes (ABS) feature in order to blank a particular proportion of the subframes. Most of the previous work assumes a duty cycling approach for LTE-WiFi coexistence (e.g., [15–17]), however, this approach would allow WiFi transmissions only when LTE is silent. On the other hand, LBT allows the sharing of the unlicensed band in an asynchronous manner, as opposed to a scheduled one. Moreover, the 3rd Generation Partnership Project (3GPP) has identified LBT as the direction for standardizing a global solution for LTE-U and hence meeting the LBT regulatory requirements in markets such as Europe and Japan [18]. Nevertheless, there has been a limited amount of work that considers LBT for the coexistence of LTE and WiFi in the unlicensed band [19–23]. The exponential backoff of CSMA/CA is adopted for LTE-U cells in [19], however, this technique leads to a degradation in performance due to the unnecessary retransmissions where nodes consider all failed transmissions as collisions whereas only part of them are real collisions. In [20–22] the authors suggest a LBT scheme with a random backoff mechanism employing a fixed contention window size where an LTE-U cell chooses a random backoff counter uniformly from a predefined fixed contention window. This mechanism, however, does not take into account the load level and activity of LTE-U cells and WLAN and does not adapt to the changing traffic load and thus neglects the fairness metric between the two technologies. The

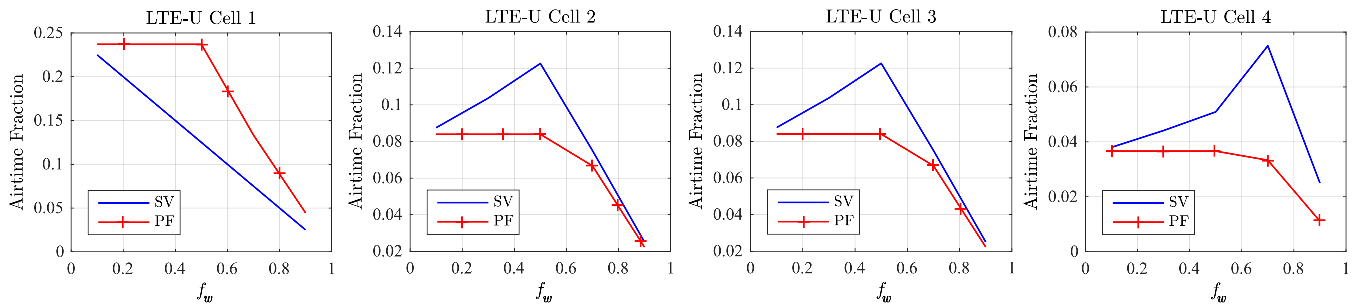

Figure 2: Example scenario: Airtime fraction with respect to f_w.

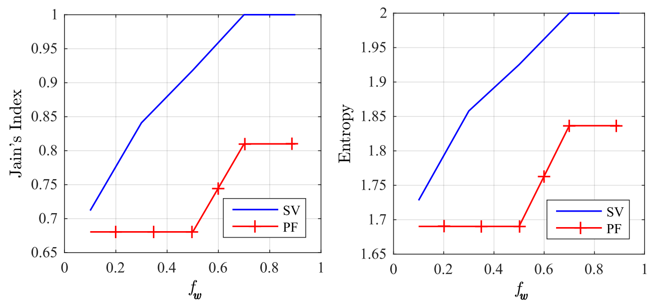

Figure 3: Example scenario: Jain's index and entropy with respect to f_w.

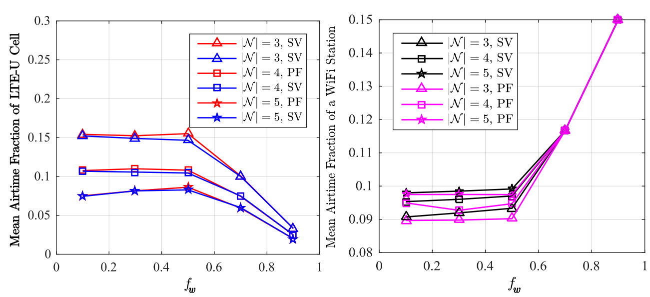

Figure 4: Comparison of SV and PF: Mean airtime fraction with respect to f_w for increasing numbers of LTE-U cells.

