Models of networks

Readings: D&A, chapter 7.

Networks of neurons

• Neurons are organized in large networks. A typical neuron is cortex receives thousands of inputs.

• Aim of modeling networks: explore the computational potential of such connectivity.

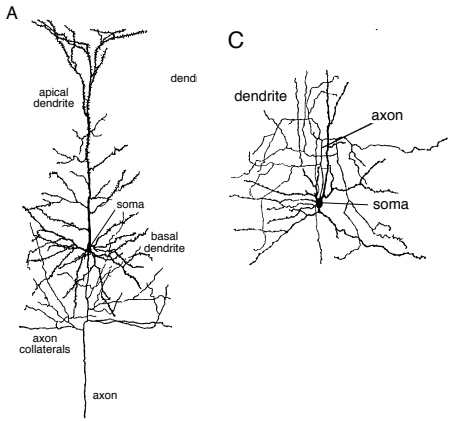
- What computations? (e.g. gain modulation, integration, selective amplification of some signal, memory etc..)
- What dynamics ? (e.g. spontaneous acticity, variability, oscillations)

•Tools:

- models of neurons and synapses : spiking neurons (IAF) or firing rate
- analytical solutions, numerical integration

What's a network ?

• In cortex, ~80% excitatory cells (pyramidal neurons), ~20% inhibitory neurons (smooth stellate + large variety of other types)/ a.k.a interneurons.

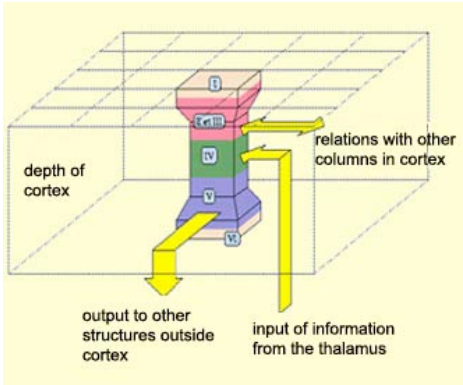


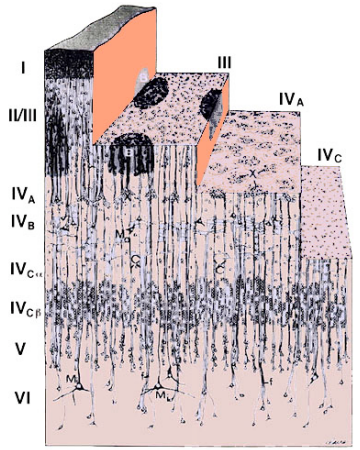
What's a network ?

• Laminar Organization.

Cortex is divided into 6 layers.

Models usually pool all layers together.

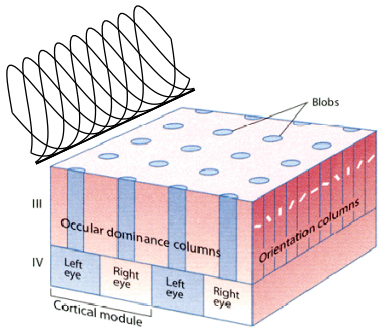




What's a network ?

• Columnar Organization.

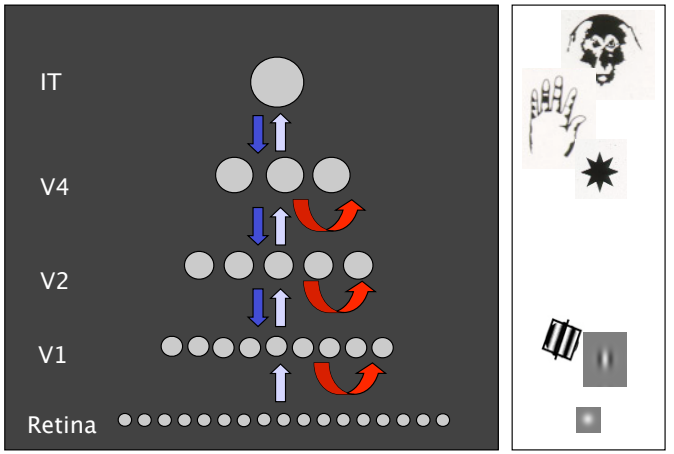
Neurons in small (30-100 micrometers) columns perpendicular to the layers (across all layers) respond to similar stimulus features.

