The Reachability Problem for Petri Nets
Where do we stand in 2017?
(LFCS Lab Lunch Talk)

Patrick Totzke

24/01/2017
Petri Nets.
Petri Nets.
Petri Nets.
Petri Nets.

\[
\begin{pmatrix}
1 \\
2 \\
0
\end{pmatrix} + \begin{pmatrix}
-1 \\
-1 \\
+1
\end{pmatrix}
\]
Petri Nets.

\[
\begin{pmatrix}
1 \\
2 \\
0
\end{pmatrix} + \begin{pmatrix}
-1 \\
-1 \\
+1
\end{pmatrix} = \begin{pmatrix}
0 \\
1 \\
1
\end{pmatrix}
\]
\begin{align*}
\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ +1 \end{pmatrix} &= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}
\end{align*}
Petri Nets.

... e.g. representing chemical reactions:

\[
C + O_2 \rightarrow CO_2
\]

\[
CO_2 + NaOH \rightarrow NaHCO_3
\]

\[
NaHCO_3 + HCl \rightarrow H_2O + NaCl + CO_2
\]
Petri Nets: Modelling the Dining Philosophers
Petri Nets: Modelling the Dining Philosophers

Diagram of a Petri net model of the Dining Philosophers problem, showing places (circles) and transitions (diamonds) with tokens and arcs.
Petri Nets: Modelling the Alternation Bit Protocol
Petri Nets: Modelling the Alternation Bit Protocol
Petri nets: Modelling gone wrong
Vector Addition Systems with States

Definition

A $d$-VASS is a finite automaton
Definition

A $d$-VASS is a finite automaton with designated states $i, f \in Q$. 

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A \( d \)-VASS is a finite automaton with designated states \( i, f \in Q \) and alphabet \( A \subseteq \mathbb{Z}^d \).
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The step-relation between configurations in \(Q \times \mathbb{N}^d\) is

\[(q, v) \longrightarrow (q', v')\]

if there is an edge \(q \xrightarrow{a} q'\) such that \(v' = v + a\).
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Reachability
\( (i, 0) \overset{*}{\rightarrow} (f, 0) \)?

Coverability
\( (i, 0) \overset{*}{\rightarrow} (f, v) \) for some \( v \)?
On the Category of Petri Net Computations

In Memory and Dedication to my Beloved mother Liana

Vladimiro Sassone°

BRICS° – Computer Science Dept., University of Aarhus

Abstract. We introduce the notion of strongly concatenable process as a refinement of concatenable processes [3] which can be expressed axiomatically via a functor $Q[-]$ from the category of Petri nets to an appropriate category of symmetric strict monoidal categories, in the precise sense that, for each net $N$, the strongly concatenable processes of $N$ are isomorphic to the arrows of $Q[N]$. In addition, we identify a coreflection right adjoint to $Q[-]$ and characterize its replete image, thus yielding an axiomatization of the category of net computations.

Introduction

Petri nets, introduced by C.A. Petri [8] (see also [10]), are unanimously considered among the most representative models for concurrency, since they are a fairly simple and natural model of concurrent and distributed computations. However, Petri nets are, in our opinion, not yet completely understood.

Among the semantics proposed for Petri nets, a relevant role is played by the various notions of process [9, 4, 1], whose merit is to provide a faithful account of computations involving many different transitions and of the causal connections between the events occurring in a computation. However, process models, at least in their standard forms, fail to bring to the foreground the algebraic structure of nets and their computations. Since such a structure is relevant to the understanding of nets, they fail, in our view, to give a comprehensive account of net behaviours.

The idea of looking at nets as algebraic structures [10, 7, 13, 14, 2] has been given an original interpretation by considering monoidal categories as a suitable framework [6]. In fact, in [6, 3] the authors have shown that the semantics of Petri nets can be understood in terms of symmetric monoidal categories—where objects are states, arrows processes, and the tensor product and the arrow
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Definition 1.1 (Petri Nets)
A Petri net is a structure \( N = (\partial^0_N, \partial^1_N : T_N \rightarrow S^\oplus_N) \), where \( T_N \) is a set of transitions, \( S_N \) is a set of places, and \( \partial^0_N \) and \( \partial^1_N \) are functions.
A morphism of Petri nets from \( N_0 \) to \( N_1 \) is a pair \( (f, g) \), where \( f : T_{N_0} \rightarrow T_{N_1} \) is a function and \( g : S^\oplus_{N_0} \rightarrow S^\oplus_{N_1} \) is a monoid homomorphism such that \( (f, g) \) respects source and target, i.e., \( \partial_i^0 \circ f = g \circ \partial_i^0 \), for \( i = 0, 1 \).
This defines the category \( \text{Petri} \) of Petri nets.
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This defines the category \( \mathcal{P}[N] \) of Petri nets.

