- Why did you choose this course?
- Why did you choose the clothes you're wearing?
- Why are you sitting where you are?
- Why are you reading this?

- Who or what made the decision???

Decision Making

Readings: Gold and Shadlen, the neural
basis of decision making, 2007

Theoretical framework: statistical inference

- decision making can be thought of as a form of statistical inference.
- decide = select among competing hypotheses h_1, h_2 (and may be more)
- elements of this decision process:

  ✴ priors $P(h_1)$= Probability that h_1 is correct before collecting any evidence = a bias (or prejudice)

  ✴ evidence (e) = information we can collect in factor of h_1. Only useful when we know how likely it is to be true if the hypothesis is true i.e. if we have conditional probabilities such as $P(e|h_1)$ = the likelihood

  ✴ value(v) = subjective costs and benefits for each outcome.

Bayes' Theorem

- Bayes' theorem is a result in probability theory that relates conditional probabilities P(AIB) and P(BIA)
- Given the likelihood and the prior, we can compute the posterior.

Reverend Thomas
Bayes, 1702- 1761

$$P(h_1|e) = \frac{P(e|h_1)P(h_1)}{P(e)}$$

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalizing constant}}$$

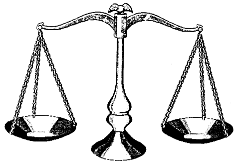
To decide, compare probabilities of each hypothesis

- Choose h_1 if:

$$P(h_1|e) = \frac{P(e|h_1)P(h_1)}{P(e)}$$

$$>$$

$$P(h_2|e) = \frac{P(e|h_2)P(h_2)}{P(e)}$$



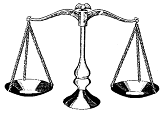
Likelihood ratio test

- Just re-organizing the terms of this inequality: - choose h_1 if:

$$\frac{P(e|h_1)}{P(e|h_2)} > \frac{P(h_2)}{P(h_1)}$$

- This is known as the likelihood ratio test = optimal decision rule.
- If the prior probabilities are equal (0.5), choose h_1 if

$$LR = \frac{P(e|h_1)}{P(e|h_2)} > 1$$



Values (1)

- It might be that the costs and benefits associated with the various outcomes are very different.
- benefit of choosing h_1 = value of choosing h_1 if h_1 is true (V_{11}) + value of choosing h_1 if h_1 is wrong (V_{12}) given the evidence.
- benefit of choosing h_2 = value of choosing h_2 if h_2 is true (V_{22}) + value of choosing h_2 if h_2 is wrong (V_{21}) given the evidence.

run or not?

- So we now want to compare:

$V_{11}P(h_1|e) + V_{12}P(h_2|e)$ with $V_{22}P(h_2|e) + V_{21}P(h_1|e)$

Values (2)

- rewriting this gives the general (optimal) rule: choose h_1 if :

$$\frac{P(e|h_1)}{P(e|h_2)} > \frac{(V_{22} - V_{12})P(h_2)}{(V_{11} - V_{12})P(h_1)}$$

- which has also the form of comparing the LR with a threshold.
- Signal detection theory: LR (or any monotonic function of it - e.g. LOG) provides an optimal 'decision variable'.

Sequential Analysis

- This framework can be extended to the situation where we have multiple pieces of evidence e_1, e_2, ..e_n observed over time.
- Here we allow the decision variable to 'accumulate the evidence' in time:

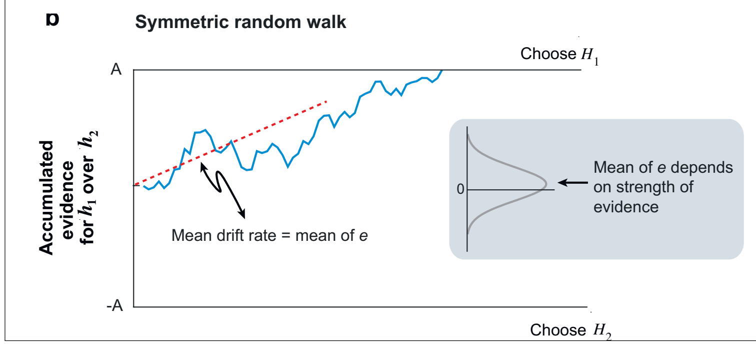
$$\log LR_{12} \equiv \log \frac{P(e_1, e_2, \ldots, e_n | h_1)}{P(e_1, e_2, \ldots, e_n | h_2)} = \sum_{i=1}^{n} \log \frac{P(e_i | h_1)}{P(e_i | h_2)}.$$