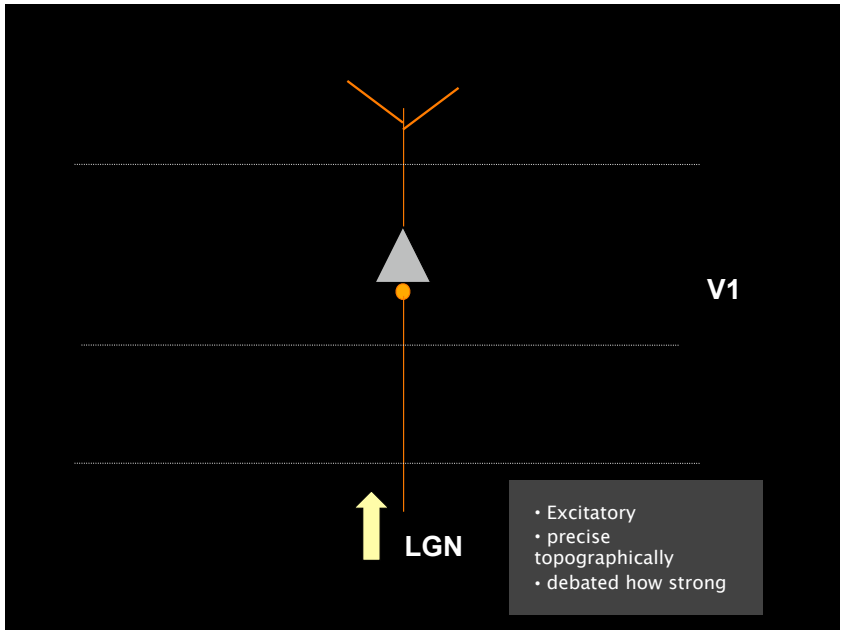


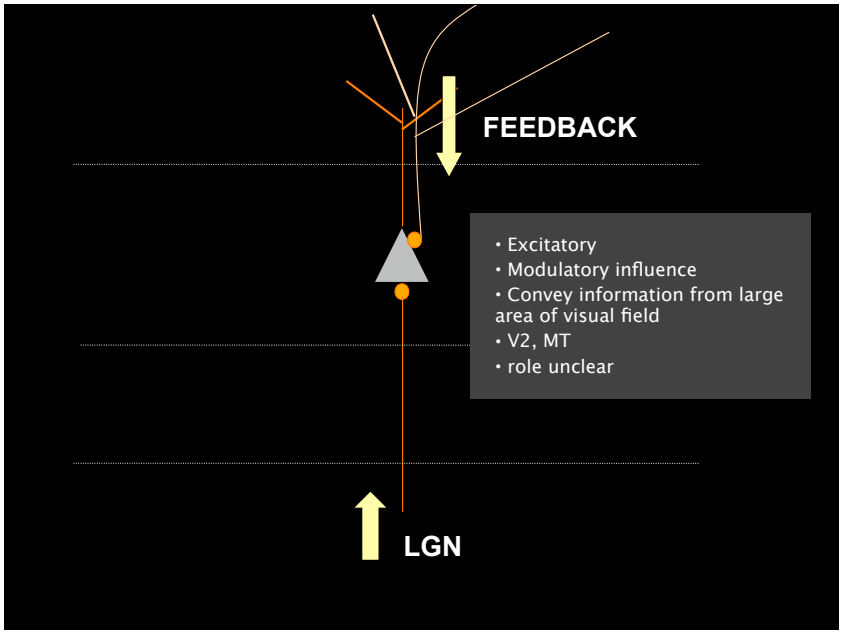
(Aus Gazzaniga et al., 1998)