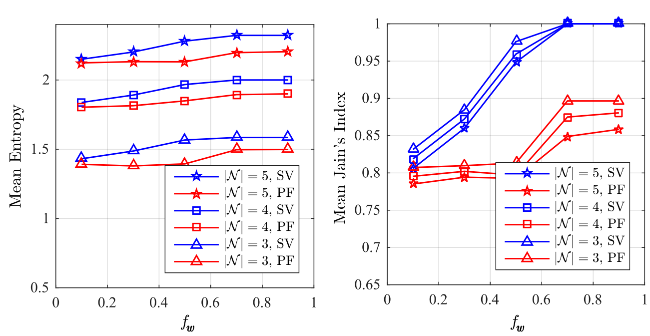

Figure 5: Comparison of SV and PF: Mean entropy and Jain's index with respect to f_w for increasing numbers of LTE-U cells.

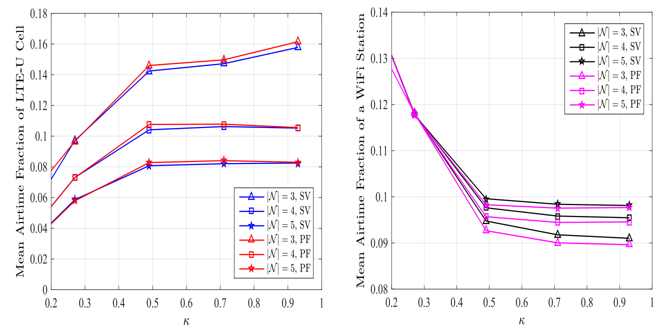

Figure 6: Comparison of SV and PF: Mean airtime fraction with respect to κ for increasing numbers of LTE-U cells.

authors in [23] suggest an adaptive LBT scheme where the LTE-U cell contention window size varies according to the available licensed BW and WiFi traffic load, but their system model is limited to a single LTE-U cell.

In contrast to the aforementioned work, the focus of this paper is on modelling the interaction between multiple co-located LTE-U small cell operators. Although the negative impact of selfishness is noted in the literature [4], there is no model that captures the cooperation among operators, especially when LBT is used as the channel access mechanism. Our work offers a theoretical framework to fill this void.

8. CONCLUSIONS

We have analyzed the coexistence problem of LTE-U and WiFi in unlicensed spectrum where LTE-U utilizes Listen-Before-Talk (LBT) as the default channel access mechanism. We have examined how LTE-U cells from different operators must adjust their contention windows in order to provide a fair coexistence both with WiFi and among themselves. We studied an altruistic model where LTE-U cells behave altruistically both among themselves and to WiFi. Interaction of LTE-U cells is studied using a coalition formation game framework which is based on Shapley value. We showed how our proposed payoff configuration scheme, which involves altruism, is always zero-monotonic, and also showed that Shapley value is also max-min fair. Comparison of Shapley value with weighted proportional fairness through numerical results demonstrate that the former is more fair while providing similar average airtime performance.

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APPENDIX

A. PROOFS

A.1 Proof of Lemma 1

Since $\leq \tilde{\alpha}_i \leq (1-f_w)(1/|\mathcal{N}|)$ and $\tilde{\alpha}_i = p_i^L \prod_{j\in\mathcal{N}\setminus i}(1-p_j^L)(1-p_w), \forall i \in \mathcal{N}$, the optimization problem given in equation (15) can be given by

$$\max \sum_{i\in\mathcal{N}} u_i \log\left(p_i^L \prod_{j\in\mathcal{N}\setminus i}(1-p_j^L)(1-p_w)\right)$$
$$= \max\left(\sum_{i\in\mathcal{N}} u_i \log\left(\frac{p_i^L}{1-p_i^L}\right) + \sum_{i\in\mathcal{N}} u_i \log\left(\frac{1-q_w}{(1-p_w)^{n_w-2}}\right)\right)$$

subject to

$$\frac{1-q_w}{(1-p_w)^{n_w-2}} \leq \left(\frac{1}{p_i^L}-1\right)\frac{(1-f)}{|\mathcal{N}|}, \quad \forall i \in \mathcal{N}$$
$$p_w = \frac{2(1-2q_w)}{(1-2q_w)(CW_w+1)+q_w CW_w(1-(2q_w)^{c_S^w})},$$
$$\frac{1-q_w}{(1-p_w)^{n_w-1}} = \prod_{j\in\mathcal{N}}(1-p_j^L),$$
$$0 \leq p_i^L \leq 1, \forall i \in \mathcal{N}, 0 \leq q_w \leq 1.$$