- When the DV reaches a threshold (which possibly reflects priors and values), a decision is made.
- This is known as the sequential probability ratio test (optimal rule).

$$\begin{array}{llll} e_0 \rightarrow f_0(e_0) & \Rightarrow \underset{or}{Stop} & & \\ & \swarrow & & \\ & e_1 \rightarrow & f_1(e_0, e_1) & \Rightarrow \underset{or}{Stop} \\ & & & \swarrow \end{array}$$

Random Walk model (1)

- Related to this framework are the random walk and race models of decision making developed by psychologists to explain behavioral data.
- The Decision Variable is the cumulated sum of the evidence. The bounds represent the stopping rule.
- If e is log LR, then this model = sequential prob ratio test.



Random Walk model (2)

- stochastic differential equation of the form (Wiener process)

$$\tau \frac{d}{dt} v(t) = e(t) + \eta(t)$$

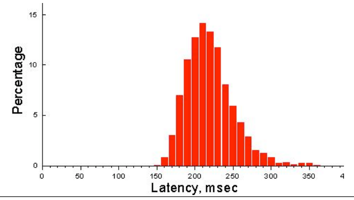
- or (Ornstein Uhlenbeck process) - similar but assume a decay or leakage in the accumulation process.

$$\tau \frac{d}{dt} v(t) = -v + e(t) + \eta(t)$$

Random Walk model (3)

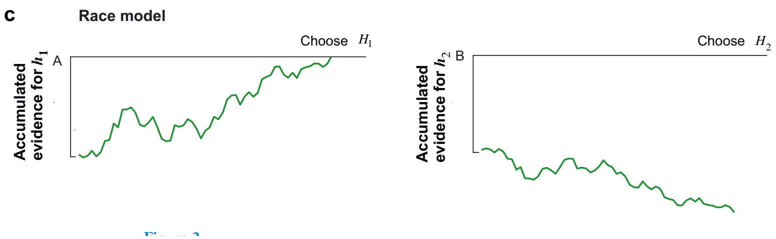
- Well-studied mathematically (diffusion processes)
- many variants (discrete time, continuous time, leaky integration)
- These models have been compared systematically and shown to successfully account for [Smith & Ratcliff, 2004]:

- Distribution of Reaction Times
- Speed-accuracy tradeoff: decreasing the boundary has the effect of increasing speed and decreasing accuracy.
- Error response RTs (sometimes error responses can be very quick..).



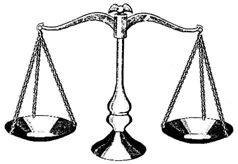
Race Model

- Another variant is the race model
- Two or more decision processes represent the accumulated evidence for each alternative.



- Different properties

- Anything like that in the brain?



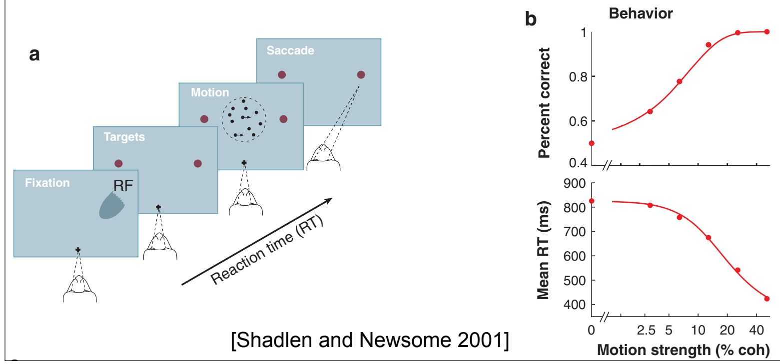
- yes

- study decision on perceptual tasks

Mike Shadlen, Paul Glimcher
(and others)

Random Dots Motion Direction Task

- monkey decides between 2 possible opposite directions, and saccade to signal his choice whenever he is ready.
- task difficulty is controlled by varying coherence level
- decision = problem of movement selection

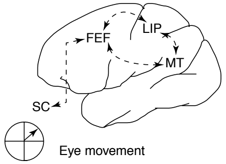


[Shadlen and Newsome 2001]

neural basis of the perceptual decision ?