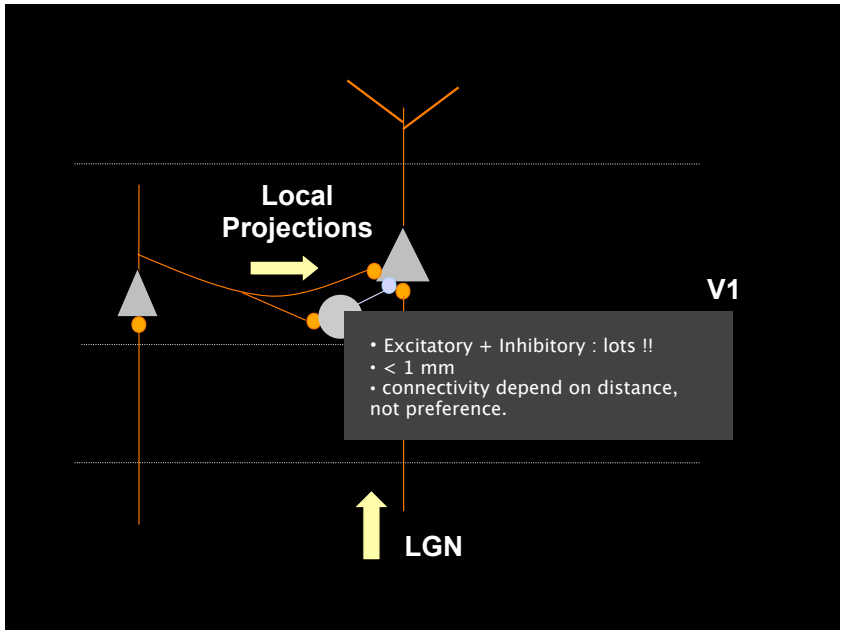
Connectivity

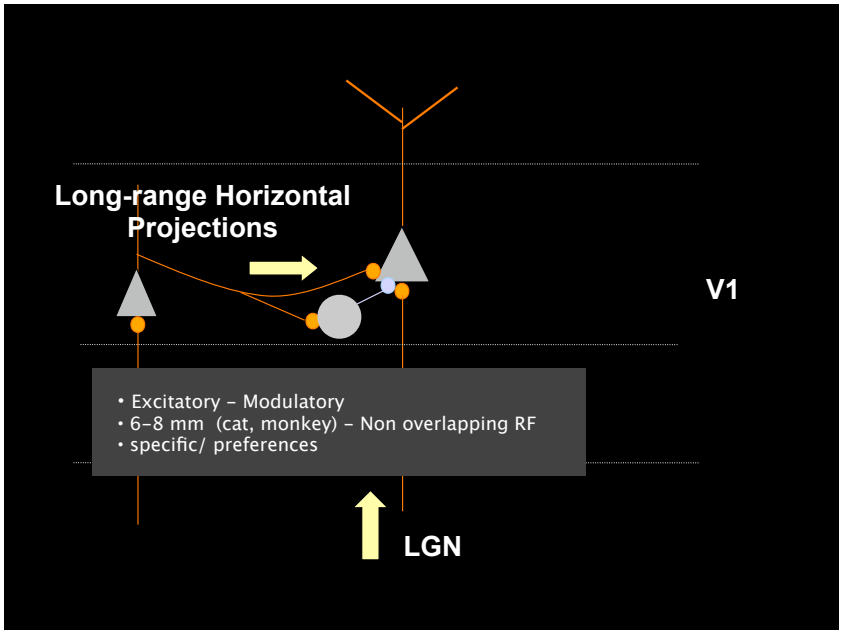
• 3 types of connections: feed-forward, recurrent (lateral), feedback.