The terms $(1-q_w)/(1-p_w)^{n_w-1}$ and $(1-q_w)/(1-p_w)^{n_w-2}$ are monotonically decreasing with respect to q_w. Since

$$(1-q_w)/(1-p_w)^{n_w-1} = \prod_{j\in\mathcal{N}}(1-p_j^L) \leq 1,$$

there is always a solution set with $p_i^L \geq p_j^L, \forall i,j \in \overrightarrow{\mathcal{N}}$ where $\overrightarrow{\mathcal{N}}$ is the ordered set according to values of p_i^L which is directly adjusted by u_i. Thus, we have also $u_i^L \geq u_j^L \Leftrightarrow p_i^L \geq p_j^L, \forall i, j \in \overrightarrow{\mathcal{N}}$.

Consider an order of players $\pi \in \Pi$ and LTE-U cell k which is in turn. Before k, let the active players, passive players, and candidates be $\mathcal{S}$, $\mathcal{E}$, and $\mathcal{N} \setminus \mathcal{P}_k^\pi$. Note that zero-monotonicity implies that the set of passive players is an empty set, i.e. $\mathcal{E} \equiv \emptyset$ which accounts for the case where in every turn, the player is better off in the new payoff configuration. This comes from the constraint $\tilde{\alpha}_i \leq \alpha_i(\mathcal{S}), \forall i \in \mathcal{S}$ entailing a new payoff configuration only if every active player and the candidate are better off. The constraint related to the airtime fraction of all LTE-U cells is given by

$$\sum_{i \in \mathcal{P}_k^\pi \cup k} \alpha_i(\mathcal{P}_k^\pi \cup k) \leq (1-f)\frac{\left|\mathcal{P}_k^\pi \cup k\right|}{|\mathcal{N}|}$$

$$\alpha_i(\mathcal{P}_k^\pi) \leq \alpha_i(\mathcal{P}_k^\pi \cup k), \quad \forall i \in \mathcal{P}_k^\pi,$$

$$\tilde{\alpha}_k \leq \alpha_k(\mathcal{P}_k^\pi \cup k).$$

The total airtime fraction before and after k is given by $\sum_{i \in \mathcal{P}_k^\pi} \alpha_i(\mathcal{P}_k^\pi) + \sum_{j \in \mathcal{E}} \tilde{\alpha}_j + \tilde{\alpha}_k$ and $\sum_{i \in \mathcal{P}_k^\pi} \alpha_i(\mathcal{P}_k^\pi \cup k) + \alpha_k(\mathcal{P}_k^\pi \cup k) + \sum_{j \in \mathcal{E}} \tilde{\alpha}_j$, respectively. The difference airtime fraction of leaving $\mathcal{E}$ and joining $\mathcal{S}$ is always positive, i.e. $\sum_{i \in \mathcal{P}_k^\pi} \alpha_i(\mathcal{P}_k^\pi \cup k) - \sum_{i \in \mathcal{P}_k^\pi} \alpha_i(\mathcal{P}_k^\pi) + \alpha_k(\mathcal{P}_k^\pi \cup k) - \tilde{\alpha}_k \geq 0$ since $\sum_{i \in \mathcal{P}_k^\pi} \alpha_i\left(\mathcal{P}_k^\pi \cup k\right) \geq \sum_{i \in \mathcal{P}_k^\pi} \alpha_i(\mathcal{P}_k^\pi)$ and $\alpha_k(\mathcal{P}_k^\pi \cup k) \geq \tilde{\alpha}_k$. That proves that the game is *zero-monotonic*.

