CCN assignment 1: Visual orientation perception and environmental statistics

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January 25, 2016

1 Introduction

Deadline is Monday February 15th 2016 at 4 pm (standard late policies apply). Please submit the pdf of your report to ITO using the command submit. Please name your document using yourname-assign1-ccn16.pdf.

Please also submit the paper copy to ITO by the deadline or just after (it will be the time of the submit command that will matter).

Report your findings. Particularly well-researched answers can receive additional points. Ideally you substantiate your explanations, for instance by additional simulations. Plots should always include axes labels and units. Figures should always have a caption. The presentation and format will count in the final mark. The report should look like a scientific report – no need to include any code.

Copying results is not allowed. It’s OK to ask for help from your friends. However, this help must not extend to copying code or written text that your friend has written, or that you and your friend have written together. I assess you on the basis of what you are able to do by yourself. It’s OK to help a friend. However, this help must not extend to providing your friend with code or written text. If you are found to have done so, a penalty will be assessed against you as well.

Email me (pseries@inf.ed.ac.uk) the Matlab script that you used. I will not assess the programming style, but I might check them if results are unexpected. I will also run plagiarism detectors on them.

2 Background Information

This assignment is inspired from [1]. In this paper, the authors study the visual biases related to the perception of orientation, in particular the fact that orien-
tation judgements are more accurate at cardinal (vertical and horizontal) orientations and that judgements made under conditions of uncertainty are strongly biased toward cardinal orientations. They show that such biases are consistent with participants using a Bayesian prior favoring cardinal orientations. They further estimate observers’ individual prior based on their visual performance and compare whether this prior matches the statistics of the environment, in particular the distribution of local orientation that can be measured from a large set of photographs. They found that this was qualitatively the case. They finally propose a simple neural model to explain how this prior could be implemented in the visual cortex.

In this assignment, we construct a model similar to theirs to explore the same idea.

3 Neural model

3.1 Model of the population of neurons

We consider a population of $N = 60$ neurons, possibly located in V1, with tuning curves $f_i(\theta)$ describing the mean spike count of each neuron in 1 second as a function of the stimulus direction $\theta$. The cells have preferred orientations $\theta_i$ equally spaced between $-90$ deg and $90$ deg. The tuning curves are circular normal distributions defined by

$$f_i(\theta) = G \cdot \exp(\beta(\cos(2(\theta - \theta_i)) - 1)) + K$$

where $G$ is the maximal firing rate ($G = 50$ spikes), $\beta = 4$, the concentration parameter, controls the width of the tuning curves and $K$ denotes spontaneous activity ($K = 5$ spikes). The variability of the spike count is Poisson. We denote by $r(\theta) = \{r_1(\theta), ..., r_N(\theta)\}$ the response of the population of neurons on a given trial of 1 sec for a stimulus $\theta$.

- Plot the mean response $f(\theta_0)$ of the population of neurons to stimulus $\theta_0 = 0^\circ$.
- Plot an example of the population response $r(\theta_0)$ to stimulus $\theta_0 = 0^\circ$ for one trial.

Tip: You will need to use a Poisson random number generator in matlab, for eg. [http://homepages.inf.ed.ac.uk/pseries/CCN/poiav.m](http://homepages.inf.ed.ac.uk/pseries/CCN/poiav.m)

These functions are similar to Gaussian functions but they are periodic, so that they wrap around the circle of stimulus orientations naturally. Please remember that the $\cos$ and $\sin$ matlab functions take radians not degrees.
3.2 Decoding the orientation of the stimulus: population vector

To decode the orientation of the stimulus based on the responses of the neurons, we decide to use a population vector. In general, the population vector estimate $\hat{\theta}$ of the stimulus $\theta$ is defined as:

$$\vec{P}(\theta) = \sum_{i=1}^{i=60} r_i(\theta) \vec{p}_i$$

$$\hat{\theta} = \text{angle}(\vec{P}(\theta))$$

where each $p_i$ is a vector associated with neuron $i$ pointing in the direction $\theta_i$ and of unit length.