Network modeling strategies (1)

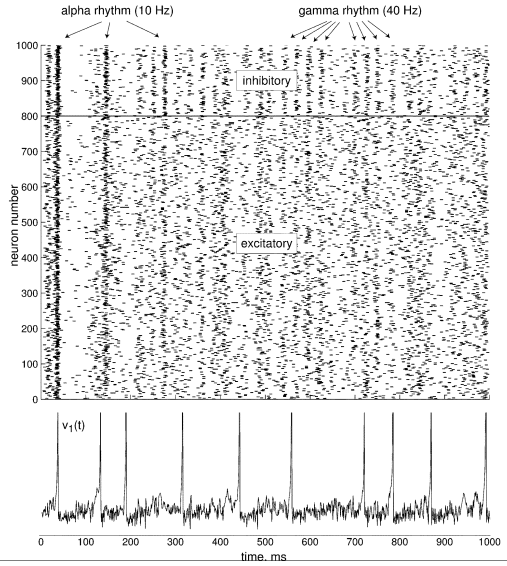
- method 1: spiking neurons, e.g. integrate and fire neurons

$$C_m \frac{dV_i(t)}{dt} = -\sum_j g_{ij}\left(t-\tau_{ij}\right)\left(V_i(t)-E_{\text{EXCIT}}\right) - \sum_j g_{ij}\left(t-\tau_{ij}\right)\left(V_i(t)-E_{\text{INHIB}}\right)$$
$$-g_{\text{LEAK}}\left(V_i(t)-E_{\text{LEAK}}\right) - g_{\text{AHP}}\left(t\right)\left(V_i(t)-E_{\text{AHP}}\right).$$

$$g_{ij}(t) = \bar{g}_{ij}\sum_l \left[t-t_j^l\right]^+ \left(\frac{e}{\tau_{\text{peak}}}\right)\exp\left(-\frac{t-t_j^l}{\tau_{\text{peak}}}\right).$$

- up to 10,000 neurons.
- advantage: comparison with electrophysiology, a system where all neurons can be 'recorded' at all times.
- difficulties: lots of parameters/assumptions, long simulations, analysis difficult.

Network modeling strategies (2)



spike raster

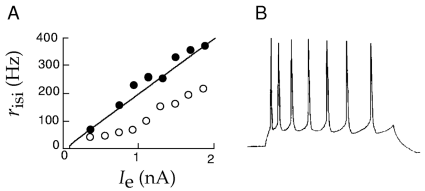
neuron trace

[Izhikevitch, 2003]

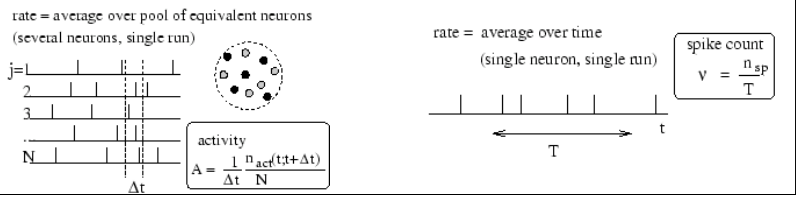
Network modeling strategies (3)

- method 2: reduce the description to describe only rate of spiking r(t), instead of Vm(t).

$$\tau_r \frac{dr_i(t)}{dt} = -r_i(t) + \text{input}(t)$$



- Interpretation: average over time, average over equivalent neurons

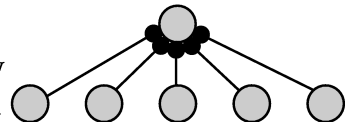


Firing rate model (1)

- each neuron is described at time t by a firing rate v(t).

$$\tau_r \frac{dv_i(t)}{dt} = -v_i(t) + F(\sum_{j=1}^{j=N} w_{ij}u_j)$$

output v
weights $\mathbf{w}$
input $\mathbf{u}$



- In absence of input, the firing rate relaxes to 0 with a time constant t_r - which also determines how quickly the neuron responds to input.
- The input from a presynaptic neuron is proportional to its firing rate *u*
- The weight w_{ij} determines the strength of connection of neuron j to neuron i
- The total input current is the sum of the input from all external sources.

Firing rate model (2)

- each neuron is described at time t by a firing rate v(t).