A.2 Proof of Lemma 2

For any order of players π the airtime fraction given by $\alpha = \alpha(\mathcal{N}) \in V(\mathcal{N})$ is always a hyperplane that can be defined as $H = \left\{\alpha : \sum_{i \in \mathcal{N}} \lambda_i \alpha_i = v^\lambda(\mathcal{N})\right\}$ which is basically a set described by a single scalar product equality. On the other hand, the Shapley value of player i can also be defined in the following form:

$$Sh_i(\mathcal{N}, v) = \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus i} \frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!}\left(v^\lambda(\mathcal{S} \cup i) - v^\lambda(\mathcal{S})\right)$$

For sake of simplicity, let us write separately the part of player i as following:

$$Sh_i(\mathcal{N}, v) = \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus i : |\mathcal{S}|>1} \frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!}\left(v^\lambda(\mathcal{S} \cup i) - v^\lambda(\mathcal{S})\right) + \sum_{j \in \mathcal{N} \setminus i} \frac{(|\mathcal{N}|-2)!}{|\mathcal{N}|!}\left(v^\lambda(j,i) - v^\lambda(j)\right) + \frac{(|\mathcal{N}|-1)!}{|\mathcal{N}|!} v^\lambda(i)$$

Recall that from (14), we have $\lambda_i \alpha_i(\mathcal{N}) = Sh_i(\mathcal{N}, v^\lambda)$. A solution of this problem can be obtained by assuming that for any $\mathcal{S} \subseteq \mathcal{N}, \alpha_i(\mathcal{S}) = \zeta \tilde{\alpha}_i, \forall i \in \mathcal{S}$ where note that $\zeta \geq 1$ due to the assumption that $\alpha_i(\mathcal{S}) \geq \tilde{\alpha}_i, \forall \mathcal{S} \subseteq \mathcal{N}$. Thus, $\lambda_i \zeta \tilde{\alpha}_i = Sh_i(\mathcal{N}, v^\lambda)$. In that case, $Sh_i(\mathcal{N}, v)$ becomes

$$\sum_{\mathcal{S} \subseteq \mathcal{N} \setminus i : |\mathcal{S}|>1} \frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!}\left(\sum_{k \in \mathcal{S} \cup i} \lambda_k \alpha_k(\mathcal{S} \cup i) - \sum_{k \in \mathcal{S}} \lambda_k \alpha_k(\mathcal{S})\right) + \sum_{j \in \mathcal{N} \setminus i} \frac{(|\mathcal{N}|-2)!}{|\mathcal{N}|!}\left(\lambda_i \alpha_i(i,j) + \lambda_j \alpha_j(i,j) - \lambda_j \tilde{\alpha}_j\right) + \frac{(|\mathcal{N}|-1)!}{|\mathcal{N}|!} \lambda_i \tilde{\alpha}_i$$

$$= \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus i : |\mathcal{S}|>1} \frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!}\left(\sum_{k \in \mathcal{S}} \lambda_k(\zeta \tilde{\alpha}_k) + \lambda_i(\zeta \tilde{\alpha}_i) - \sum_{k \in \mathcal{S}} \lambda_k(\zeta \tilde{\alpha}_k)\right) + \sum_{j \in \mathcal{N} \setminus i} \frac{(|\mathcal{N}|-2)!}{|\mathcal{N}|!}\left(\lambda_i(\zeta \tilde{\alpha}_i) + \lambda_j\left(\zeta \tilde{\alpha}_j\right) - \lambda_j \tilde{\alpha}_j\right) + \frac{(|\mathcal{N}|-1)!}{|\mathcal{N}|!} \lambda_i \tilde{\alpha}_i$$

which results in

$$Sh_i(\mathcal{N}, v) = \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus i : |\mathcal{S}|>1} \frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!} \lambda_i \zeta \tilde{\alpha}_i + \sum_{j \in \mathcal{N} \setminus i} \frac{(|\mathcal{N}|-2)!}{|\mathcal{N}|!}\left(\lambda_i \zeta \tilde{\alpha}_i + \lambda_j \zeta \tilde{\alpha}_j - \lambda_j \tilde{\alpha}_j\right) + \frac{(|\mathcal{N}|-1)!}{|\mathcal{N}|!} \lambda_i \tilde{\alpha}_i.$$