When decoding orientation (which is $\pi$-periodic as opposed to direction which is $2\pi$-periodic), $\hat{\theta}$ can be obtained using:

$$\hat{\theta} = \frac{1}{2} \arctan \frac{p_{\sin}(\theta)}{p_{\cos}(\theta)}$$

where $p_{\cos}(\theta) = \sum_i r_i \cos(2\theta_i)$; $p_{\sin}(\theta) = \sum_i r_i \sin(2\theta_i)$

- Implement the population vector and try it out on a few trials. Does it work? (tip: the matlab function atan2 can be useful).
- Vary the stimulus orientation $\theta$ from -90 to 90 deg and for each stimulus direction, compute the stimulus estimate for 100 repetitions of the stimulus. Comment.
- Plot the bias as a function of $\theta$ (i.e. the difference between the estimate of the stimulus and the real stimulus, averaged over all repetitions). Comment. (Tip: circular statistics such as the matlab inbuilt function circ_mean might be useful).

3.3 Decoding the direction of the stimulus: maximum likelihood

We now wish to compare the population vector decoding with maximum likelihood decoding.

- Using the definition of Poisson variability, write the mathematical expression for the log likelihood, $\ln P[r|\theta]$ for the present model, as a function of stimulus orientation $\theta$.  

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• Implement the maximum likelihood decoder algorithm (Tip: matlab optimization functions such as fminsearch can be useful) and try it out on a few trials.

• Vary the stimulus orientation $\theta$ from -90 to 90 deg and for each stimulus direction, compute the stimulus estimate for 100 repetitions of the stimulus. Comment.

• Plot the bias as a function of $\theta$ (i.e. the difference between the estimate of the stimulus and the real stimulus, averaged over all repetitions). Comment.

• Compare the variance of the estimation with that of the population vector. Which decoding method is most accurate?

3.4 If the brain used a Bayesian prior ...

Girshick et al show that human performances are compatible with humans using a prior representation which would favour the cardinal orientations.

• Choose a function to model this prior and parameters such that the resulting prior approximates figure 5A (different choices are possible). Write the equation and plot it.

• Implement the maximum a posteriori decoder algorithm using this prior and try it out on a few trials.

• Vary the stimulus orientation $\theta$ from -90 to 90 deg and for each stimulus direction, compute the stimulus estimate for 100 repetitions of the stimulus. Comment.

• Plot the bias as a function of $\theta$ (i.e. the difference between the estimate of the stimulus and the real stimulus, averaged over all repetitions). Comment.

• Plot again the variance and – using the expression given in the slides of the course or in [2] – the discrimination threshold.

• Does this model have the potential to explain the kind of biases found in psychophysics?

3.5 Effect of cell heterogeneities on estimation and discrimination performances

It is well known that vertical and horizontal orientations are more represented in the visual cortex. We now assume that the representation of orientation preferences is not uniform. We can use the same model for this heterogeneity as
in the paper by Girshick et al: “Nonuniform preferred orientations were drawn from a von Mises distribution modified to peak at 0 and 90 with a standard deviation of 35 deg\(^2\).

- Write the equation for this function (or the approximation you are using) and plot it.
- Draw 60 samples from this distribution, check that you get the desired heterogeneity and use it to define the preferred orientations of the neurons in the population.
- Use the population vector decoding method again. Plot the mean estimates, variance and discrimination threshold of the estimates. What do you find?
- How does the performance compare to that of 3.4. Can you tune the parameters better so that those two situations become very similar? what changes would you need to make?

**Conclusion:**

- Can you comment on the validity of the model proposed by [1].

**References**


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\(^2\)This is meant to be understood as “a concentration parameter which would correspond to a standard deviation of 35 deg if the model were gaussian” – since the concentration parameter doesn’t come in degrees at all. You can actually approximate this function with a Gaussian if you find it easier to sample from using matlab existing commands.