CCN assignment 1: The Encoding-Decoding model of Perception, Visual Orientation Biases and Environmental Statistics.

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1 Introduction

Deadline is Friday October 27th 2017 at 4 pm (standard late policies apply). Please submit the pdf of your report to ITO using the command submit. Please name your document using yourname-assign1-ccn17.pdf. The command will thus be of the form: submit ccn 1 yourname_assign1_ccn17.pdf

Please also submit the *paper copy* to ITO by the deadline or just after (it will be the time of the submit command that will matter).

- Your report should look like a scientific report – no need to include any code. Report your findings. Particularly well-researched answers can receive additional points. Plots should always include axes labels and units. Figures should always have a caption and be references in the text. The presentation and format will count in the final mark. Be concise and precise in how you report your results. Don't include a million graphs: you can superimpose different graphs in the same plot. The maximum length for the report is **6 pages**.
- Copying results is not allowed. It's OK to ask for help from your friends. However, this help must not extend to copying code or written text that your friend has written, or that you and your friend have written together. I assess you on the basis of what you are able to do by yourself. It's OK to help a friend. However, this help must not extend to providing your friend with code or written text. If you are found to have done so, a penalty will be assessed against you as well.
- Email me (pseries@inf.ed.ac.uk) the **Matlab script** that you used. I will not assess the programming style, but I might check them if results are unexpected. I will also run plagiarism detectors on them.

2 Background Information

This assignment explores issues arround encoding, decoding and Bayesian models of perception.

It is inspired from the seminal article of [1].

In this paper, the authors study the visual biases related to the perception of orientation, in particular the fact that orientation judgements are more accurate at cardinal (vertical and horizontal) orientations and that judgements made under conditions of uncertainty are strongly biased toward cardinal orientations. They show that such biases are consistent with participants using a Bayesian prior favoring cardinal orientations. They further estimate observers' individual prior based on their visual performance and compare whether this prior matches the statistics of the environment, in particular the distribution of local orientation that can be measured from a large set of photographs. They found that this was qualitatively the case. They finally propose a simple neural model to explain how this prior could be implemented in the visual cortex.

In this assignment, we construct a model similar to theirs to explore the same ideas.

3 Neural model

3.1 Model of the population of neurons

We consider a population of $N = 100$ neurons, possibly located in V1, with tuning curves $f_i(\theta)$ describing the mean spike count of each neuron in 1 second as a function of the stimulus direction θ. The cells have preferred orientations θ_i equally spaced between -90 deg and 90 deg. The tuning curves are circular normal distributions defined by [1]:

$$f_i(\theta) = G.\exp(\beta(\cos(2(\theta - \theta_i)) - 1)) + K \tag{1}$$

where G is the maximal firing rate ($G = 50$ spikes/s), $\beta = 4$, the concentration parameter, controls the width of the tuning curves and K denotes spontaneous activity ($K = 5$ spikes/s). The variability of the spike count is Poisson. We denote by $\mathbf{r}(\theta) = \{r_1(\theta), .., r_N(\theta)\}$ the response of the population of neurons on a given trial of 1 sec for a stimulus θ.

- Plot the mean response $\mathbf{f}(\theta_0)$ of the population of neurons to stimulus $\theta_0 = 0°$ and stimulus $\theta_1 = 45°$.

[1] These functions are similar to Gaussian functions but they are periodic, so that they wrap around the circle of stimulus orientations naturally. Please remember that the `cos` and `sin` matlab functions take radians not degrees.

- Plot an example of the population response $\mathbf{r}(\theta_\mathbf{0})$ to stimulus $\theta_0 = 0°$ for one trial. Do the same thing for a stimulus $\theta_1 = 45°$.
  Tip: You will need to use a Poisson random number generator in matlab, for eg. `http://homepages.inf.ed.ac.uk/pseries/CCN/poirv.m`

3.2 Setting the stage: winner take-all decoding

To decode the orientation of the stimulus based on the responses of the neurons, we decide to first use the simplest method, a winner take all mechanism (WTA). The WTA estimate $\hat{\theta}$ of the stimulus θ simply defined as the preferred orientation of the neuron that responds maximally on this trial.

