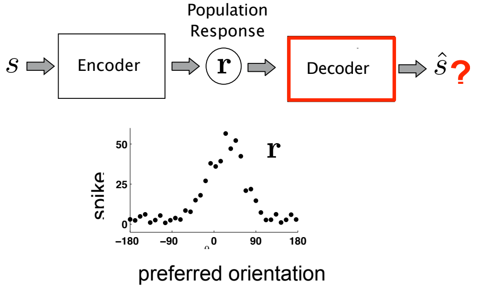
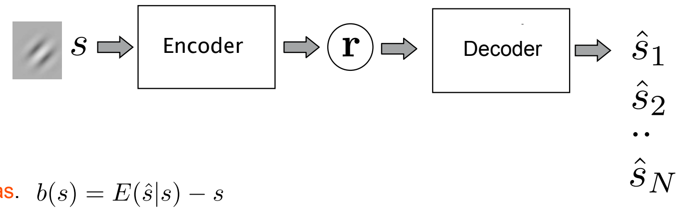
Decoding: Summary of Last Lecture

- Decoding: for neuro-prostheses and/or for understanding the relationship between the brain's activity and perception or action

- Different strategies are possible: optimal decoders (e.g. ML, MAP) vs simple decoders (e.g. winner take all, population vector), depending on what we know about the encoding model, and constraints.



How good is our estimation of *s*? a bit of estimation theory ...



- Bias. $b(s) = E(\hat{s}|s) - s$

  If $E(\hat{s}|s) = s$ the estimator is said to be unbiased.

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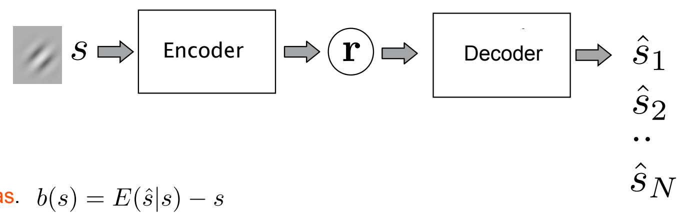

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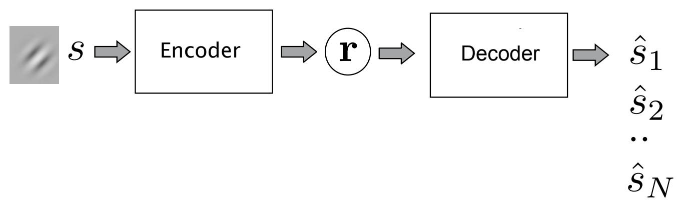
- Variance $var(\hat{s})$

  Estimation theory tells us that, knowing the encoder model P[r|s], there is a lower bound on the variance that can be achieved by any decoder. This quantity is known as the Cramer-Rao Bound. The denominator is known as Fisher Information.

$$var(\hat{s}) \geq \frac{(1 + b'(s))^2}{I_F(s)}$$

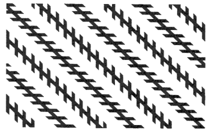
From Population Codes to Psychophysics

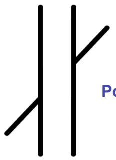


- Now imagine the stimulus is an oriented Gabor patch;
- The encoder is a set of noisy tuning curves, e.g. in V1;
- *r* is a population response;
- The decoder is the model we choose for this (e.g optimal);
- *s_hat* is the percept of the grating;
- **How can we relate this model of perception and the statistics of the estimates with psychophysical performance?**

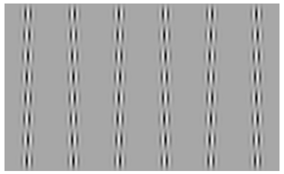
Estimation tasks

✳ The measured quantity is the difference between the perceived orientation and the real orientation: $< \hat{s} > - s$

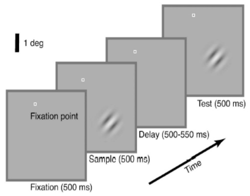




Fraser Illusion



Discrimination Tasks

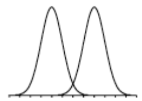


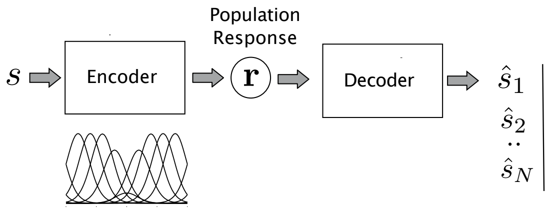
✳ **Is the second grating of the same orientation as the first grating, or a different orientation?**

✳ The measured quantity is the Discrimination Threshold a.k.a Just Noticeable difference (JND)

Discrimination threshold for stimulus 1 and 2 depends on the overlap between the internal 'representation' of the 2 stimuli: $p[r|s_1]$ and $p[r|s_2]$:

- The bias of the internal representation (expansion/contraction of the 'distance' between the stimuli)
- How noisy the internal representation is (the variance of the estimates)