- a sensory stage where the evidence is collected. MT seems to fulfill the role.
- a decision stage ‘reading-out’ the sensory stage.
- These neurons must accumulate the information over time to explain performance accuracy
- A sustained activity is needed to compare alternatives presented successively in time.
- neurons in parietal and frontal ‘association’ cortex
- possibly the neurons that are linked to the specific behavioral response (= the preparation of the saccade)

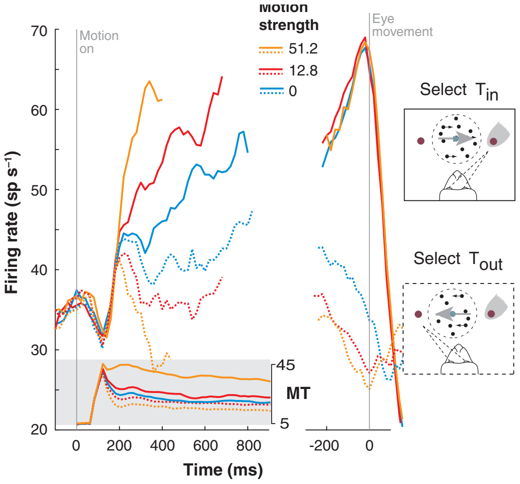
Accumulation of Evidence in LIP (1)



- LIP receives inputs from MT and MST, outputs in FEF and SC (generation of saccades)
- LIP is implicated in selection of saccade targets, working memory, intention etc..
- Record neurons which have one of the choice targets in the response field and the other outside.

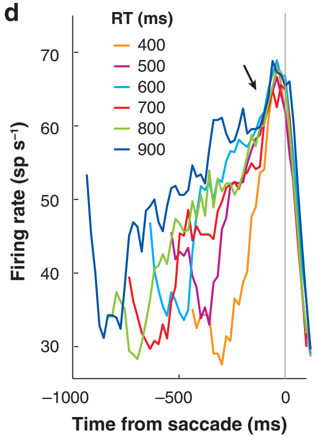
- After ~ 220 ms, response reflects decision - faster rise for easier choices, decrease for opposite direction.
- Aligning responses to saccade initiation reveals correlate of commitment: a threshold rate at which the decision is made, ~ 70 msec before saccade initiation.

Accumulation of Evidence in LIP (2)



Accumulation of Evidence in LIP (3)

- Responses grouped by RT
- Responses achieve a common level of activity ~ 70 msec before saccade initiation
- When the monkey chooses other direction, another set of neurons (with chosen target in their RFs) behave similarly
- as if the fact that they reach a threshold value ‘determines the termination of the decision process’



[Gold and Shadlen 2007]

Accumulation of Evidence in LIP (4)

- pattern of LIP activity matches prediction of diffusion/race models.
- rise of activity appears to reflect accumulation of evidence
- evidence could come from a difference in activity of pools of MT neurons with opposite direction preferences, which was suggested to approximate the LogLR (Gold & Shadlen, 2001)

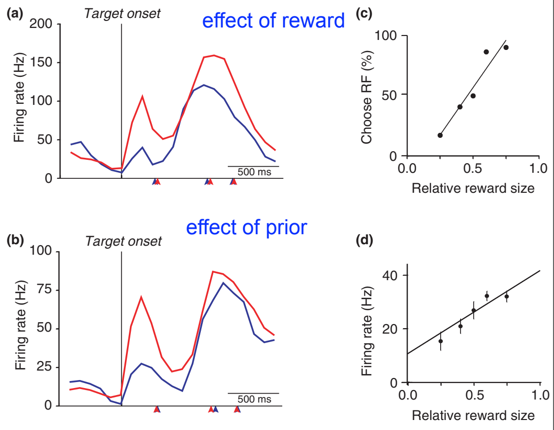
- suggests that LIP neurons represent the decision variable ?
- implements a logLR test?
- How is the criterion / threshold set and what happens when it is reached?