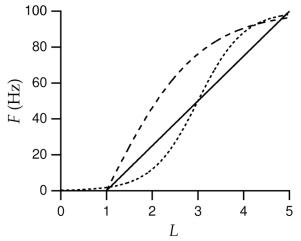
$$\tau_r \frac{dv_i(t)}{dt} = -v_i(t) + F(\sum_{j=1}^{j=N} w_{ij}u_j) = -v_i(t) + F(\mathbf{w}.\mathbf{u})$$

dot-product

- F determines the steady state r as a function of input
- F is called the activation function
- F can be taken as a saturating function, e.g. sigmoid
- F is often chosen to be threshold linear

$$F(I) = \frac{r_{max}}{1 + \exp(g(I_{1/2} - I)}$$

$$F(I) = G[I - I_0]_+$$



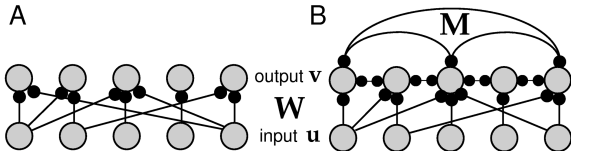
Network Architectures

- A: Feedforward

$$\tau_r \frac{dv_i(t)}{dt} = -v_i(t) + F(\sum_{j=1}^{N} W_{ij}u_j(t))$$

- B: Recurrent

$$\tau_r \frac{dv_i(t)}{dt} = -v_i(t) + F(\sum_{j=1}^{N} W_{ij}u_j(t) + \sum_{k=1}^{N} M_{ik}v_k(t))$$



Excitatory - Inhibitory Network

- Some models have a single population of neurons and the weights are allowed to be positive and negative.
- Other models represent the excitatory and inhibitory population separately. (more 'biological' + richer dynamics).
- 4 weight matrices, M_{EE}, M_{IE}, M_{II}, M_{EI}

$$\tau_E \frac{d\mathbf{v}_E}{dt} = -\mathbf{v}_E + \mathbf{F}_E\left(\mathbf{h}_E + \mathbf{M}_{EE}\cdot\mathbf{v}_E + \mathbf{M}_{EI}\cdot\mathbf{v}_I\right)$$

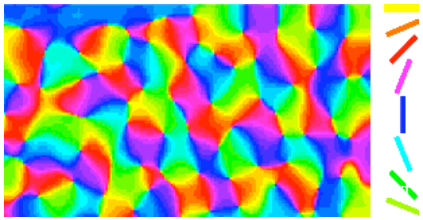
and

$$\tau_I \frac{d\mathbf{v}_I}{dt} = -\mathbf{v}_I + \mathbf{F}_I\left(\mathbf{h}_I + \mathbf{M}_{IE}\cdot\mathbf{v}_E + \mathbf{M}_{II}\cdot\mathbf{v}_I\right) .$$

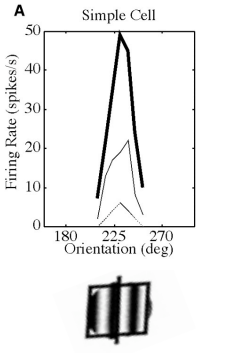
Example:

Orientation selectivity as a model problem: spiking networks and ring model

LGN neurons are not selective to orientation, V1's are: Origin of Orientation selectivity ?



V1

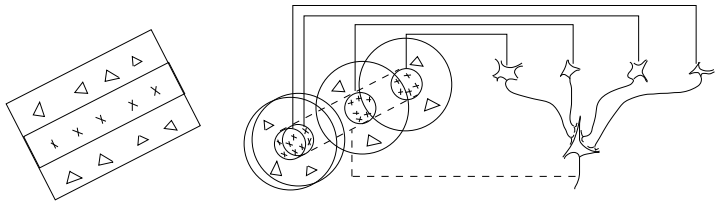


Sclar and Freeman, 1982

- Example of a computation, emergence of a new property.