Since $\lambda_i \zeta \tilde{\alpha}_i = Sh_i(\mathcal{N}, v^\lambda)$, we have to choose $\zeta = 1$, and then

$$Sh_i(\mathcal{N}, v) = \lambda_i \tilde{\alpha}_i\left(\sum_{\mathcal{S} \subseteq \mathcal{N} \setminus i : |\mathcal{S}|>1} \frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!} + \sum_{j \in \mathcal{N} \setminus i} \frac{(|\mathcal{N}|-2)!}{|\mathcal{N}|!} + \frac{(|\mathcal{N}|-1)!}{|\mathcal{N}|!}\right) = \lambda_i \tilde{\alpha}_i$$

where

$$\sum_{\mathcal{S} \subseteq \mathcal{N} \setminus i : |\mathcal{S}|>1} \frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!} + \sum_{j \in \mathcal{N} \setminus i} \frac{(|\mathcal{N}|-2)!}{|\mathcal{N}|!} + \frac{(|\mathcal{N}|-1)!}{|\mathcal{N}|!} = 1.$$

Thus, we have the solution imposing that for any $i \in \mathcal{N}$, the Shapley value becomes $Sh_i(\mathcal{N}, v^\lambda) = \tilde{\alpha}_i$ if we set $\lambda_i = 1, \forall i \in \mathcal{N}$. Moreover, v^λ corresponds to the total airtime fraction of LTE-U cells.

REMARK 3. The proof of Lemma 2 is intuitive. It imposes that we need only to know the individual payoffs of players. Thus, we can set $\zeta = 1$, and the mechanism $\mathbb{M}$ involves only the complexity of calculating the individual payoffs of players given in equation (15).

A.3 Proof of Lemma 3

A feasible allocation of airtime fraction is max-min fair if for each LTE-U cell i, α_i cannot be increased without decreasing α_j, where $\alpha_j \leq \alpha_i$. Let us define the order mapping $\mu : \Re^{|\mathcal{N}|} \to \Re^{|\mathcal{N}|}$ as the mapping which sorts in non-decreasing order: $\mu(\alpha_1, \ldots, \alpha_{|\mathcal{N}|}) = (\alpha_{(1)}, \ldots, \alpha_{(|\mathcal{N}|)})$ with $\alpha_{(1)} \leq \alpha_{(2)} \ldots \leq \alpha_{(|\mathcal{N}|)}$. *Lexicographic ordering* of vectors in $V(\mathcal{N})$ is defined by $\alpha \overset{\text{lex}}{>} \alpha'$ if and only if there exists an i such that $\alpha > \alpha'$ and for all $j < i$, $\alpha_j = \alpha'_j$. Also, it is said that $\alpha \overset{\text{lex}}{\geq} \alpha'$ if and only if $\alpha \overset{\text{lex}}{>} \alpha'$ or $\alpha = \alpha'$. Vector α is *leximin larger* than or equal to α' if $\mu(\alpha) \overset{\text{lex}}{\geq} \mu(\alpha')$. Thus, $\alpha \in V(\mathcal{N})$ is *leximin maximal* on set $V(\mathcal{N})$ if for all $\alpha' \in V(\mathcal{N})$, we have $\mu(\alpha) \overset{\text{lex}}{\geq} \mu(\alpha')$. If a max-min fair vector exists on $V(\mathcal{N})$, then it is the unique leximin maximal vector on $V(\mathcal{N})$. Due to the zero-monotonicity, in every turn, each player is better off, i.e. $\tilde{\alpha}_i \leq \alpha_i(\mathcal{S}) \leq \alpha_i(\mathcal{S}') \leq 1, \forall i \in \mathcal{S}', \forall \mathcal{S}' \subseteq \mathcal{N}$ where $\mathcal{S} \subset \mathcal{S}'$. That is enough for ensuring the leximin maximal property of α. This concludes the proof that the *NTU Shapley value is max-min fair* in the proposed payoff configuration.