Platt & Glimcher 1999 (1)

- monkeys cued by a color of a fixation stimulus to saccade on 1 of 2 targets
- change the reward associated with each target (value)
- vary the probability that a saccade to a target will be required (prior)
- offset of the responses of LIP neurons before and during presentation of the saccade target
- suggests that behavioral outcome and priors are also encoded.

Platt & Glimcher 1999 (2)

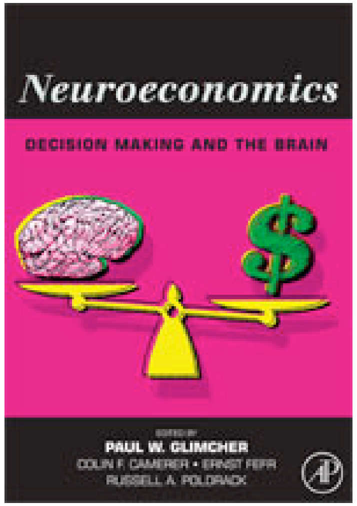
Neural correlates of behavioral value. **(a)** Average firing rate of a single LIP neuron plotted as a function of time, on trials in which a saccade in the preferred direction (RF) of the neuron was cued. Neuronal activity was greater when a large reward was associated with the cued saccade (red curve) than when a small reward was associated with the same movement (blue curve). Arrows indicate, successively, mean times of instruction cue onset, central fixation stimulus offset, and saccade onset in high (red) and low (blue) reward blocks. **(b)** Neuronal activity for a second LIP neuron was greater when the cued movement was more probable (red curve) than when the same movement was less probable (blue curve). Conventions as in (a). **(c)** When free to choose, monkeys shift gaze to the target associated with the larger reward. Relative reward size reflects the volume of juice available for a saccade in the neuron's preferred direction, divided by the total volume of juice available from both possible saccades, within a block of trials. Data are from a single experiment. **(d)** Average activity (± standard error) of a single LIP neuron measured after target onset and plotted as a function of relative reward size, for trials in which the monkey shifted gaze in the neuron's preferred direction. Data are from the same experiment as in (c). Adapted with permission from [60]. RF, response field.



"understand the processes that connect sensation and action by revealing the neurobiological mechanisms by which decisions are made"

...

"an emerging transdisciplinary field that uses neuroscientific measurement techniques to identify the neural substrates associated with economic decisions"



Neuroeconomics

- Add neural data to the Study of Economic Decisions. For e.g., what do you prefer: 45 pesos or a gamble with a 50% chance of 100 pesos and 50% chance of nothing?

- Utility Theory: subjective desirability
- Games Theory: John von Neumann and Oscar Morgenstern (1944)

mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others. E.g. Prisoner's Dilemna.

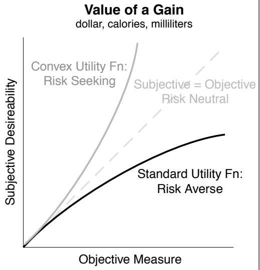


Fig. 1. Bernoulli's notion of subjective value or utility. The black line plots the typical relationship between objective and subjective valuations of an action. As the objective value of a gain increases, the subjective desirability, or utility, grows more slowly. Bernoulli demonstrated that this relationship could account for the observation that humans are typically risk-averse. The solid gray line plots a condition in which subjective value grows more quickly than objective value, a preference structure that would yield risk-seeking behavior.

Games Theory: The prisoner's dilemma

- 2 suspects are arrested. Police have insufficient evidence for conviction, and visit each of them separately to offer the same deal.

- If one testifies (defects) against the other and the other remains silent (cooperates), betrayer goes free and the silent accomplice receives the full 10-year sentence.

- If both remain silent, both prisoners are sentenced to only 6 months in jail.

- If each betrays the other, each receives a 5-year sentence.

- What would you do?

Summary

- a decision = process that weights priors, evidence, and value to generate a commitment
- Signal detection theory and sequential analysis provide a theoretical framework for understanding how decisions are formed
- Studies that combine behavior and neurophysiology have begun to uncover how the elements of decision formation are implemented in the brain
- Perceptual tasks are used to distinguish evidence and decision variable.
- comparing a decision variable to a given threshold seems to be the basic mechanism of decision making
- Many open questions though ...