Model of Hubel and Wiesel (1962)

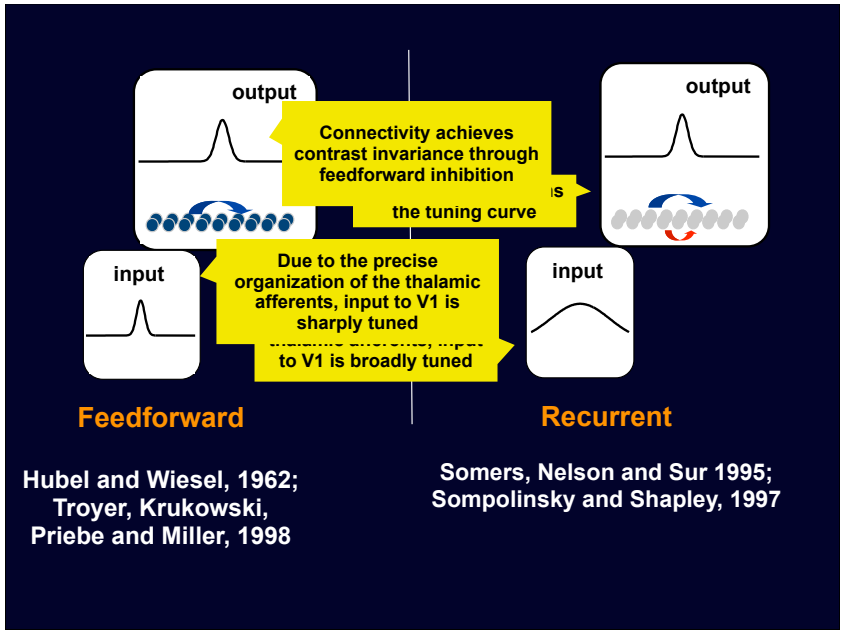
- Hubel and Wiesel (1962) proposed that the oriented fields of V1 neurons could be generated by summing the input from appropriately selected LGN neurons.
- The model accounts for selectivity in V1 on the basis of a purely feedforward architecture.

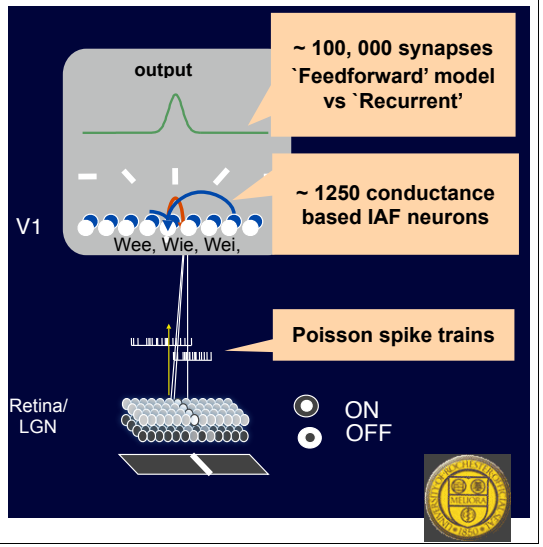


V1

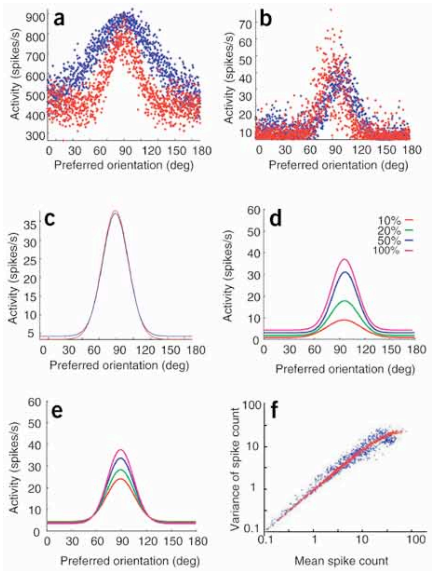
LGN neurons

V1 simple cell





Seriès, Latham and Pouget - *Nature Neuroscience* 2004



• **Explore physiological and anatomical plausibility**:
- cortical connectivity scheme,
- thalamocortical connectivity,
- properties of inhibition in Cx (inactivation)
…
(Sompolinsky and Shapley, 1997; Ferster and Miller, 2000).

• **Coding efficiency**
(are these models making different predictions in terms of information transmission?)