Linking the statistics of the model and psychophysics

$b(\hat{s}) = < \hat{s} > - s$ ⟷ perceptual bias

$\mathrm{var}(\hat{s})$

$$\mathrm{threshold}(\hat{s}) = \frac{\mathrm{std}(\hat{s})}{1 + b'(\hat{s})}$$ ⟷ discrimination threshold (76% correct)
just noticeable difference

From the Cramer Rao Bound, knowing the encoder model and independently of the decoder, it is known that the threshold is bounded by the sqrt of Fisher information:

$$\mathrm{threshold}(\hat{s}) \geq \frac{1}{\sqrt{I_F(s)}}$$

Fisher information: the best possible discrimination performance for a given encoder model

✳ Fisher information: gives the discrimination threshold that would be obtained (asymptotically) by an optimal decoder, for eg. ML (units of var ^-1)
✳ is expressed in terms of the encoding model P[r|s], i.e. in terms of the tuning curves and the noise

$$I_F(s) = - < \frac{\partial^2 \ln P[r|s]}{\partial s^2} >$$

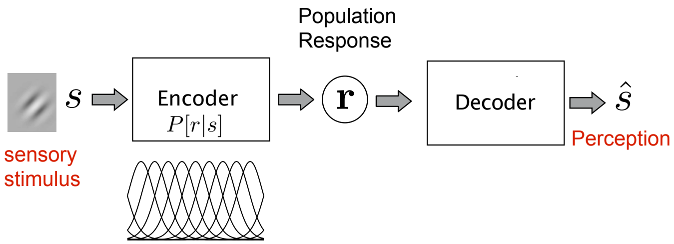
✳ Interpreted as a measure of 'information' in the responses;
✳ a useful tool to relate directly the properties of the neural responses with discrimination performance.

From Population Responses to Psychophysics

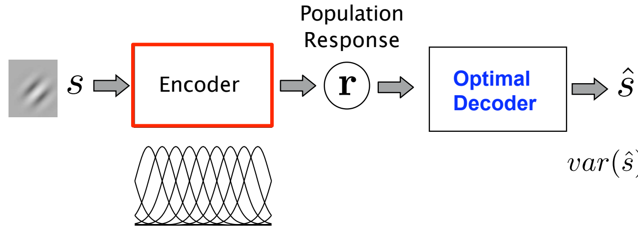
Two strategies:

✶ Assume the decoder is optimal: Compute Fisher information from P[r|s]. We know that this will give us the minimal possible variance of any unbiased decoder, and the minimal threshold of any decoder (biased or unbiased).

✶ Construct explicitly the decoder (e.g. population vector). Compute explicitly bias, variance, and threshold of estimates.



From Population Responses to Psychophysics



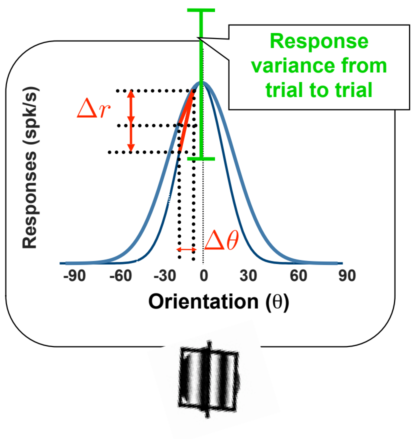
$$var(\hat{s}) \simeq \frac{1}{I_F(s)}$$

$$\mathrm{thres}(\hat{s}) \simeq \frac{1}{\sqrt{I_F(s)}}$$

What changes in encoder would increase discrimination performances?

- Number of neurons?
- Tuning curves shape ?
- Noise correlations ?

What are the factors that control performance?



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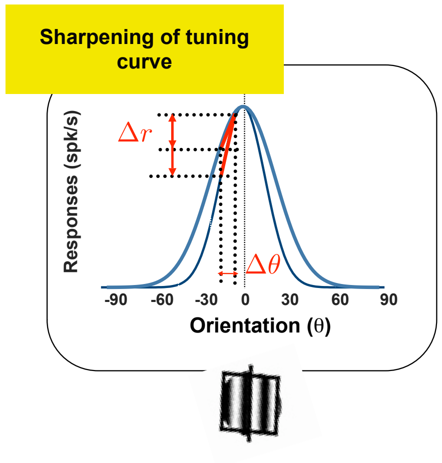

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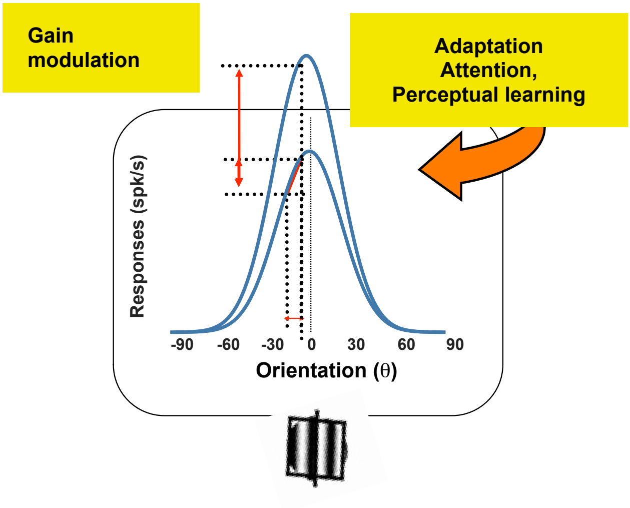