- Implement the WTA and using simulated population responses like in 3.1 with $\theta_0 = 0°$ or $45°$, try it out on a few trials to check that it works: can you recover the orientation of the stimulus?
- Vary the stimulus orientation θ from -90 to 90 deg and for each stimulus direction, compute the stimulus estimates for 150 repetitions of the stimulus and the average over the 150 repetitions.
- Plot the **bias** of the estimator as a function of θ (i.e. the difference between the estimate of the stimulus and the real stimulus, averaged over all repetitions). Comment. (Tip: circular statistics such as the matlab inbuilt function `circ_mean` will be useful. Note that the problem being entirely circular symmetrical, we don't expect any "discontinuities" at -90 and 90 (which correspond to the same orientation), you might have to find some "hacks" to make sure the space is circular symmetric).
- Plot the **variance** of the estimator as a function of θ. Comment.

3.3 Towards a better estimate of the orientation of the stimulus: population vector

We next decide to use a population vector as decoding technique. In general, the population vector estimate $\hat{\theta}$ of the stimulus θ is defined as:

$$\vec{P}(\theta) = \sum_{i=1}^{i=100} r_i(\theta)\vec{p}_i$$

$$\hat{\theta} = \text{angle}(\vec{P}(\theta))$$

where each p_i is a vector associated with neuron i pointing in the direction θ_i and of unit length.

When decoding orientation (which is π-periodic as opposed to direction which is 2π-periodic), $\hat{\theta}$ can be obtained using:

$$\hat{\theta} = \frac{1}{2} arctan \frac{p_{sin}(\theta)}{p_{cos}(\theta)}$$

where $p_{cos}(\theta) = \sum_i r_i cos(2\theta_i)$; $p_{sin}(\theta) = \sum_i r_i sin(2\theta_i)$

- Implement the population vector and check it works. Vary the stimulus orientation θ from -90 to 90 deg and for each stimulus direction, compute the stimulus estimates for 150 repetitions of the stimulus and the average over the 150 repetitions. Plot the bias and the variance of the estimator as a function of θ. How does it compare to the WTA?

3.4 Maximum Likelihood (ML) estimator and Maximum a Posteriori (MAP)

We now wish to compare the population vector decoding with maximum likelihood (ML) decoding.

- Using the definition of Poisson variability (cf. lecture slides), write the mathematical expression for the log likelihood, $lnP[r|\theta]$ for the present model, as a function of $f_i(\theta)$ and r_i.
- Implement the maximum likelihood decoder algorithm (Tip: matlab optimization functions such as `fminsearch` can be useful) and try it out on a few trials to check that it works. Vary the stimulus orientation θ from -90 to 90 deg and for each stimulus direction, compute the stimulus estimate for 50 repetitions of the stimulus. Plot the bias as a function of θ. Comment. Compare the variance of the estimation with that of the population vector and WTA. Which decoding method is most accurate? Does that depend on the number of neurons in the population?
- How would you test if the minimum possible variance is achieved by any of those decoding methods?

Girshick et al show that human performances are compatible with humans using a prior representation which would favour the cardinal orientations.

- Choose a function that is suitable to model this prior and parameters such that the resulting prior approximates figure 5A of the article (different choices are possible). Write the equation and plot it.
- Implement the maximum *a posteriori* (MAP) decoder algorithm using this prior and try it out on a few trials. Check that the prior influences the estimation (you might have to tweak things for this to happen, report on this if that's the case).

- Show the bias and variance. Does this model have the potential to explain the kind of biases found in psychophysics?

3.5 Effect of cell heterogeneities on estimation and discrimination performances

It is well known that vertical and horizontal orientations are more represented in the visual cortex. For this reason, in this section, we assume that the representation of orientation preferences is not uniform. We can use the same model for this heterogeneity as in the paper by Girshick et al: "Nonuniform preferred orientations were drawn from a von Mises distribution modified to peak at 0 and 90 with a standard deviation of 35 deg" (this corresponds to a concentration parameter β of about 3.3).

- Plot this distribution (or your approximation of it).
- Draw 100 samples from this distribution that are going to correspond to the preferred orientations of your new population of neurons, check that you get the desired heterogeneity. Mention how you do the sampling.
- Use the population vector decoding method again. Plot the mean estimates and variance of the estimates. How does the performance compare to that of 3.4 (MAP)? Can you tune the parameters better so that those two situations become very similar? what changes would you need to make?

3.6 Conclusion

- Do your experiments validate the model proposed by [1]? Comment. Do you think this model offers a general view on how priors could be represented in the brain?

References

[1] Ahna R Girshick, Michael S Landy, and Eero P Simoncelli. Cardinal rules: visual orientation perception reflects knowledge of environmental statistics. *Nat Neurosci*, 14(7):926–932, Jul 2011.