CCN Exercise sheet 2 - Neurons and Bayes

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Introduction

The aim of the first exercise is to learn some of the basic tools of computational neuroscience: models of spiking neurons and synapses. The aim of the second exercise is to learn some elements of Bayesian modeling.

Figures should have labelled axis, legends and a caption. Report/ Describe your findings. The format and presentation of the results will count in the final mark.

If you find some important typos or missing information in those exercises, please let me know. Don't hesitate to discuss your results with your friends, but I expect that the reports and codes will be done individually (it's not okay to share reports and code).

1 Exercise 1: The Integrate-and-Fire Neuron

The integrate-and-fire neuron is the model that is classically used to describe spiking neurons.

Build a model integrate-and-fire neuron defined by:

$$\tau_m \frac{dV}{dt} = E_L - V + R_m I_e \tag{1}$$

Use $E_L = -65$ mV, $R_m = 90 M\Omega$, $\tau_M = 30$ msec. When the membrane potential reaches $V_{th} = -50$ mV, make the neuron fire a spike and reset the potential to $V_{reset} = -65$ mV .

- Show sample voltage traces (with spikes) for a 300-ms-long current pulse (choose a reasonable current I_e) centered in a 500-ms-long simulation.
- Determine the firing rate of the model for various magnitudes of constant I_e and compare with figure 1 (Figure 5.6 of [2]) .

- Include an extra current in the integrate-and-fire model to introduce spike-rate adaptation:

  $$\tau_m \frac{dV}{dt} = E_L - V - r_m g_{sra}(V - E_K) + R_m I_e \tag{2}$$

  This conductance relaxes to 0 exponentially with time constant τ_{sra} through the equation

  $$\tau_{sra} \frac{dg_{sra}}{dt} = -g_{sra} \tag{3}$$

  Whenever the neuron fires a spike, g_{sra} is increased by an amount Δg_{sra} : $g_{sra} \rightarrow g_{sra} + \Delta g_{sra}$. Use the parameters of Figure 1 (C).

- Check how things change and comment about the comparison with Figure 1. Parameters can be changes to allow a better fit if needed.

Synaptic inputs

In realistic conditions, neurons are activated not by injected currents, but by a bombardment of synaptic inputs.

- First simulate a presynaptic Poisson spike train. The rate of the Poisson spike train is 100 Hz (= 100 spikes/seconds). Simulate 1 second of this spike train. Illustrate and comment. Tips: check out the method described in the class (first lecture: encoding) to do this.

- Add an excitatory synaptic conductance to the integrate-and fire model by adding the extra synaptic conductance term

  $$\tau_m \frac{dV}{dt} = E_L - V - r_m g_s P_s(V - E_s) + R_m I_e \tag{4}$$

  with $E_s = 0$. Set the external curent to zero, $I_e = 0$. Use $r_m g_s = 0.35$ and describe P_s using the following equations:

  $$\tau_s \frac{dP_s}{dt} = -P_s \tag{5}$$

  making the replacement $P_s \rightarrow 1$ immediately after each presynaptic action potential.

- Visualize the membrane potential of this neuron for a constant rate presynaptic Poisson spike train with frequency 10 Hz. Use τ_s =10 msec.

- Increase the frequency of the presynaptic spike train so as to make the cell fire (the presynaptic spike train might in fact represent the combined spiking of multiple neurons synapsing on the recorded cell). Illustrate.

(from exercises 5.3 and 5.4 of [2])