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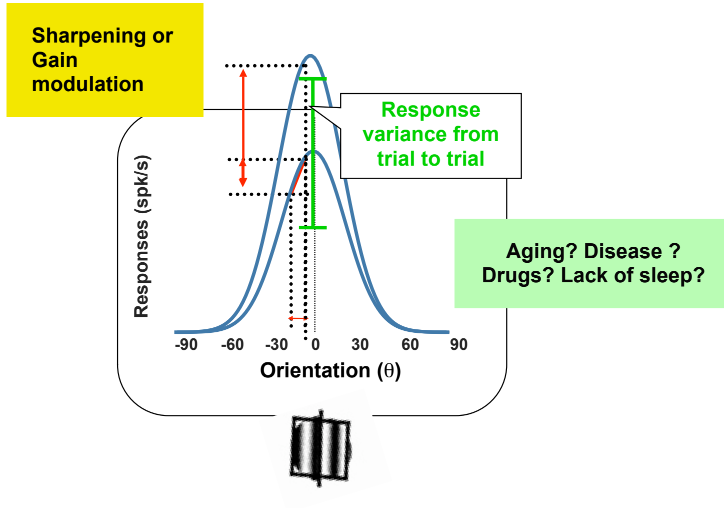

What are the factors that control performance?

- Fisher information formalizes intuition and provides a tool to explore these questions precisely.
- For Poisson noise:

$$I_i(s) = \frac{f_i'(s)^2}{f_i(s)}$$

$\frac{\textbf{Slope}^2}{\textbf{variance}}$

$$I(s) = \sum_i \frac{f_i'(s)^2}{f_i(s)}$$

For independent neurons, FI of the population is the sum of each neurons' FI

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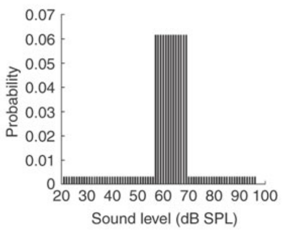
- For Gaussian correlated noise:

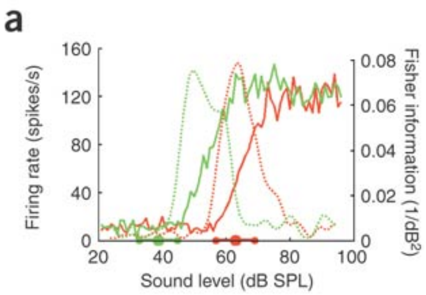
$$I_F(s) = \mathbf{f}'(\mathbf{s})\mathbf{Q}^{-1}(\mathbf{s})\mathbf{f}'(\mathbf{s}) + \frac{1}{2}\mathrm{Trace}[\mathbf{Q}^{-1}(\mathbf{s})\mathbf{Q}'(\mathbf{s})\mathbf{Q}^{-1}(\mathbf{s})\mathbf{Q}'(\mathbf{s})]$$

For correlated neurons, FI is modulated by correlations.

Research questions

✳ What would be the ‘optimal’ shape for tuning curves?
✳ Are adaptation, attention and learning a step towards more ‘optimal’ tuning curves for the attended/trained stimulus ?



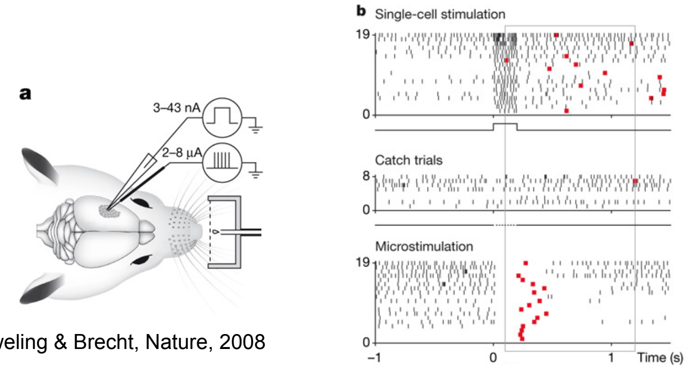


Neurons in auditory midbrain of the guinea pig adjust their response to improve the accuracy of the code close to the region of most commonly occurring sound levels.

[Dean, Harper & McAlpine, Nature Neuro, 2005]

Research questions

✳ How many neurons participate in a psychophysical task ? (see also, lab 1) 1, 10, 100, 10000? How can we find out ?
✳ comparing performance (e.g. MT: Britten et al 1992). stimulating (MT: Salzman, Britten, Newsome 1990).



Houweling & Brecht, Nature, 2008

Research questions

✳ Pooling from large populations of neurons thought to be a way to average out the noise.
✳ Pairs of neurons show correlations in their variability: does pooling more and more neurons increases (linearly) the accuracy of the representation?
or Is information saturating over a certain number of neurons ?
[Zohary et al 1994]

Research questions

✳ Can the study of illusions inform us on the type of ‘decoder’ that is used in the brain? (cf assignment 1/ visual adaptation)