Definition 1.3 (The Category \( \mathcal{P}[N] \))
The category \( \mathcal{P}[N] \) is the monoidal quotient of \( \mathcal{F}(N) \), the symmetric strict monoidal category whose monoid of objects is \( S_N^\oplus \) and whose arrows are freely generated from the transitions of \( N \), modulo the axioms

\[
\begin{align*}
\gamma_{a,b} &= id_{a \otimes b} & \text{if } a, b &\in S_N \text{ and } a \neq b, \\
(t; (id_u \otimes \gamma_{a,a} \otimes id_v)) &= t & \text{if } t &\in T_N \text{ and } a \in S_N, \\
(id_u \otimes \gamma_{a,a} \otimes id_v); t &= t & \text{if } t &\in T_N \text{ and } a \in S_N,
\end{align*}
\]

where \( \gamma \) is the symmetry isomorphism of \( \mathcal{F}(N) \).
Some Equivalent Decision Problems

Database Theory

Satisfiability of $\mathit{FO}^2(\sim, <, +1)$ over (finite and infinite) data words.
Some Equivalent Decision Problems

Database Theory
Satisfiability of $FO^2(\sim, <, +1)$ over (finite and infinite) data words.

Formal languages
Emptiness of $\sqcap \sqcap (R_1) \cap R_2$, for regular languages $R_1, R_2$. 
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Validity for the $!$-Horn Fragment of Linear Logic.
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Program Verification
fair-PTL model checking of communicating processes $C \times U^n$. 
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2-VASS have effectively semilinear reachability sets.
Hopcroft and Pansiot ’79
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- Finite control costs 3 extra dimensions: $d$-VASS can be simulated by $(d + 3)$-VAS.

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  - Their reachability problem is decidable
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This is not true for dimensions $d \geq 3$: 
Finite control costs 3 extra dimensions: $d$-VASS can be simulated by $(d + 3)$-VAS.

2-VASS have effectively semilinear reachability sets.
- Their reachability problem is decidable

This is not true for dimensions $d \geq 3$: 

```
(0, 0, 0)  
(0, 1, -1)  
(0, -1, 2)  
(1, 0, 0)  
```

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Howell, Rosier, Huynh, and Yen ’86
“A Multiparameter Analysis of the Boundedness Problem for VAS”
“Some complexity bounds for problems concerning finite and two-dimensional VASS”

- Reachability is $NP$-hard for 2-VASS
- The Hopcroft and Pansiot algorithm works in $2^{2^O(|T| \cdot \| T \|)}$ time.
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Linear Path Schemes

A language $\Lambda = \alpha_0 \beta_1^* \alpha_1 \beta_2^* \ldots \beta_k^* \alpha_k$, is called a *linear path scheme*. 

Leroux and Sutre '04
Every 2-VASS is flattable.
Linear Path Schemes

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A VAS $L \subseteq (\mathbb{Z}^d)^*$ is called *flattable* if

$$L \rightarrow \ orall \ S$$

for a finite union $S$ of LPSs.
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$$\xrightarrow{L} = \xrightarrow{S}$$

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Leroux and Sutre ’04
Every 2-VASS is flattable.
2-dim. VASS are flattable

A language $\Lambda = \alpha_0 \beta \alpha_1 \beta \ast \ldots \beta \ast \alpha_k \beta$, is called a linear path scheme.

A VASS $L \subseteq (\mathbb{Z}^d)^\ast$ is called flattable if $L \longrightarrow = S \longrightarrow$ for a finite union $S$ of LPSs.

Leroux and Sutre '04

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where $\|T\|$ is the maximal absolute value of any integer in $T$. 

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What is the size of the input?

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A language $\Lambda = \alpha_0 \beta^* \alpha_1 \beta^* \ldots \beta^*_k \alpha_k$, is called a linear path scheme.

A VAS $L \subseteq (\mathbb{Z}^d)^*$ is called flattable if $L \rightarrow S \rightarrow$ for a finite union $S$ of LPSs.

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For any regular $L \subseteq (\mathbb{Z} \times \mathbb{Z})^*$, there exist finitely many LPSs $\Lambda_1, \Lambda_2, \ldots, \Lambda_k \subseteq L$ such that

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2. $|\Lambda_i| \leq (\|L\| + |L|)^{O(1)}$ for all $1 \leq i \leq k$. 

Witnesses have the form $\alpha_0 \beta_{n_1} \alpha_1 \beta_{n_2} \cdots \beta_{n_m} \alpha_m$ for small $m$. 

Small solutions lemmas from linear programming lead to a $2^{|L|} \cdot \|L\|^{O(1)}$ bound on the $n_i$ and thus shortest witnesses. 

PSPACE upper bound for 2-VASS Reachability (binary) 

NP upper bound for 2-VASS Reachability (unary)
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\[ \xrightarrow{L} \text{ for context-free } L \subseteq \mathbb{Z}^d \]

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tokens a carry “datum” from an infinite domain.
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