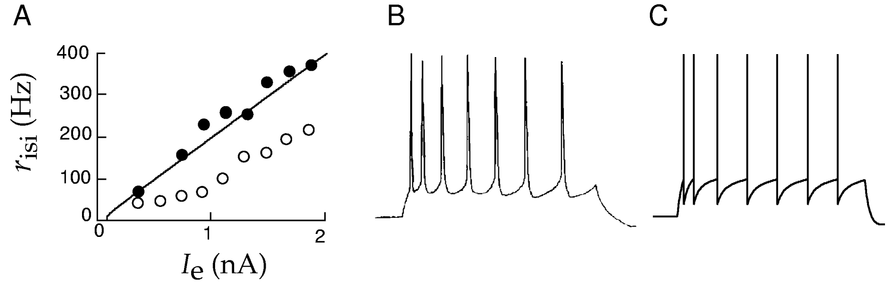

Figure 1: A) Comparison of firing rates as a function of injected current for an integrate and fire model and a cortical neuron measure in vivo. The line gives r_{isi} for a model neuron with $\tau_m = 30$ msec, $E_L = V_{reset} = -65$ mV, $V_{th} = -50$ mV and $R_m = 90M\Omega$. The data points are from a pyramidal cell in primary visual cortex of a cat. The filled circles show the inverse of the inter-spike interval for the first two spikes fired, while the open circles show the steady-state interspike-interval firing rate after spike-rate adaptation. B) A recording of the firing of a cortical neuron under constant current injection showing spike-rate adaptation. C) Membrane voltage trajectory and spikes for an integrate and fire model with an added current with $r_m \Delta g_{sra} = 0.6$, $\tau_{sra} = 100$ ms and $E_K = -70$ mV. (Figure 5.6 of [2])

2 Exercise 2: Bayesian Cue Combination

2.1 Background

We are interested in understanding how humans localize sound sources, when they can use both hearing and vision. We are also interested in illusions that arise in situations where there is a conflict between the two signals, for example as in ventriloquism [1].

We first present subjects with visual stimuli v: a low contrast Gaussian blob, very quickly flashed at some position in space x_v. The subjects are asked to localize the stimulus position x. We find that the subjects estimates are unbiased on average but display some variability from trial to trial. Their estimates $P(x|v)$ can be modelled using a gaussian distribution with mean x_v and variability $\sigma_v = 4$ degree of visual angle.

We then remove the visual stimuli and present subjects with very briefs auditory stimuli a (“clicks”) originating from different positions in space x_a. We again ask the subjects to localize the source of the sound, and find that their estimates $P(x|a)$ can be modelled using a gaussian distribution with mean x_a and variability $\sigma_a = 3$ degrees of visual angle.

2.2 Cue Combination

Now we present the visual and auditory stimuli at the same time and at the same location in space.

- If humans are Bayesian optimal in integrating the information from both sources, how precise are they going to be in their localization performance now ? Express mathematically the distribution of the subjects' estimates – explaining how this is derived.
- Plot the three distributions (based on vision alone, based on audition alone, and based on both vision and audition) on the same graph.

Now we trick the subjects. We tell them that the 2 stimuli come from the same point in space and corresponded to a single event (like a ball hitting the screen), but actually we introduce a small displacement between the stimuli: the visual stimulus is displaced 5 deg rightwards and the auditory stimulus displaced 5 deg leftwards ($x_V - x_A = 10$ deg, where x_v and x_a are the spatial positions of the visual and auditory stimuli)

- How is this affecting the response? Plot the 3 distributions on the same graph.
- Now we keep the auditory stimulus unchanged but vary the blurry-ness of the visual stimulus, we use first a very precise stimulus v_1 and repeat the experiment. We measure in this case $\sigma_{v1} = 1$ deg. Then we try a very blurry stimulus v_2 for which we measure $\sigma_{v2} = 20$ deg. Where do subjects localize the source in these 2 cases ? Illustrate these examples and comment on your results.

These predictions were precisely tested by [1] and they found that human behaviour is consistent with the Bayesian predictions. As described in class, this seems to be a general result, with evidence from a number of experimental protocols, in different modalities [1, 3, 4].

- If you were to construct an artificial system which can also achieve such optimal combination of different cues (based on information for two types of captors), how would you do? What questions/ challenges are you going to face? Do you have ideas how this could be implemented with neurons and networks of neurons? What's the difficulty / the implications ? Discuss.

References

[1] David Alais and David Burr. The ventriloquist effect results from near-optimal bimodal integration. *Curr Biol*, 14(3):257–262, Feb 2004.

[2] Peter Dayan and Larry F Abbott. *Theoretical Neuroscience*. MIT Press, 2001.

[3] M.O. Ernst and M.S. Banks. Humans integrate visual and haptic information in a statistically optimal fashion. *Nature*, 415:327–348, 2002.

[4] David C Knill and Alexandre Pouget. The bayesian brain: the role of uncertainty in neural coding and computation. *Trends Neurosci*, 27(12):712–719, Dec 2004.